I. USP MINI-COURSE ON NEUTRINOS

The slides I used in the course will be uploaded in the webpage:
http://fmatrm.if.usp.br/ minakata/index.html

A. Problems for the first week

During the course it was shown that the oscillation probability of $\nu_\beta \to \nu_\alpha$ is given by ($\Delta m_{ji} \equiv m_j^2 - m_i^2$)

$$P(\nu_\beta \to \nu_\alpha) = -4\sum_{j>i} \mathrm{Re}[U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}] \sin^2\left(\frac{\Delta m^2_{ji}L}{4E}\right)$$
$$-2\sum_{j>i} Im[U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}] \sin^1\left(\frac{\Delta m^2_{ji}L}{4E}\right), \tag{1}$$

for $\alpha \neq \beta$ (called appearance channel), and for the same flavor (called disappearance channel) by

$$P(\nu_\alpha \to \nu_\alpha) = 1 - 4\sum_{j>i} |U_{\alpha i}|^2|U_{\alpha j}|^2 \sin^2\left(\frac{\Delta m^2_{ji}L}{4E}\right). \tag{2}$$

(1) Use the unitarity relation

$$\sum_\alpha U_{\alpha i}U^*_{\alpha j} = 0 \qquad (i \neq j) \tag{3}$$

to show that in the three generation case, $J^{ij}_{\alpha\beta} \equiv Im[U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}]$ is unique up to the sign. (Formally, there are $6 \times 6 = 36$.) For example, $J^{ij}_{e\mu} = -J^{ij}_{\tau\mu} = J^{ij}_{\mu\tau}$.

(2) Show that CP (or T) violating term in (3) ($\propto J^{ij}_{\alpha\beta}$) disappears in the limit $\Delta m^2_{21} \to 0$.

(3) Compute explicitly $J^{12}_{e\mu}$ (or any other ones) by using the standard parametrization of the mixing matrix.

$$U = U_{23}U_{13}U_{12} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}, \tag{4}$$

where δ stands for the CP violating phase.

(4) Compute $P(\nu_e \to \nu_e)$ in the limit $\Delta m^2_{21} \to 0$, and express the result in terms of the three mixing angles defined above.

Neutrino Propagation in Vacuum (2 Flavor)

- 弱い相互作用によって weak (gauge) eigenstate ν_α のニュートリノが生成されたとする。この ν_α は "mass eigenstate" ν_i と

$$\nu_\alpha = U_{\alpha i}\nu_i$$

と関係づけられている。

Freedom of U matrix = flavormixing exists in the SM (see later)

- mass eigenstate の発展は、真空中では、

$$-i\frac{d}{dx}\begin{bmatrix}\nu_1\\ \nu_2\end{bmatrix} = \begin{bmatrix}p_1 & 0\\ 0 & p_2\end{bmatrix}\begin{bmatrix}\nu_1\\ \nu_2\end{bmatrix}$$

(Dirac方程式の positive energy solution から出発して forward-going solution を選ぶと

$-i\frac{d}{dx}\nu = p\nu_1$　を示す部は省略)

① （ν_α は二つの mass eigenstate の重ね合せで書くとき
エネルギーは共通のはず．）

↙ ↓

$$-i\frac{d}{dx}\begin{bmatrix}\nu_1\\ \nu_2\end{bmatrix} = \begin{bmatrix}\sqrt{E^2-m_1^2} & 0\\ 0 & \sqrt{E^2-m_2^2}\end{bmatrix}\begin{bmatrix}\nu_1\\ \nu_2\end{bmatrix}$$

$$\simeq \begin{bmatrix}E-\frac{m_1^2}{2E} & 0\\ 0 & E-\frac{m_2^2}{2E}\end{bmatrix}\begin{bmatrix}\nu_1\\ \nu_2\end{bmatrix}$$

- Redefinition of neutrino's wave function

$$\begin{bmatrix}\nu_1\\ \nu_2\end{bmatrix} = e^{+iEX}\begin{bmatrix}\tilde{\nu}_1\\ \tilde{\nu}_2\end{bmatrix}$$

→

$$+i\frac{d}{dx}\begin{bmatrix}\tilde{\nu}_1\\ \tilde{\nu}_2\end{bmatrix} = \begin{bmatrix}\frac{m_1^2}{2E} & 0\\ 0 & \frac{m_2^2}{2E}\end{bmatrix}\begin{bmatrix}\tilde{\nu}_1\\ \tilde{\nu}_2\end{bmatrix}$$

$$\begin{bmatrix}\nu_e\\ \nu_\mu\end{bmatrix} = \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\nu_1\\ \nu_2\end{bmatrix} = \underset{\uparrow}{e^{-iEX}}\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\tilde{\nu}_1\\ \tilde{\nu}_2\end{bmatrix}$$

共通の phase → unobservable

◦以下 $\tilde{\nu}_i$ の ~ を省略して，$\tilde{\nu}$ basis で議論。

2世代振動確率の計算(真空中)

$$i\frac{d}{dx}\begin{bmatrix}\nu_e\\ \nu_\mu\end{bmatrix} = U\begin{bmatrix}0 & 0\\ 0 & \frac{\Delta m^2}{2E}\end{bmatrix}U^+\begin{bmatrix}\nu_e\\ \nu_\mu\end{bmatrix}$$ $\Delta m^2 = m_2^2 - m_1^2$

- これは UH_dU^+ を Hamiltonian とする Schrödinger 方程式と同じ
- S行列を $\nu_\alpha(x) = S_{\alpha\beta}\,\nu_\beta(0)$ と定義すると

$$S(x) = Te^{-i\int_0^x dx' H(x')}$$ (一般論)

今は H が x-independent である事に注意すると

$$S(x) = e^{-iHx}$$

- 今 Hamiltonian は特殊な構造をしている; $H = UH_dU^+$

$$S(x) = e^{-iUH_dU^+x} = Ue^{-iH_dx}U^+$$

- $$e^{-iH_dx} = \begin{bmatrix}e^{-iH_{d11}x} & \\ & e^{-iH_{d22}x}\end{bmatrix}$$ (H_d = 対角なので sum が計算できる)

- 2世代では $U = \begin{bmatrix}c & s\\ -s & c\end{bmatrix}$

$c = \cos\theta$
$s = \sin\theta$

$$\therefore S = \begin{bmatrix}c & s\\ -s & c\end{bmatrix}\begin{bmatrix}1 & \\ & e^{-i\frac{\Delta m^2}{2E}x}\end{bmatrix}\begin{bmatrix}c & -s\\ s & c\end{bmatrix}$$

$$= \begin{bmatrix}c & s\\ -s & c\end{bmatrix}\begin{bmatrix}c & -s\\ se^{-i2\Delta x} & ce^{-i2\Delta x}\end{bmatrix}$$ $\Delta \equiv \frac{\Delta m^2}{4E}$

$$= \begin{bmatrix}c^2+s^2e^{-i2\Delta x} & cs(-1+e^{-i2\Delta x})\\ cs(-1+e^{-2i\Delta x}) & s^2+c^2e^{-i2\Delta x}\end{bmatrix}$$

・従って $S_{e\mu} = cs \cdot e^{-i\Delta x}\left[-e^{i\Delta x} + e^{-i\Delta x}\right]$

$$= -2i\sin\Delta x \cdot \cos\theta \sin\theta\, e^{-i\Delta x}$$

$$= \sin 2\theta \sin\Delta x \underline{(-i)\, e^{-i\Delta x}}$$

← pure phase

∴ $P_{\mu\to e} = |S_{e\mu}|^2 = \sin^2 2\theta \sin^2\Delta x$

$$= \sin^2 2\theta \cdot \sin^2\left(\frac{\Delta m^2 x}{4E}\right)$$

・$S_{ee} = c^2 + s^2 e^{-2i\Delta x}$

$$= e^{-i\Delta x}\left(c^2 e^{i\Delta x} + s^2 e^{-i\Delta x}\right)$$

$P_{ee} = |S_{ee}|^2 = \left(c^2 e^{-i\Delta x} + s^2 e^{i\Delta x}\right)\left(c^2 e^{i\Delta x} + s^2 e^{-i\Delta x}\right)$

$= c^4 + s^4 + c^2 s^2\left(e^{2i\Delta x} + e^{-2i\Delta x}\right)$

$= 1 \underset{-2c^2s^2}{\overset{\wedge}{+}} c^2 s^2\, 2\, \cancel{\sin} \cos 2\Delta x$ も $\cos 2\Delta x = 1 - 2\sin^2\Delta x$

$= \cancel{1 + 2c^2 s^2 (1 - 2\sin^2\Delta x)}$

$= 1 - 2c^2 s^2 \underbrace{(1 - \cos 2\Delta x)}_{= 2\sin^2\Delta x}$

$= 1 - \sin^2 \underset{2}{\wedge}\theta\, \sin^2\Delta x$

∴ $P_{ee} + P_{\mu e} = 1$ <u>OK</u> ← unitarity

〈note〉 $c^4 + s^4 = (c^2+s^2)^2 - 2c^2 s^2 = 1 - 2c^2 s^2$

Vacuum Oscillation の L/E dependence

$$\frac{\Delta m^2}{4E} L = 1.27 \left(\frac{\Delta m^2}{10^{-3} eV^2}\right)\left(\frac{L}{1000\ km}\right)\left(\frac{E}{1\ GeV}\right)^{-1}$$ atm ν / accelerator ν

$$= 1.27 \left(\frac{\Delta m^2}{1 eV^2}\right)\left(\frac{L}{1\ km}\right)\left(\frac{E}{1\ GeV}\right)^{-1}$$ LSND

$$= 1.27 \left(\frac{\Delta m^2}{10^{-5} eV^2}\right)\left(\frac{L}{100\ km}\right)\left(\frac{E}{1\ MeV}\right)^{-1}$$

KamLAND

$$= 1.27 \left(\frac{\Delta m^2}{10^{-3} eV^2}\right)\left(\frac{L}{1 Km}\right)\left(\frac{E}{1 MeV}\right)^{-1}$$

原子炉 θ_{13} 実ケン
or "CHOOZ"

3 Generation ν Oscillation in Vacuum

$$i\frac{d}{dx}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix} = \begin{bmatrix} m_1^2/2E & & \\ & m_2^2/2E & \\ & & m_3^2/2E \end{bmatrix}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix}$$

or $$i\frac{d}{dx}\nu_i(x) = \frac{m_i^2}{2E}\nu_i(x) \Rightarrow \nu_i(x) = e^{-i\frac{m_i^2}{2E}x}\,\nu_i(0)$$

"neutrino propagation by mass eigenstate is simple"

- Translate the mass-eigenstate description to that of flavor eigenstate

$$\nu_\alpha(x) = U_{\alpha i}\,\nu_i(x) = U_{\alpha i}\, e^{-i\frac{m_i^2}{2E}x}\,\underline{\nu_i(0)}$$

$$\text{"} \quad (U^\dagger)_{i\beta}\,\nu_\beta(0)$$

$$= U_{\alpha i}\,U^*_{\beta i}\, e^{-i\frac{m_i^2}{2E}x}\,\nu_\beta(0)$$

$\therefore$

$$S_{\alpha\beta} = U_{\alpha i}\,U^*_{\beta i}\, e^{-i\frac{m_i^2}{2E}x}$$

$$P(\underset{x=0}{\nu_\beta} \to \underset{x=x}{\nu_\alpha}) = |S_{\alpha\beta}|^2$$

Now let us compute "flavor oscillation" probability $P(\nu_\beta \to \nu_\alpha)$

$P(\nu_\beta \to \nu_\alpha)$ の計算

$$= \sum_i \sum_j U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j} \; \underline{e^{-i\frac{m_i^2}{2E}x} e^{+i\frac{m_j^2 x}{2E}}}$$

$$\parallel$$

$$\Delta m_{ji}^2 \equiv m_j^2 - m_i^2 \Rightarrow \qquad e^{i\frac{\Delta m_{ji}^2 x}{2E}}$$

- Let us decompose:

$$U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j} = \mathrm{Re}(UUUU) + i\,\mathrm{Im}(UUUU)$$

$$e^{i\frac{\Delta m_{ji}^2 x}{2E}} = \cos\left(\frac{\Delta m_{ji}^2 x}{2E}\right) + i \sin\left(\frac{\Delta m_{ji}^2 x}{2E}\right)$$

$$P(\nu_\beta \to \nu_\alpha) = \sum_{i=j}\sum |U_{\alpha i}|^2 |U_{\beta i}|^2$$

$$+ \sum_{i \neq j} \mathrm{Re}\left(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}\right) \cos\frac{\Delta m_{ji}^2 x}{2E}$$

$$\cancel{+}\,- \sum_{i \neq j} \mathrm{Im}\left(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}\right) \sin\frac{\Delta m_{ji}^2 x}{2E}$$

<note>

$\mathrm{Re}(UUUU) \times \sin\left(\frac{\Delta m_{ji}^2 x}{2E}\right)$ vanish ∵ ij sym × ij asym

$\mathrm{Im}(UUUU) \times \cancel{\sin} \cos\left(\frac{\Delta m_{ji}^2 x}{2E}\right)$ vanish //

From Unitarity

$$\sum_i U_{\alpha i} U^*_{\beta i} = \delta_{\alpha\beta} \qquad \text{or}$$

$$\sum_i \sum_j U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j} = \delta_{\alpha\beta}$$

$$\sum_{i=j} |U_{\alpha i}|^2 |U_{\beta i}|^2 + \sum\sum_{i\neq j} U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j} = \delta_{\alpha\beta}$$

$$\therefore \sum_{i=j} |U_{\alpha i}|^2 |U_{\beta i}|^2 = \delta_{\alpha\beta} - \sum_{i\neq j} U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}$$

$$P(\nu_\beta \to \nu_\alpha) = \sum_i \sum_j U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}\, e^{i\frac{\Delta m^2_{ji} x}{2E}}$$

$$= \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2 + \sum_{i\neq j} \cancel{Re}\left(U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}\right) \times e^{i\frac{\Delta m^2_{ji} x}{2E}} \cancel{\cos\left(\frac{\Delta m^2_{ji} x}{2E}\right)}$$

$$\cancel{-\sum_{i\neq j} Im}$$

$$= \delta_{\alpha\beta} + \sum_{i\neq j} \left(U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}\right)\left(e^{i\frac{\Delta m^2_{ji} x}{2E}} - 1\right)$$

ここで UUUU = Re(UUUU) + i Im(UUUU) と分解

$$e^{i\frac{\Delta m^2_{ji} x}{2E}} - 1 = \cos\left(\frac{\Delta m^2_{ji} x}{2E}\right) - 1 + i \sin\left(\frac{\Delta m^2_{ji} x}{2E}\right)$$

$$= -2\sin^2\left(\frac{\Delta m^2_{ji} x}{4E}\right) + i \sin\left(\frac{\Delta m^2_{ji} x}{2E}\right)$$

∵

$$P(\nu_\beta\to\nu_\alpha) = \delta_{\alpha\beta}$$

$$- 2\sum_{i\neq j}\mathrm{Re}\left(U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}\right)\sin^2\left(\frac{\Delta m^2_{ji}x}{4E}\right)$$

$$- \sum_{i\neq j}\mathrm{Im}\left(U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}\right)\sin\left(\frac{\Delta m^2_{ji}x}{2E}\right)$$

//

この後の議論を明快にするため

$$\sum_{i\neq j} = 2\sum_{j>i}$$

3 Flavor oscillation probability in vacuum

CP-violation

$$P(\bar{\nu}_\beta\to\bar{\nu}_\alpha) = P\left(\nu_\beta\to\nu_\alpha\ ;\ U_{\alpha\beta}\to U^*_{\alpha\beta}\right)$$

$$= \delta_{\alpha\beta} - 2\sum_{i\neq j}\underbrace{\mathrm{Re}\left(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right)}_{=\ \mathrm{Re}\left(U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}\right)}\sin^2\left(\frac{\Delta m^2_{ji}x}{4E}\right)$$

$$+ \sum_{i\neq j}\mathrm{Im}\left(U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}\right)\sin\left(\frac{\Delta m^2_{ji}x}{2E}\right)$$

∵

$$P(\nu_\beta\to\nu_\alpha) - P(\bar{\nu}_\beta\to\bar{\nu}_\alpha) = -2\sum_{i\neq j}\mathrm{Im}\left(U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}\right)$$

$$\times\sin\left(\frac{\Delta m^2_{ji}x}{2E}\right)$$

T-violation

$$T P(\nu_\beta \to \nu_\alpha) = P(\nu_\alpha \to \nu_\beta) = P(\nu_\beta \to \nu_\alpha : \alpha \leftrightarrow \beta)$$

$$= \delta_{\alpha\beta} - 2 \sum_{i \neq j} \mathrm{Re}\left(U_{\alpha i}^{*} U_{\beta i}^{*} U_{\alpha j}^{*} U_{\beta j}\right) \sin^2\left(\frac{\Delta m_{ji}^2 x}{4E}\right)$$

$$+ \sum_{i \neq j} \mathrm{Im}\left(U_{\alpha i} U_{\beta i}^{*} U_{\alpha j}^{*} U_{\beta j}\right) \sin\left(\frac{\Delta m_{ji}^2 x}{2E}\right)$$

<note) by $\alpha \leftrightarrow \beta$ transformation

$$U_{\alpha i} U_{\beta i}^{*} U_{\alpha j}^{*} U_{\beta j} \to U_{\beta i} U_{\alpha i}^{*} U_{\beta j}^{*} U_{\alpha j}$$

$$= \left(U_{\alpha i} U_{\beta i}^{*} U_{\alpha j}^{*} U_{\beta j}\right)^{*}$$

Hence,

$$P(\nu_\beta \to \nu_\alpha) - \underbrace{T P(\nu_\beta \to \nu_\alpha)}_{\overset{\|}{P(\nu_\alpha \to \nu_\beta)}} = -2 \sum_{i \neq j} \mathrm{Im}\left(U_{\alpha i} U_{\beta i}^{*} U_{\alpha j}^{*} U_{\beta j}\right) \times \sin\left(\frac{\Delta m_{ji}^2 x}{2E}\right)$$

$\Delta P_{CP} = \Delta P_T$: consequence of CPT theorem

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$$-2 \sum_{j>i} \mathrm{I}m[U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j}] \sin^1 \left(\frac{\Delta m_{ji}^2 L}{4E} \right), \quad (1)$$

for $\alpha \neq \beta$ (called appearance channel), and for the same flavor (called disappearance channel) by

$$P(\nu_\alpha \to \nu_\alpha) = 1 - 4 \sum_{j>i} |U_{\alpha i}|^2 |U_{\alpha j}|^2 \sin^2 \left(\frac{\Delta m_{ji}^2 L}{4E} \right). \quad (2)$$

(1) Use the unitarity relation

$$\sum_\alpha U_{\alpha i} U_{\alpha j}^* = 0 \qquad (i \neq j) \quad (3)$$

to show that in the three generation case, $J_{\alpha\beta}^{ij} \equiv \mathrm{I}m[U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j}]$ is unique up to the sign. (Formally, there are $6 \times 6 = 36$.) For example, $J_{e\mu}^{ij} = -J_{\tau\mu}^{ij} = J_{\mu\tau}^{ij}$.

(2) Show that CP (or T) violating term in (3) ($\propto J_{\alpha\beta}^{ij}$) disappears in the limit $\Delta m_{21}^2 \to 0$.

(3) Compute explicitly $J_{e\mu}^{12}$ (or any other ones) by using the standard parametrization of the mixing matrix.

$$U = U_{23} U_{13} U_{12} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} \end{bmatrix}, \quad (4)$$

where δ stands for the CP violating phase.

(4) Compute $P(\nu_e \to \nu_e)$ in the limit $\Delta m_{21}^2 \to 0$, and express the result in terms of the three mixing angles defined above.

"Atmospheric oscillation dominant Regime" ← この方が適してる名前だろう!

One-mass scale dominance approximation

$$P(\nu_\alpha \to \nu_\alpha) = 1 - 4 \sum_{j>i} |U_{\alpha i}|^2 |U_{\alpha j}|^2 \sin^2\left(\frac{\Delta m^2_{ji} x}{4E}\right)$$

· Region I : $\frac{\Delta m^2_{32} x}{4E} \sim O(1)$ $\frac{\Delta m^2_{21} x}{4E} \ll 1$

$$\boxed{\frac{\Delta m^2_{21}}{\Delta m^2_{31}} \simeq \frac{1}{30}}$$

expansion parameter $\frac{\Delta m^2_{21}}{\Delta m^2_{32}} \equiv \varepsilon$

$$\cdot P(\nu_\alpha \to \nu_\alpha) = 1 - 4|U_{\alpha 1}|^2 |U_{\alpha 3}|^2 \sin^2 \frac{\Delta m^2_{31} x}{4E}$$
$$- 4|U_{\alpha 2}|^2 |U_{\alpha 3}|^2 \sin^2 \frac{\Delta m^2_{32} x}{4E}$$
$$- 4|U_{\alpha 1}|^2 |U_{\alpha 2}|^2 \sin^2 \frac{\Delta m^2_{21} x}{4E} \quad \leftarrow O(\varepsilon^2)$$

$$\frac{\Delta m^2_{31} x}{4E} = \frac{\Delta m^2_{32} x}{4E} + \frac{\Delta m^2_{21} x}{4E} = \frac{\Delta m^2_{32} x}{4E} (1+\epsilon) \quad + O(\epsilon^2)$$

$$\sin \frac{\Delta m^2_{31} x}{4E} = \sin \frac{\Delta m^2_{32} x}{4E} + \cos\left(\frac{\Delta m^2_{32} x}{4E}\right)\left(\frac{\Delta m^2_{32} x}{4E}\right) \cdot \varepsilon \quad + O(\epsilon^2)$$

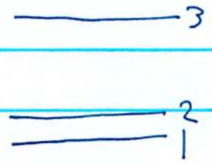
∴

$$\sin^2\left(\frac{\Delta m^2_{31} x}{4E}\right) = \sin^2 \frac{\Delta m^2_{32} x}{4E} + \sin\left(\frac{\Delta m^2_{32} x}{2E}\right)\left(\frac{\Delta m^2_{32} x}{4E}\right) \cdot \varepsilon \quad + O(\epsilon^2)$$

☆

$$P(\nu_\alpha \to \nu_\alpha) = 1 - 4|U_{\alpha 3}|^2 \left(|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2\right) \sin^2 \frac{\Delta m^2_{32} x}{4E}$$
$$- 4|U_{\alpha 1}|^2 |U_{\alpha 3}|^2 \cdot \varepsilon \left(\frac{\Delta m^2_{32} x}{4E}\right) \cdot \sin\left(\frac{\Delta m^2_{32} x}{2E}\right) + O(\epsilon^2) \quad //$$

"normal type"



$\Delta m^2_{31} = m_3^2 - m_1^2$

$\Delta m^2_{32} = m_3^2 - m_2^2$

$\Delta m^2_{31} - \Delta m^2_{32} = m_2^2 - m_1^2 = \Delta m^2_{21} \Rightarrow \Delta m^2_{31} = \Delta m^2_{32} + \Delta m^2_{21}$

- Using unitarity

$$P(\nu_\alpha \to \nu_\alpha) = 1 - 4\,|U_{\alpha 3}|^2 \left(1 - |U|^2_{\alpha 3}\right) \sin^2\!\left(\frac{\Delta m^2_{32} L}{4E}\right) + O(\epsilon)$$

"2 flavor form"

$\boxed{\text{Examples}}$

$\alpha = e$ $\qquad U_{e3} = s_{13} e^{-i\delta} \qquad |U_{e3}|^2 = s^2_{13}$

$$P(\nu_e \to \nu_e) = P(\bar\nu_e \to \bar\nu_e)$$

$$= 1 - 4 s^2_{13} c^2_{13} \sin^2\!\left(\frac{\Delta m^2_{32} L}{4E}\right) + O(\epsilon)$$

$$= 1 - \sin^2 2\theta_{13} \sin^2\!\left(\frac{\Delta m^2_{32} L}{4E}\right) + O(\epsilon)$$

short baseline reactor experiments

"2flavor form" [illegible]

$\alpha = \mu$ $\qquad U_{\mu 3} = s_{23} c_{13} \qquad |U_{\mu 3}|^2 = s^2_{23} c^2_{13}$

$$1 - |U_{\mu 3}|^2 = 1 - s^2_{23} c^2_{13} = 1 - s^2_{23}(1 - s^2_{13}) = \underbrace{1 - s^2_{23}}_{c^2_{23}} + s^2_{23} s^2_{13}$$

☺

$$4|U_{\mu 3}|^2 (1 - |U_{\mu 3}|^2) = 4 s^2_{23} c^2_{13} \left(c^2_{23} + s^2_{23} s^2_{13}\right)$$

$$= c^2_{13} \sin^2 2\theta_{23} + 4 c^2_{13} s^2_{13} s^4_{23}$$

$$= c^2_{13} \sin^2 2\theta_{23} + s^4_{23} \sin^2 2\theta_{13}$$

☺

$$P(\nu_\mu \to \nu_\mu) = 1 - \left(c^2_{13} \sin^2 2\theta_{23} + s^4_{23} \sin^2 2\theta_{13}\right) \sin^2\!\left(\frac{\Delta m^2_{32} L}{4E}\right) + O(\epsilon)$$

"2 flavor form"

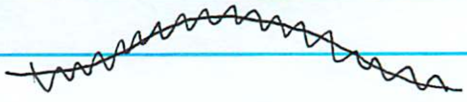
numerically this is $\sim O(\epsilon)$

$\sin^2 2\theta_{13} \simeq 0.1$ (not quite)

Solar-scale Oscillation Dominant Regime

・ $\frac{\Delta m^2_{21} x}{4E} \sim O(1)$ の領域では $\frac{\Delta m^2_{32} x}{4E} \simeq \frac{\Delta m^2_{31} x}{4E} \sim 30 \gg 1$

よって atm. scale oscillation term は solar scale oscillation long-wa に wiggle として乗っかっているにすぎない。



・ Average over atm-scale oscillation

$$\sin^2\left(\frac{\Delta m^2_{31} x}{4E}\right) \simeq \sin^2\left(\frac{\Delta m^2_{32} x}{4E}\right) \simeq \frac{1}{2}$$

$\rightarrow$

$$P(\nu_\alpha \to \nu_\alpha) = 1 - 2|U_{\alpha 1}|^2 |U_{\alpha 3}|^2 - 2|U_{\alpha 2}|^2 |U_{\alpha 3}|^2 - 4|U_{\alpha 1}|^2 |U_{\alpha 2}|^2 \sin^2\left(\frac{\Delta m^2_{21} x}{4E}\right)$$

$$= 1 - 2|U_{\alpha 3}|^2 (1 - |U_{\alpha 3}|^2) - 4|U_{\alpha 1}|^2 |U_{\alpha 2}|^2 \sin^2\left(\frac{\Delta m^2_{21} x}{4E}\right)$$

Example $P(\nu_e \to \nu_e)$

$$|U_{e3}|^2 (1 - |U_{e3}|)^2 = c_{13}^2 s_{13}^2 = \frac{1}{4} \sin^2 2\theta_{13}$$

$$|U_{\alpha 1}|^2 |U_{\alpha 2}|^2 = c_{12}^2 c_{13}^2 s_{12}^2 c_{13}^2 = \frac{1}{4} \sin^2 2\theta_{12} \cdot c_{13}^4$$

$$P(\nu_e \to \nu_e) = 1 - \underline{\frac{1}{2} \sin^2 2\theta_{13}} - \sin^2 2\theta_{12} \cdot c_{13}^4 \sin^2\left(\frac{\Delta m^2_{21} x}{4E}\right)$$

↑ small correction を除いて "2 flavor form"

$\sin^2\theta_{12} = 0.30$

$\sin^2 2\theta_{12} = 0.84$

$c_{13}^4 = 0.95$

$$\therefore \overline{P}_{ee} \simeq 1 - \frac{1}{2} \times 0.95 \times 0.84 \simeq 0.6$$

物質中の ν propagation : 屈折率の効果

・物質中での電子との弾性散乱のために ν_e は 物質中で "potential" を感じる. ← 屈折率

$$n = \frac{c}{v} = \frac{E}{p}$$

$$= \frac{\sqrt{p^2+m^2}+V}{p} \simeq 1 + \frac{m^2}{2p^2} + \frac{V}{p}$$

☺ $\frac{m^2}{2E}$ に対して V の 補正がつく.

☺ $$i\partial_x \begin{bmatrix} \nu_e \\ \nu_\mu \end{bmatrix} = \begin{bmatrix} V(x) & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{2E}\cos 2\theta \end{bmatrix} \begin{bmatrix} \nu_e \\ \nu_\mu \end{bmatrix}$$

・Bethe's evaluation of $V(x)$

$$H_{int} = \frac{G_F}{\sqrt{2}} \bar{\nu}_e \gamma^\mu (1-\gamma_5) \nu_e \cdot \bar{e} \gamma_\mu (1-\gamma_5) e$$

↖ after Fieltz

$\gamma_5 = -1$ for ν_e

$$\langle \bar{e}\gamma^0 e \rangle = N_e(x) \neq 0, \quad \langle \bar{e}\vec{\gamma} e \rangle = 0$$

$$V_{eff} = \underset{\overset{\shortparallel}{V}}{\underline{\sqrt{2}\, G_F N_e(x)}} \cdot \nu_e^\dagger \nu_e$$

No.

Date

Numerical Value of $a_e(x)$: Iso-Singlet Charge-Neutral Media

$$a_e(x) = \sqrt{2}\, G_F\, N_e(x)$$

$$G_F = 1.17 \times 10^{-5}\ \text{GeV}^{-2} = 1.17 \times 10^{-23}\ \text{eV}^{-2}$$

$$N_e = N_A Y_e \quad (g/cm^3)$$

$$= 6.02 \times 10^{+23} \times 0.5$$

$$= 3.01 \times 10^{23}\ \text{cm}^{-3}$$

$$\therefore\ a_e = 1.414 \times 1.17 \times 10^{-23} \times 3.01 \times 10^{23}\ (\text{eV}\cdot\text{cm})^{-3} \cdot \text{eV}$$

$$= 4.98\ (\text{eV}\cdot\text{cm})^{-3}\ \text{eV}$$

$$= 4.98 \times 7.68 \times 10^{-15}\ \text{eV}$$

$$= 3.82 \times 10^{-14}\ \text{eV}$$

$$\therefore\ \boxed{a_e = 3.8 \times 10^{-14} \left(\frac{\rho}{1\ g\,cm^{-3}}\right) \text{eV}}$$

〈conversion constant〉

$$\hbar c = 197.3\ \text{MeV}\cdot\text{fm} = 1.97 \times 10^{2}\, 10^{6}\, 10^{-13}\ \text{eV}\cdot\text{cm} = 1$$

$$\therefore\ \text{eV}\cdot\text{cm} = 0.507 \times 10^{5} \qquad (\text{eV}\cdot\text{cm})^{3} = 0.130 \times 10^{15}$$

Vacuum vs. matter effects in the Sun

$$\cdot\, a(x) = 3.8\times10^{-14}\left(\frac{\rho}{1\,g/cm^3}\right)eV = 3.8\times10^{-12}\left(\frac{\rho}{100\,g/cm^3}\right)eV$$

$$\frac{\Delta m_{21}^2}{2E} = \frac{7.5\times10^{-5}\,eV^2}{2\times10\,MeV} = \frac{7.5\times10^{-5}\,eV^2}{2\times10^7\,eV} = 3.8\times10^{-12}\,eV$$

よって solar matter 中で $\left(\frac{\text{vacuum effect}}{\text{matter effect}}\right)$ は comparable

"neutrino 振動は とてつもなく 小さい energy scale を probe している"

"ν propagation に与える matter の ~~影~~ potential は
影響を
小さいが vaccum oscillation に有効な energy と comparable !

〈Consistency Check〉

$$\frac{aL}{\Delta_{31}} = \frac{\frac{1}{\sqrt{2}}G_F N_e L}{\frac{\Delta m^2 L}{4E}} = \frac{\sqrt{2}G_F N_e}{\frac{\Delta m^2}{2E}} = \frac{0.27}{1.27}\left(\frac{\rho}{2.8\,g/cm^3}\right)\left(\frac{10^{-3}\,eV^2}{\Delta m^2}\right)\times\left(\frac{E}{1\,GeV}\right)$$

$\rho = 100\,g$
$E = 10\,MeV$

$$= \frac{0.27}{1.27}\cdot\frac{100}{2.8}\cdot\frac{10^{3}}{7.5\times10^{5}}\cdot\frac{1}{10^{\cancel{3}2}} = \frac{27}{1.27\times2.8\times7.5} \fallingdotseq 1.01$$

↑

上記計算と consistent !

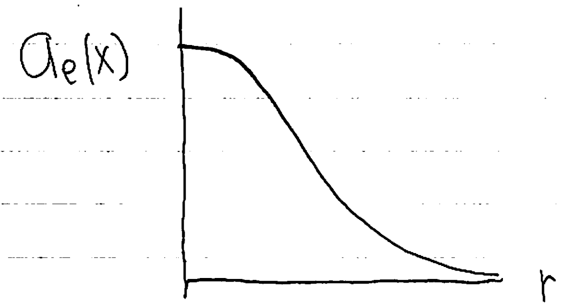
MSW効果

・物質中のν propagation (２世代)

$$i\partial_x \begin{bmatrix} \nu_e \\ \nu_\mu \end{bmatrix} = \begin{bmatrix} a_e(x) & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{2E}\cos 2\theta \end{bmatrix} \begin{bmatrix} \nu_e \\ \nu_\mu \end{bmatrix}$$

$$a_e(x) = a_{\nu_e e}(x) - a_{\nu_\mu e}(x) = \sqrt{2}\, G_F N_e(x)$$

・太陽，超新星 等の 与えられた 天体の $N_e(x)$ 分布によって



・与えられた profile に対して．

$$|a_e(x)|_{max} > \frac{\Delta m^2}{2E}\cos 2\theta > 0$$

なる不等式をみたす領域に $\begin{pmatrix} \nu \text{ mixing parameter} \\ \text{energy } E \end{pmatrix}$

が存在するとする。 ⇒ $x = x_r$ で

・ $a_e(x_r) = \frac{\Delta m^2}{2E}\cos 2\theta$ なる x_r 存在．
共鳴点という

・$x=x_r$ で対角要素縮退：

$$\begin{bmatrix} 1 & \varepsilon \\ \varepsilon & 1 \end{bmatrix} \rightarrow$$ 45° rotation diagonalize the "Hamiltonian" matrix

→ Effective large mixing arises even if $\theta \ll 1$!

Adiabatic Picture

・Suppose that the change in $\underbrace{\text{matter density}}_{}^{\text{along the } \nu \text{ trajectory}}$ is so slow that the adiabaticity condition holds. Then, it is meaningful to diagonalize Hamiltonian matrix locally in matter, and talk about the "adiabatic" basis.

$\boxed{\text{2x2 行列の対角化}}$ を参照して、

$$\sin 2\theta_m = \frac{2\cdot\frac{\Delta m^2}{4E}\sin 2\theta}{\sqrt{\left[\frac{\Delta m^2}{2E}\cos 2\theta - a_e(x)\right]^2 + 4\left(\frac{\Delta m^2}{4E}\sin 2\theta\right)^2}}$$

$$= \underline{\underline{\frac{\sin 2\theta}{\sqrt{\left[\cos 2\theta - \frac{2E}{\Delta m^2}a_e(x)\right]^2 + \sin^2 2\theta}}}} \quad //$$

- At resonance point $(x = x_r)$, $\sin 2\theta_m = 1 \rightarrow \theta_m = \frac{\pi}{4}$.

- In vacuum $a_e = 0$

↓ $\sin 2\theta_m = \sin 2\theta$ $\theta_m = \theta$: vacuum mixing angle

- $x = x_r$ $\theta_m = \frac{\pi}{4}$

↓

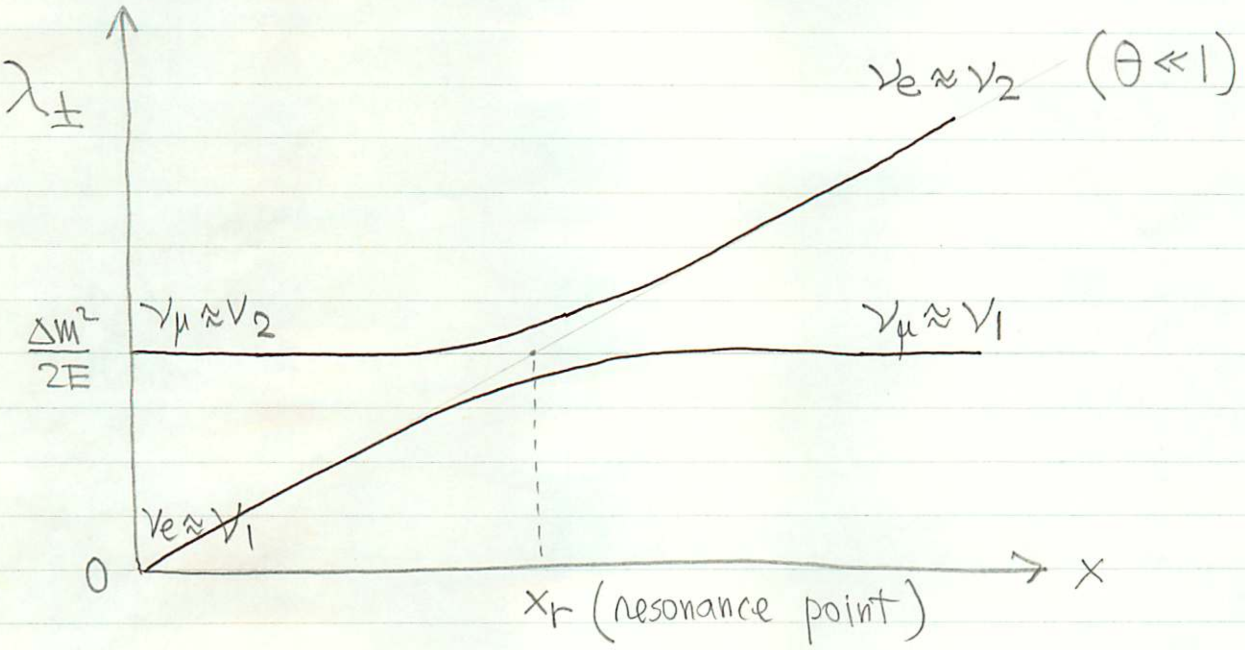
- $x = 0$: (center of the star)

Let us assume $a_e(x) \gg \frac{\Delta m^2}{2E} \cos 2\theta$

$\rightarrow \sin 2\theta_m = 0 \rightarrow \theta_m = 0$ or $\underline{\frac{\pi}{2}}$

- Eigenvalue

$$\lambda_\pm = \frac{1}{2}\left[\left(\frac{\Delta m^2}{2E}\cos 2\theta + a_e\right) \pm \sqrt{\left(\frac{\Delta m^2}{2E}\cos 2\theta - a_e\right)^2 + \left(\frac{\Delta m^2}{2E}\sin 2\theta\right)^2}\right]$$



"Level crossing diagram"

Adiabatic picture → complete conversion of $\nu_e \to \nu_\mu$

$$\cos 2\theta_m = \frac{\cos 2\theta - \frac{2E}{\Delta m^2} a_e(x)}{\sqrt{\left[\cos 2\theta - \frac{2E}{\Delta m^2} a_e(x)\right]^2 + \sin^2 2\theta}} \xrightarrow[a \to \infty]{} -1$$

$$\theta_m = \frac{\pi}{2}$$

Adiabaticity Condition

$$\Delta E \gg \delta r^{-1}$$

↑ level splitting at resonance

↑ perturbation due to density change (masured as energy)

$$\Delta E = \lambda_+ - \lambda_- \Big|_{res} = \frac{\Delta m^2}{2E} \sin 2\theta$$

- how to evaluate δr ?

" The expression of $\sin 2\theta_m$ tells us that mixing is sizable ($\theta_m \sim \frac{\pi}{4}$) at around resonance point ($\cos 2\theta = \frac{2E}{\Delta m^2} a_e$) with width $\sin 2\theta$ "

∴

$$\frac{2E}{\Delta m^2} \delta a_e = \sin 2\theta \qquad a_e = \sqrt{2} G_F N_e$$

$$\frac{2E}{\Delta m^2} \sqrt{2} G_F \, \delta N_e = \underset{\overset{\parallel}{\cos 2\theta}}{\boxed{\sqrt{2} G_F N_e \frac{2E}{\Delta m^2}}} \frac{\delta N_e}{N_e} = \sin 2\theta$$

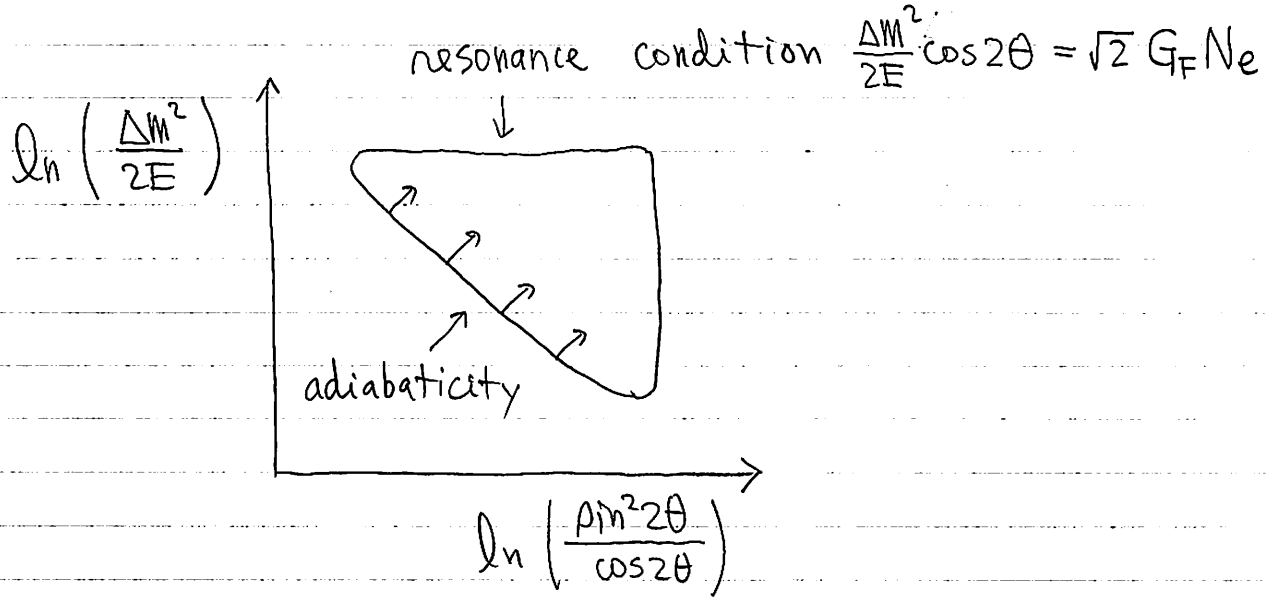
∴

$$\delta r = \frac{\delta N_e}{\left|\frac{\delta N_e}{\delta r}\right|} = \frac{N_e \tan 2\theta}{\left|\frac{dN_e}{dr}\right|}$$

Thus we obtain adiabaticity condition

$$\frac{\Delta m^2}{2E} \sin 2\theta \gg \frac{\left|\frac{dN_e}{dr}\right|}{N_e \tan 2\theta}$$

$$\left(\frac{\Delta m^2}{2E}\right)\left(\frac{\sin^2 2\theta}{\cos 2\theta}\right) \cdot \frac{N_e}{\left|\frac{dN_e}{dr}\right|} \gg 1$$



Adiabatic MSW ~~Transformation~~ Flavor Conversion

- If adiabaticity hold, mass eigenstates evolve independently even with changing matter density

$$\nu_i(x) = e^{-i\int_0^x dx\,\lambda_i(x)}\,\nu_i(0)$$

- gauge vs mass eigenstate

$$\nu_\alpha(x) = U_{\alpha i}\,\nu_i(x)$$

$$\nu_i(x) = (U^\dagger)_{i\alpha}\,\nu_\alpha(x) = U^*_{\alpha i}\,\nu_\alpha(x)$$

$$\nu_\beta(x) = U_{\beta i}\,\nu_i(x)$$

$$= U_{\beta i}\,e^{-i\int_0^x dx\,\lambda_i(x)}\,\nu_i(0)$$

$$= U_{\beta i}\,e^{-i\int_0^x dx\,\lambda_i(x)}\,U^*_{\alpha i}\,\nu_\alpha(0)$$

∴ transition amplitude $\nu_\alpha \to \nu_\beta$ is

$$T_{\beta\alpha} = U_{\beta i}\,U^*_{\alpha i}\,e^{-i\int_0^x dx\,\lambda_i(x)}$$

- Transition probability (next page)

$$P(\nu_\alpha \to \nu_\beta) = \left| \sum_i U_{\beta i} U^*_{\alpha i}\, e^{-\int_0^x dx\, \lambda_i(x)} \right|^2$$

$$= \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2 + 2 \sum_{j>i} U_{\alpha i} U^*_{\beta i} U^*_{\beta j} U_{\alpha j}$$

$$\times \exp\left\{ i \int_0^x dx \left[\lambda_i(x) - \lambda_j(x) \right] \right\}$$

$\swarrow$

the oscillating phase factor can be averaged to zero!

$\because$

$$|\lambda_i - \lambda_j| > |\lambda_i - \lambda_j|\Big|_{\text{at resonance}}$$

$$\simeq \frac{\Delta m^2}{2E} \sin 2\theta$$

$$\gg \frac{\left|\frac{dN_e}{dr}\right|}{N_e} \frac{1}{\tan 2\theta} \simeq \frac{10}{r_\odot} \frac{1}{\tan 2\theta} \simeq 10^{-10}\ \text{cm}^{-1}$$

$$N_e(r) \simeq N_e(0)\, e^{-r/h}$$

$$h \simeq \frac{1}{10} r_\odot$$

$$r_\odot \simeq 7 \times 10^{10}\ \text{cm}$$

$$\frac{\Delta m^2}{2E}\Big|_{\text{solar}} \simeq \frac{10^{-4}\ \text{eV}^2}{\underset{\uparrow \atop 1\,\text{MeV}}{10^6\ \text{eV}}} = 10^{\overset{-10}{\cancel{-12}}}\ \text{eV} = 5 \times 10^{\overset{-6}{\cancel{-8}}}\ \text{cm}^{-1} \gg 10^{-10}\ \text{cm}^{-1}$$

$$\therefore \int_0^h dx\, |\lambda_i - \lambda_j| \simeq \frac{1}{10}\, 7 \times 10^{10} \cdot 5 \times 10^{-6} \simeq \underline{\underline{3.5 \times 10^4}}$$

$5\ \text{GeV}^{-1} = 10^{-13}\ \text{cm}$ $\quad$ $\text{GeV}\cdot\text{cm} = \cancel{0.2 \times 10}\ 5 \times 10^{13}$ $\quad$ $\text{eV}\cdot\text{cm} = 5 \times 10^4$

(ii)

$$P(\nu_\alpha \to \nu_\beta) = \sum_i |U_{\alpha i}(0)|^2 |U_{\beta i}(x)|^2$$

$$\underset{\text{solar core}}{\uparrow} \qquad \underset{\text{solar surface}}{\uparrow}$$

- mixing angles at (solar core ; solar surface = vacuum)

$\boxed{\text{2 Flavor}}$

$$U(x=0) = \begin{bmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & \cos\theta_m \end{bmatrix}$$

$$U(x=r_\odot) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$P(\nu_e \to \nu_e) = |U_{e1}(0)|^2 |U_{e1}(r_\odot)|^2 + |U_{e2}(0)|^2 |U_{e2}(r_\odot)|^2$$

$$= \cos^2\theta \cos^2\theta_m + \sin^2\theta \sin^2\theta_m$$

$$= \frac{1+\cos 2\theta}{2} \frac{1+\cos 2\theta_m}{2} + \frac{1-\cos 2\theta}{2} \frac{1-\cos 2\theta_m}{2}$$

$$= \frac{1}{2}\left[1+\cos 2\theta \cdot \cos 2\theta_m\right]$$

同様に

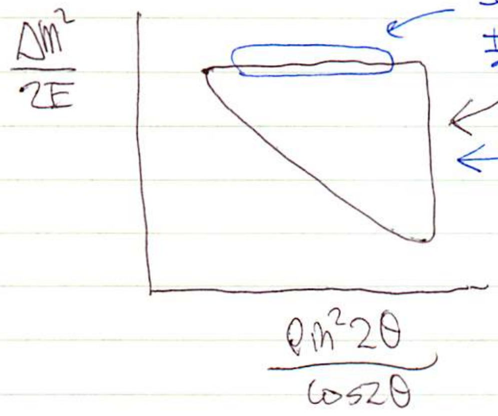
$$P(\nu_e \to \nu_\mu) = \frac{1}{2}\left[1-\cos 2\theta \cos 2\theta_m\right]$$

- Solar neutrinos produced at solar core.

$$\theta_m \simeq \frac{\pi}{2} \qquad ∵ \ \cos 2\theta_m = -1$$

∵

$$P(\nu_e \to \nu_e) = \frac{1}{2}(1-\cos 2\theta) = \sin^2\theta$$



この領域は adiabaticity はOKだが…
共鳴点がcore内にあるため $\theta_m \neq \frac{\pi}{2}$

このbranchをよく記述している。

$\theta_m \simeq \frac{\pi}{2}$:

($\sin 2\theta \ll 1$ の時には
oscillating factor
を average して zero とおく
近似が良くない.)

<note>

Adiabatic MSW formula valid if

$$\begin{cases} \text{adiabaticity OK}, & \text{oscillating factor} \approx \text{zero} \\ \text{production at far from resonance point} \to \theta_m = \frac{\pi}{2} \end{cases}$$

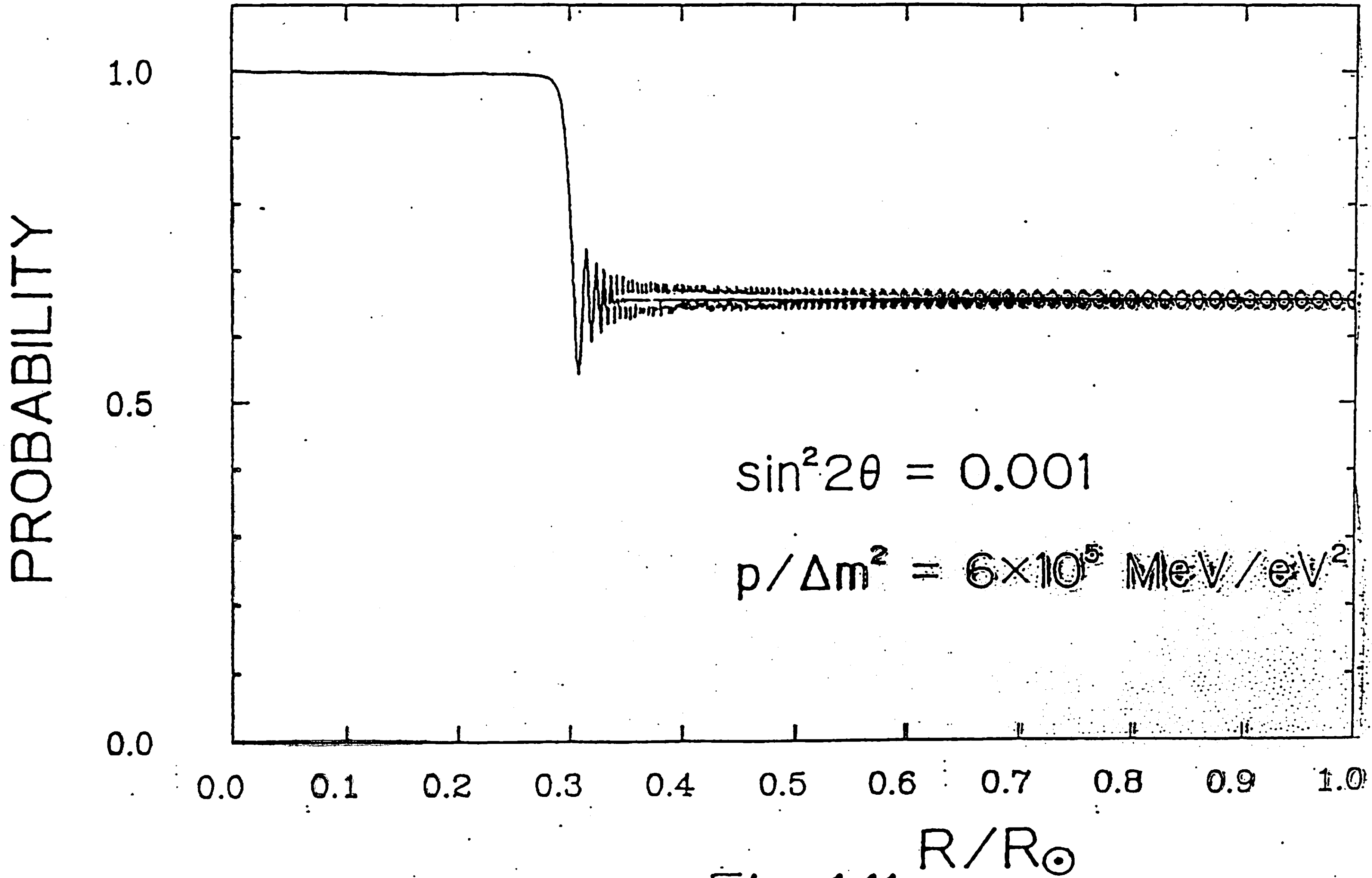
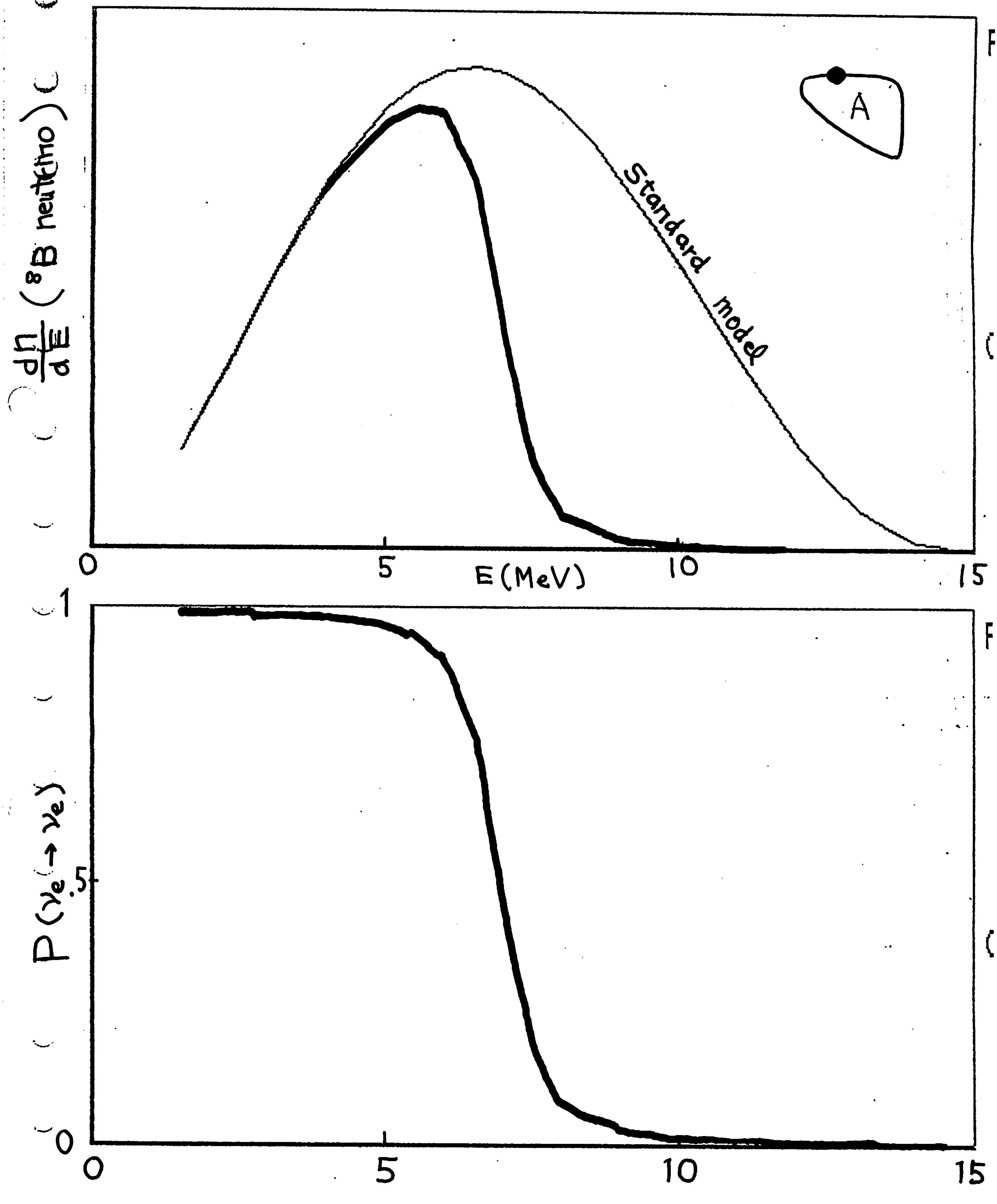

PROBABILITY
1.0
0.5
0.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
R/R⊙
$\sin^2 2\theta = 0.001$
$p/\Delta m^2 = 6\times10^5$ MeV/eV^2

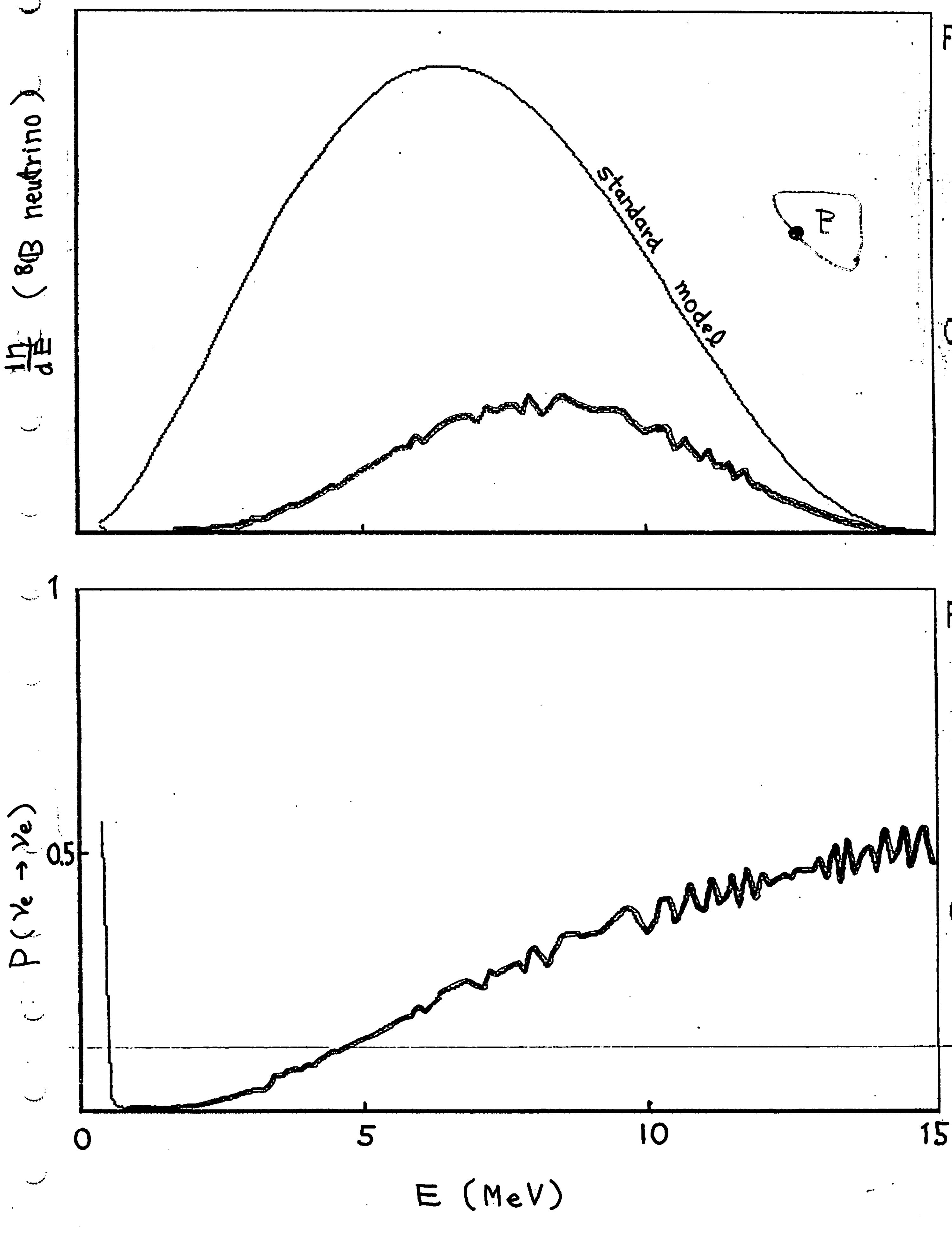
Fig. 1.11

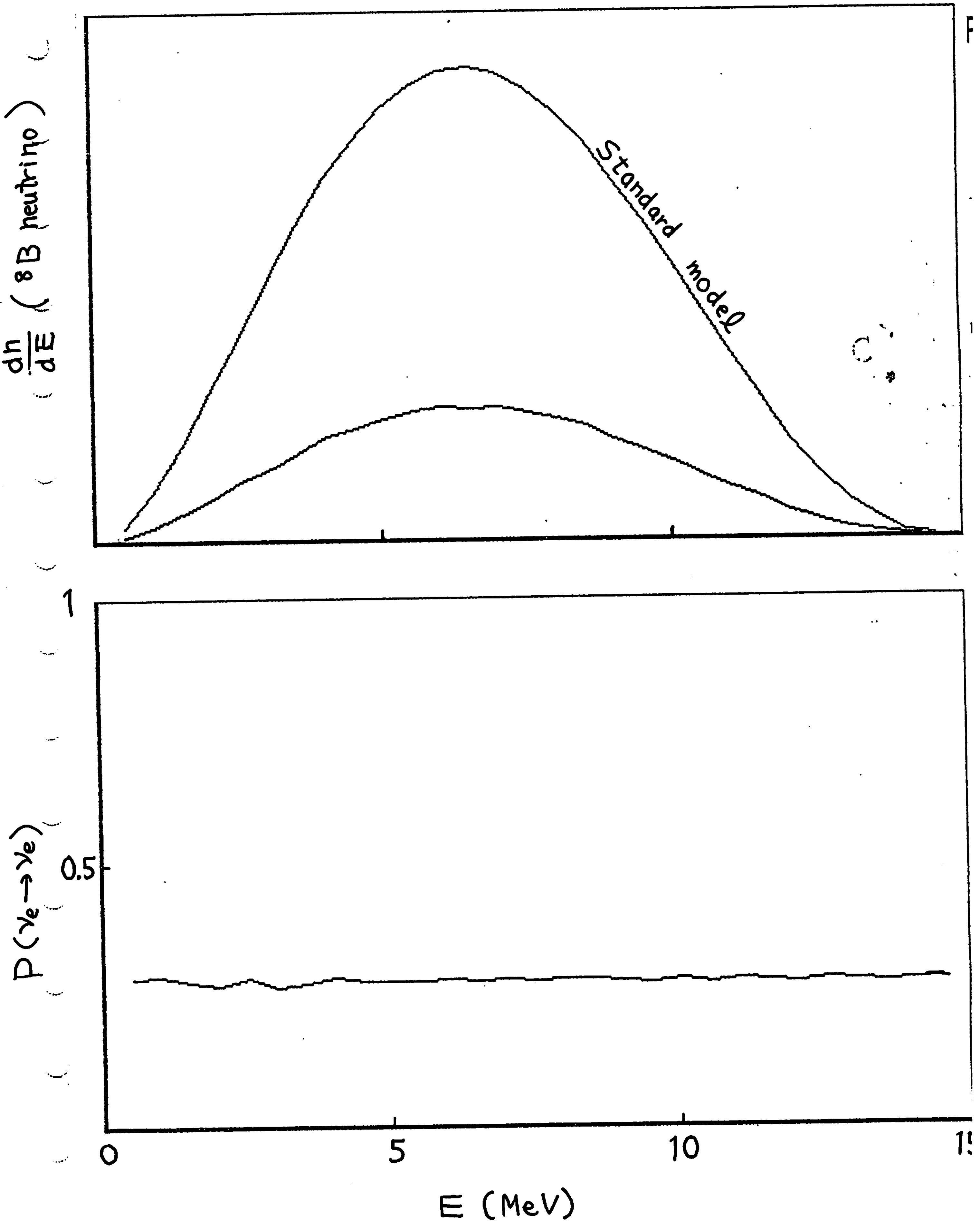
$\Delta m^2 = 6.0 \times 10^{-5}\ eV^2 \quad \sin^2 2\theta = 0.01$



$\frac{\Delta m^2}{2E}\cos 2\theta = a_e(x)$ E (MeV)

$\Delta m^2 = 3.5 \times 10^{-6}\,eV^2 \quad \sin^2 2\theta = 0.01$




$\frac{dn}{dE}$ (^{8}B neutrino)
Standard model
1
0.5
P($\nu_e \rightarrow \nu_e$)
0
5
10
1!
E (MeV)
F

京大講義パート2（6月20日）

Nenbutsu-dera in Kyoto

Hisakazu Minakata
U. Sao Paulo

Overseeing Copacabana beach
from Pão de Açúcar

All the angles are measured !
lepton CP phase δ left

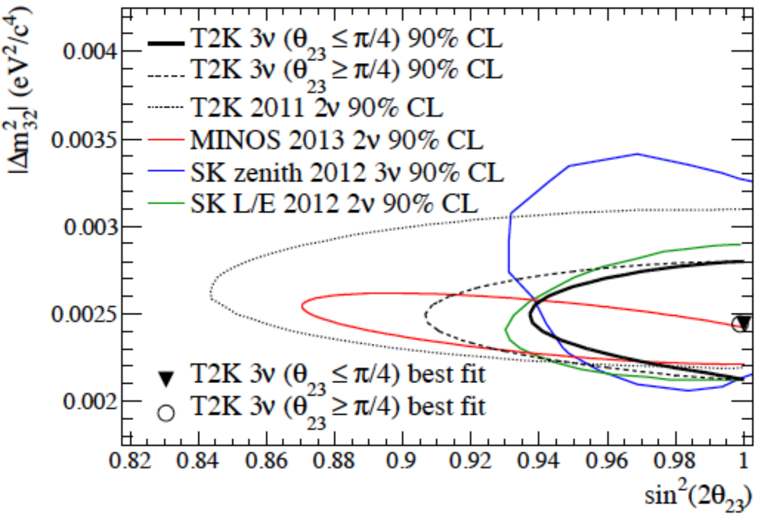
$\nu_\alpha = U_{\alpha i}\, \nu_i$

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

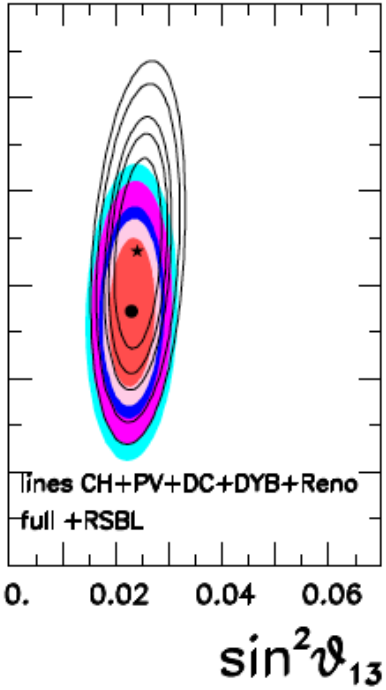
SK-atm+MINOS+T2K

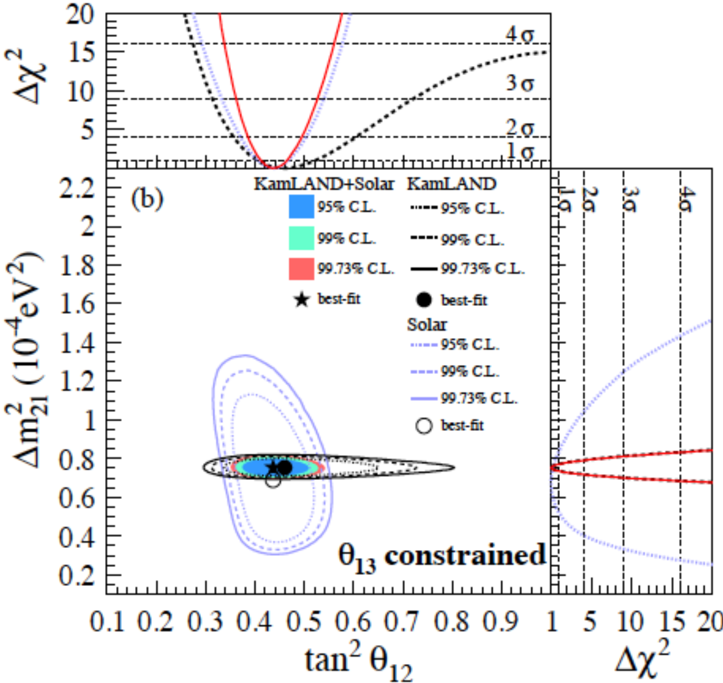
T2K-MINOS-DC-DB-RENO

solar+KamLAND



June 20, 2014



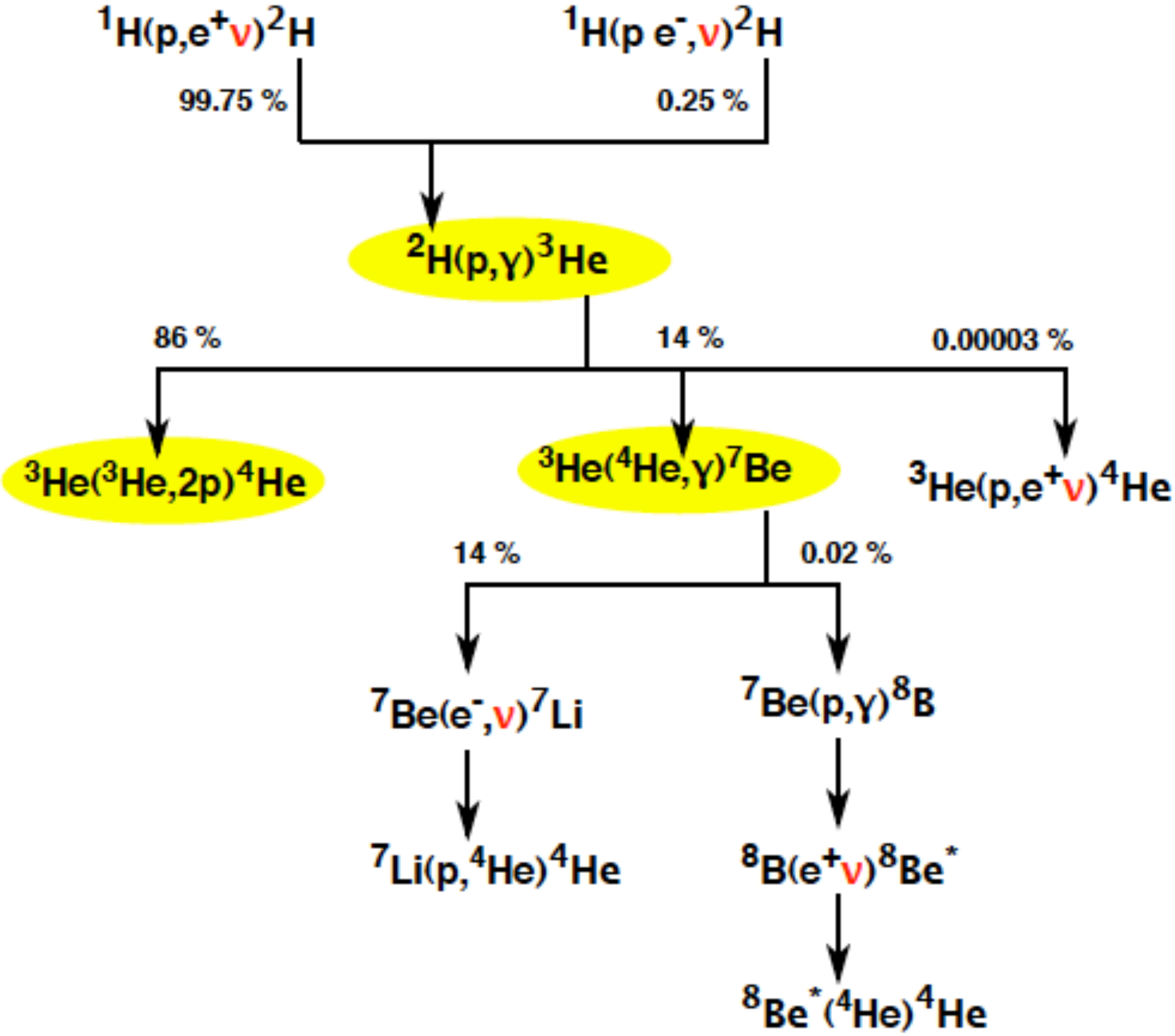


Neutrinos from the Sun

Solar ν (in a nutshell)

- Sun shines because of net reaction $pppp \rightarrow {}^4He + 2e^+ + 2\nu e + 25MeV$
- By knowing the solar constant = S_C =0.136 W/cm^2, the solar neutrino flux F at the earth is given by:
- $F = 2x\ (0.136/25)\ (J\ s/cm^2\ MeV)$
- $1\ MeV = 1.60 \times 10^{-13}\ J$
- $F = 6.8 \times 10^{10}\ (1/s\ cm^2)$

Chain of nuclear reaction in the Sun is a bit more complex



$^{1}H(p,e^{+}\nu)^{2}H$

$^{1}H(p\ e^{-},\nu)^{2}H$

99.75 %

0.25 %

$^{2}H(p,\gamma)^{3}He$

86 %

14 %

0.00003 %

$^{3}He(^{3}He,2p)^{4}He$

$^{3}He(^{4}He,\gamma)^{7}Be$

$^{3}He(p,e^{+}\nu)^{4}He$

14 %

0.02 %

$^{7}Be(e^{-},\nu)^{7}Li$

$^{7}Be(p,\gamma)^{8}B$

$^{7}Li(p,^{4}He)^{4}He$

$^{8}B(e^{+}\nu)^{8}Be^{*}$

$^{8}Be^{*}(^{4}He)^{4}He$

Chain of nuclear reaction in the Sun

pp I chain (termination 85%)

$$
\begin{aligned}
p+p &\rightarrow {}^2H+e^++\nu_e \\
p+p &\rightarrow {}^2H+e^++\nu_e \\
{}^2H+p &\rightarrow {}^3He+\gamma \\
{}^2H+p &\rightarrow {}^3He+\gamma \\
{}^3He+{}^3He &\rightarrow {}^4He+p+p \\
\text{- - - - - - - -} & \text{ - - } \text{- - - - - - - - - - - - - -} \\
p+p+p+p &\rightarrow {}^4He+2e^++2\nu_e(pp)+2\gamma
\end{aligned}
$$

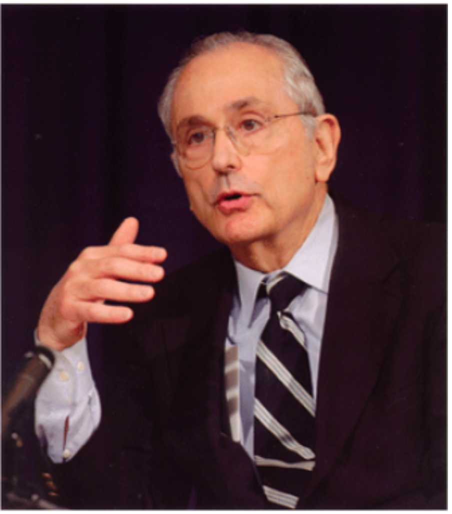
pp II chain (termination 15%)

$$
\begin{aligned}
p+p &\rightarrow {}^2H+e^++\nu_e \\
{}^2H+p &\rightarrow {}^3He+\gamma \\
{}^3He+{}^4He &\rightarrow {}^7Be+\gamma \\
{}^7Be+e^- &\rightarrow {}^7Li+\nu_e \\
{}^7Be+p &\rightarrow {}^4He+{}^4He \\
\text{- - - - - - - - - -} & \text{ - - } \text{- - - - - - - - - - - - - - - -} \\
p+p+p+p+e^- &\rightarrow {}^4He+e^++\nu_e(pp)+\nu_e({}^7Be)+2\gamma
\end{aligned}
$$

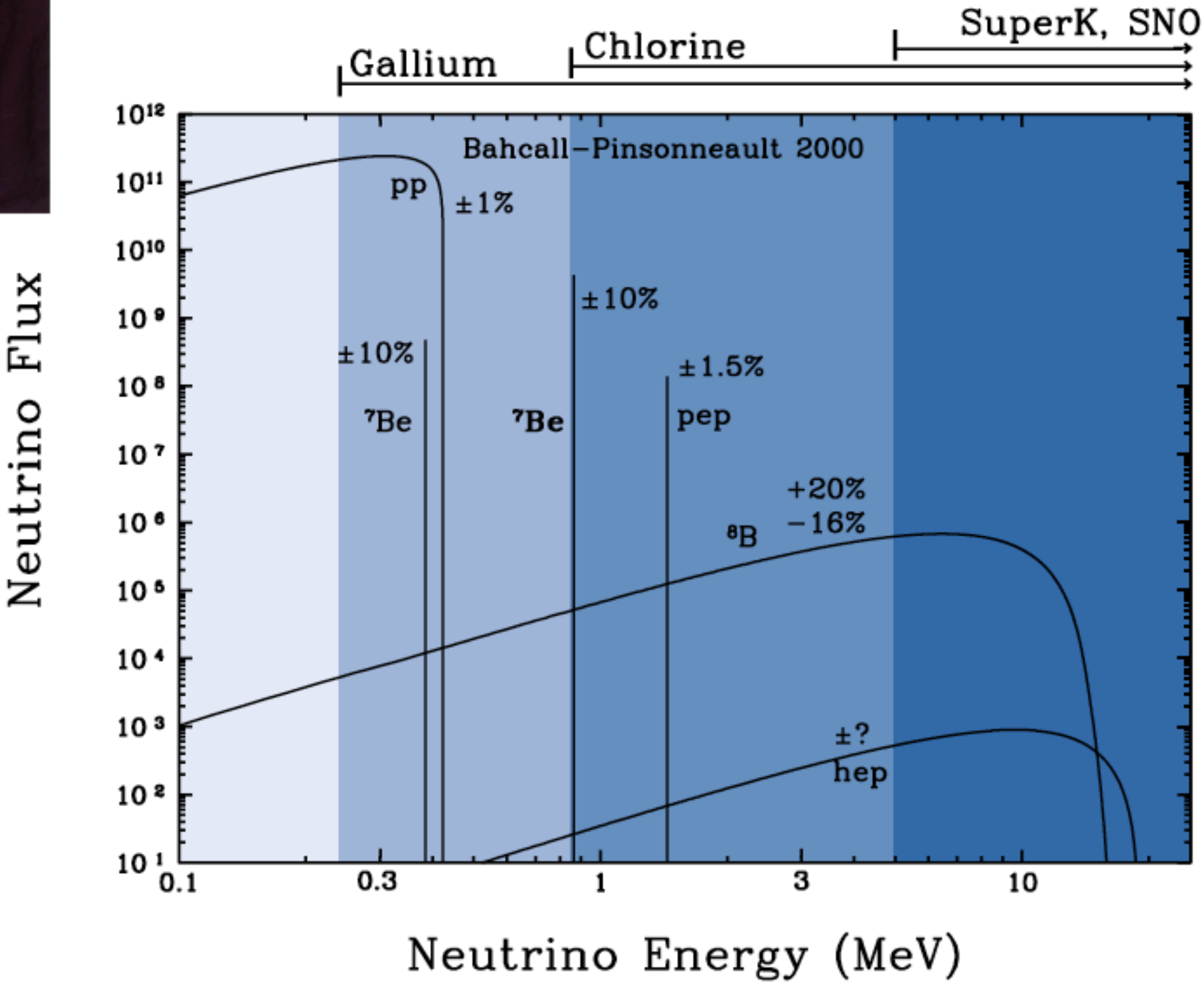
pp III chain (termination 0.02%)

$$
\begin{aligned}
p+p &\rightarrow {}^2H+e^++\nu_e \\
{}^2H+p &\rightarrow {}^3He+\gamma \\
{}^3He+{}^4He &\rightarrow {}^7Be+\gamma \\
{}^7Be+p &\rightarrow {}^8B+\gamma \\
{}^8B &\rightarrow {}^8Be^*+e^++\nu_e \\
{}^8Be^* &\rightarrow {}^4He+{}^4He \\
\text{- - - - - - - -} & \text{ - - } \text{- - - - - - - - - - - - - - - -} \\
p+p+p+p &\rightarrow {}^4He+2e^++\nu_e(pp)+\nu_e({}^8B)+3\gamma
\end{aligned}
$$

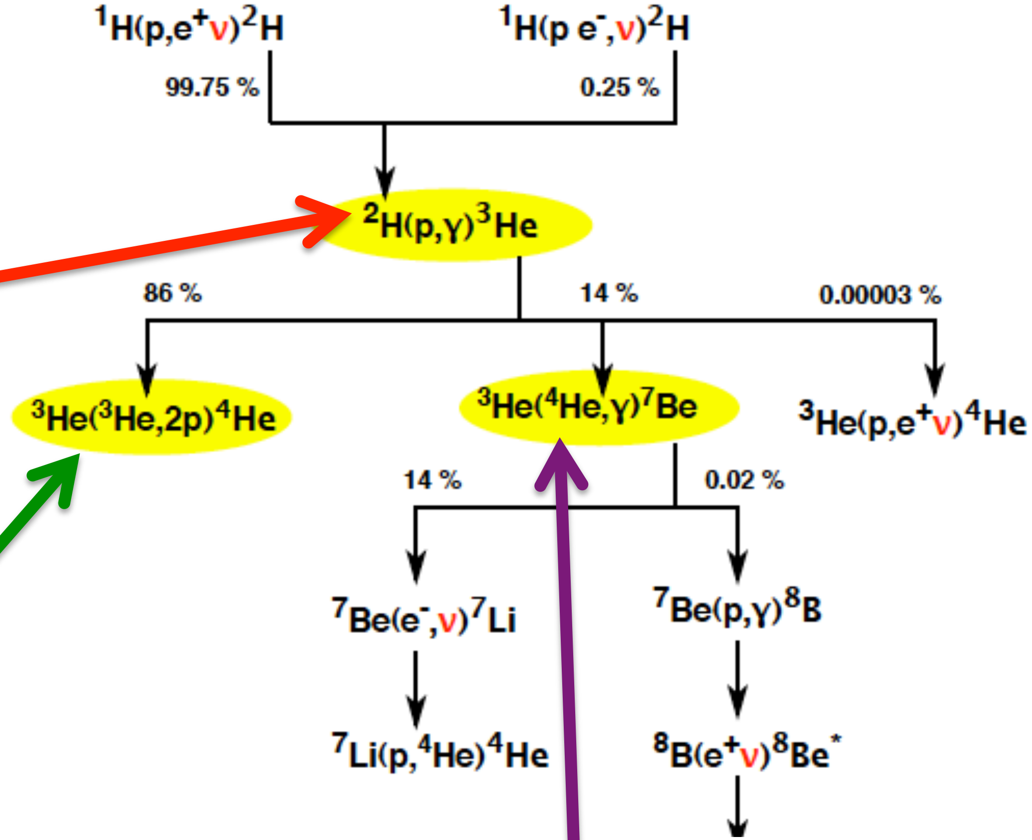
Standard solar model: one can model interior of the Sun



Calculated result of solar neutrino flux



Chain of nuclear reaction in the Sun is a bit more complex



$^1H(p,e^+\nu)^2H$ 99.75 %

$^1H(p\ e^-,\nu)^2H$ 0.25 %

$^2H(p,\gamma)^3He$

86 % $^3He(^3He,2p)^4He$

14 % $^3He(^4He,\gamma)^7Be$

0.00003 % $^3He(p,e^+\nu)^4He$

14 % $^7Be(e^-,\nu)^7Li$ → $^7Li(p,^4He)^4He$

0.02 % $^7Be(p,\gamma)^8B$ → $^8B(e^+\nu)^8Be^*$

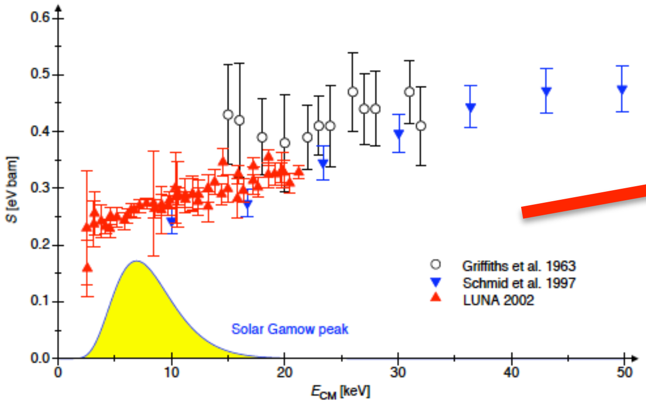


Figure 5: The $^2H(p,\gamma)^3He$ astrophysical factor $S(E)$ with the total error.

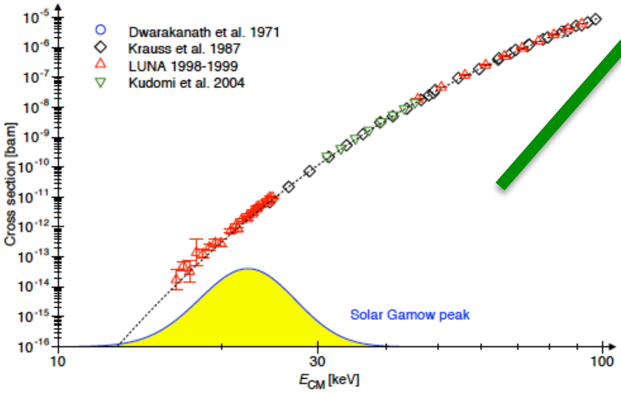


Figure 6: Cross section of the $^3He(^3He,2p)^4He$ reaction. Data from LUNA (27,28) and from other groups (59, 60, 61). The line is the extrapolation based on the measured $S(E)$-factor (28).

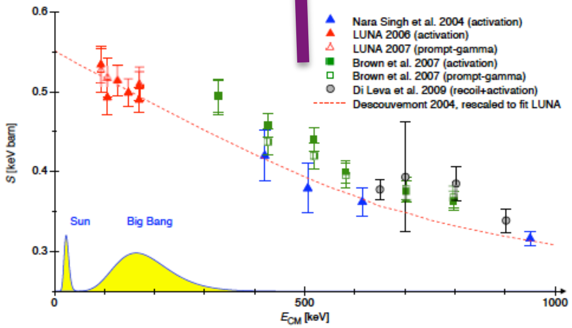


Figure 8: Astrophysical S(E)-factor for $^3He(^4He,\gamma)^7Be$. The results from the modern, high precision experiments are shown with their total error.

LENA@Gran Sasso Lab.

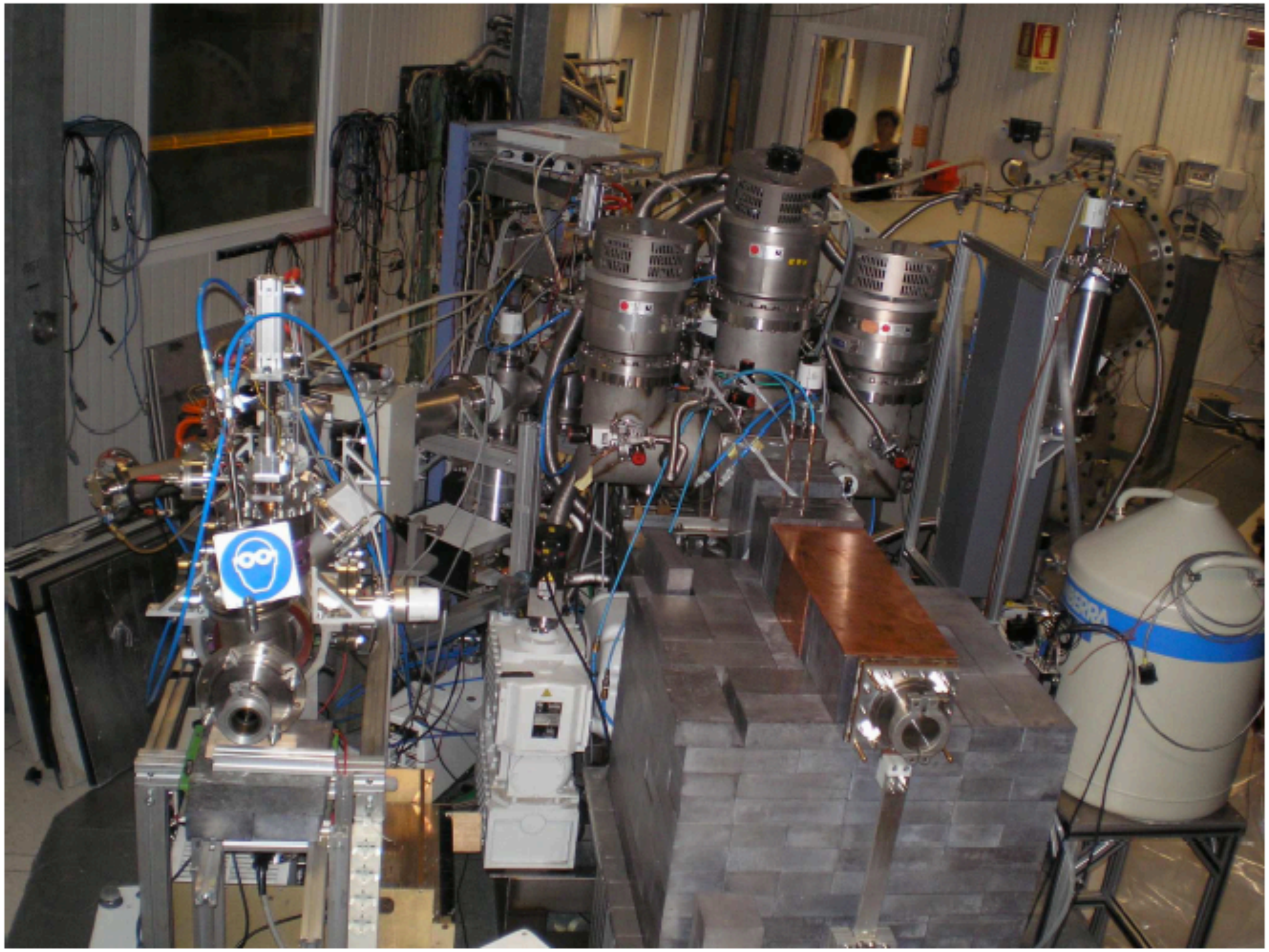


Figure 2: The LUNA set-up with the two different beam lines in the foreground and the accelerator in the back. The beam line to the left is dedicated to the measurements with solid target whereas the one on the right hosts the windowless gas target. The set-up for the study of ^{3}He(^{4}He,γ)^{7}Be is shown during installation with the shield only partially mounted.

$^{2}H(p,\gamma)^{3}He$

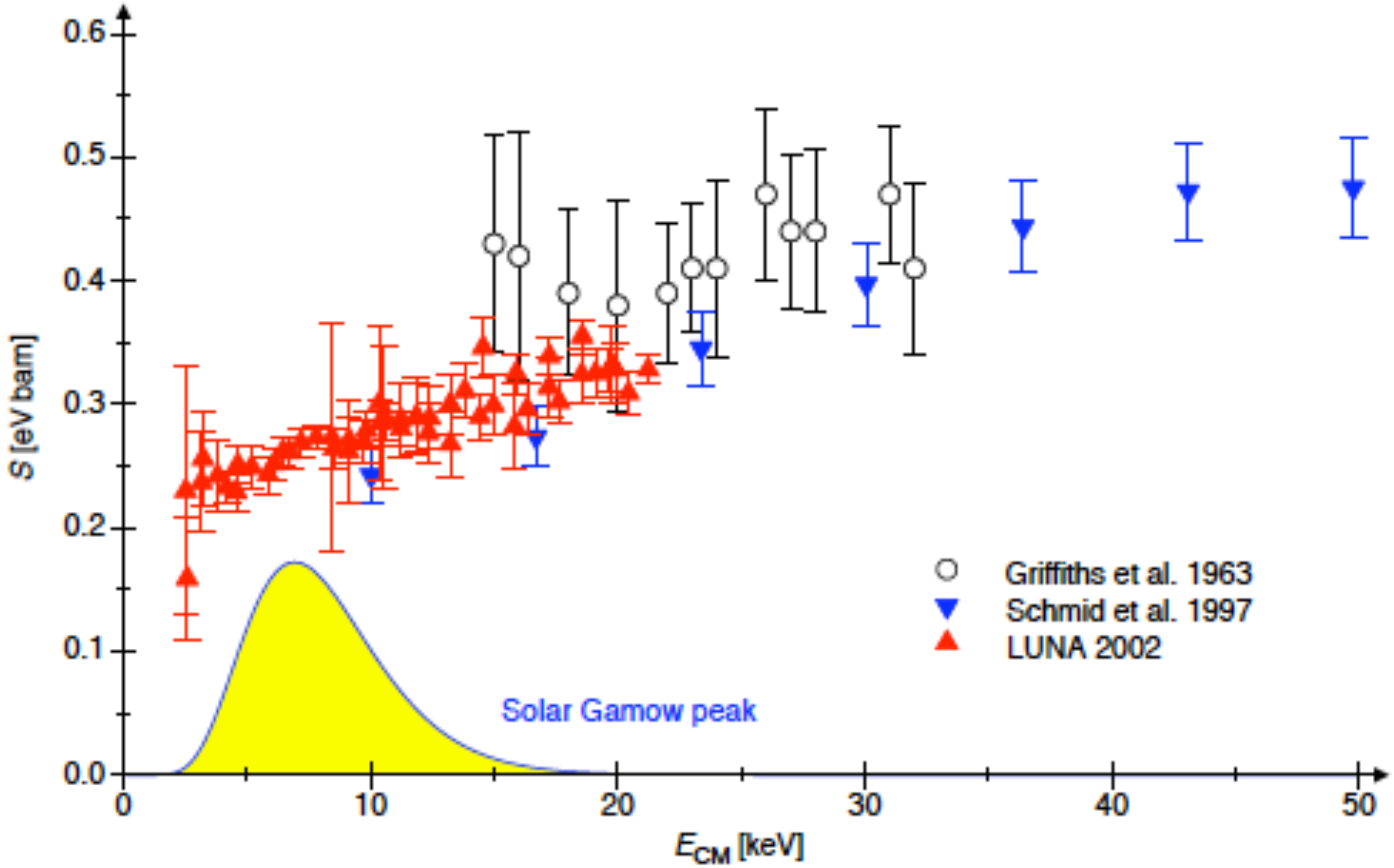


Figure 5: The $^{2}H(p,\gamma)^{3}He$ astrophysical factor $S(E)$ with the total error.

^{3}He(^{3}He,2p)^{4}He

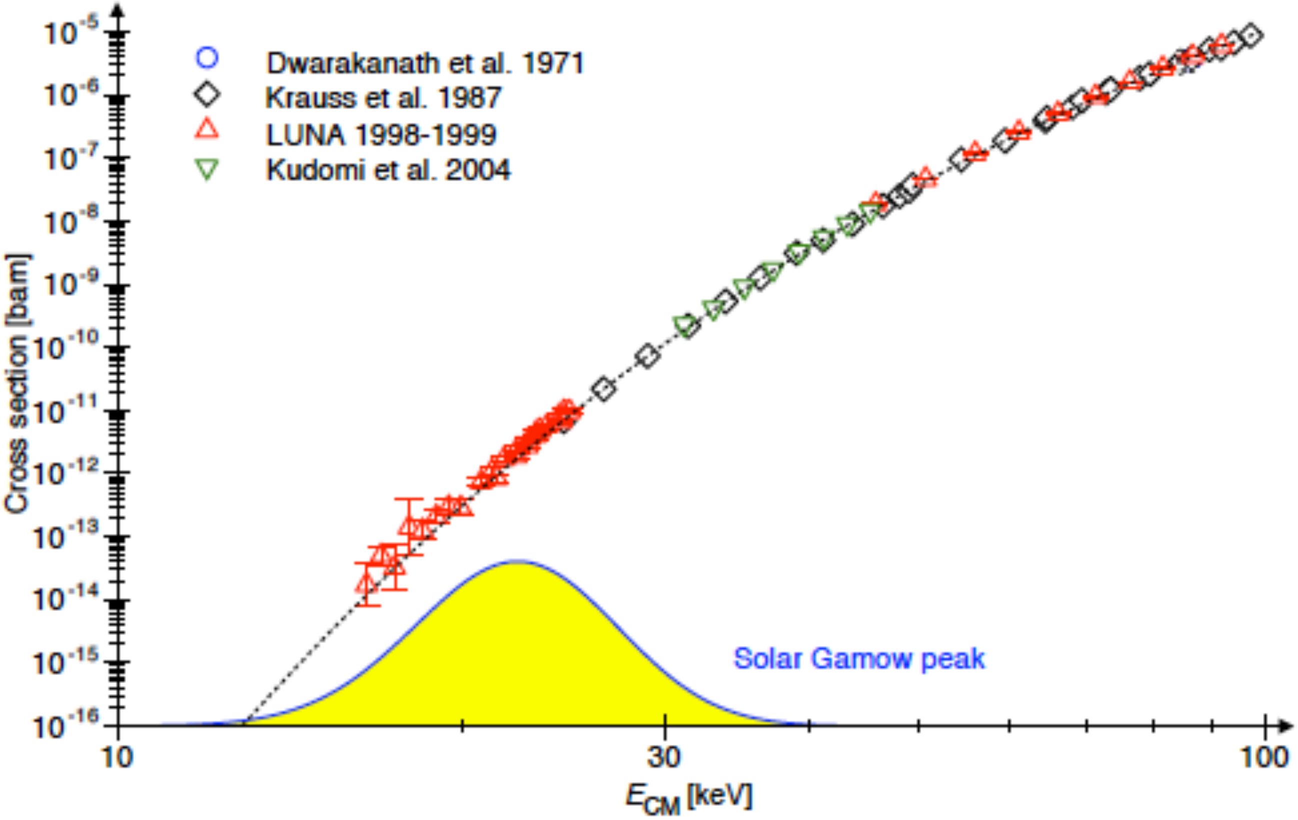


Figure 6: Cross section of the ^{3}He(^{3}He,2p)^{4}He reaction. Data from LUNA (27,28) and from other groups (59, 60, 61). The line is the extrapolation based on the measured $S(E)$-factor (28).

^{3}He(^{4}He,γ)^{7}Be.

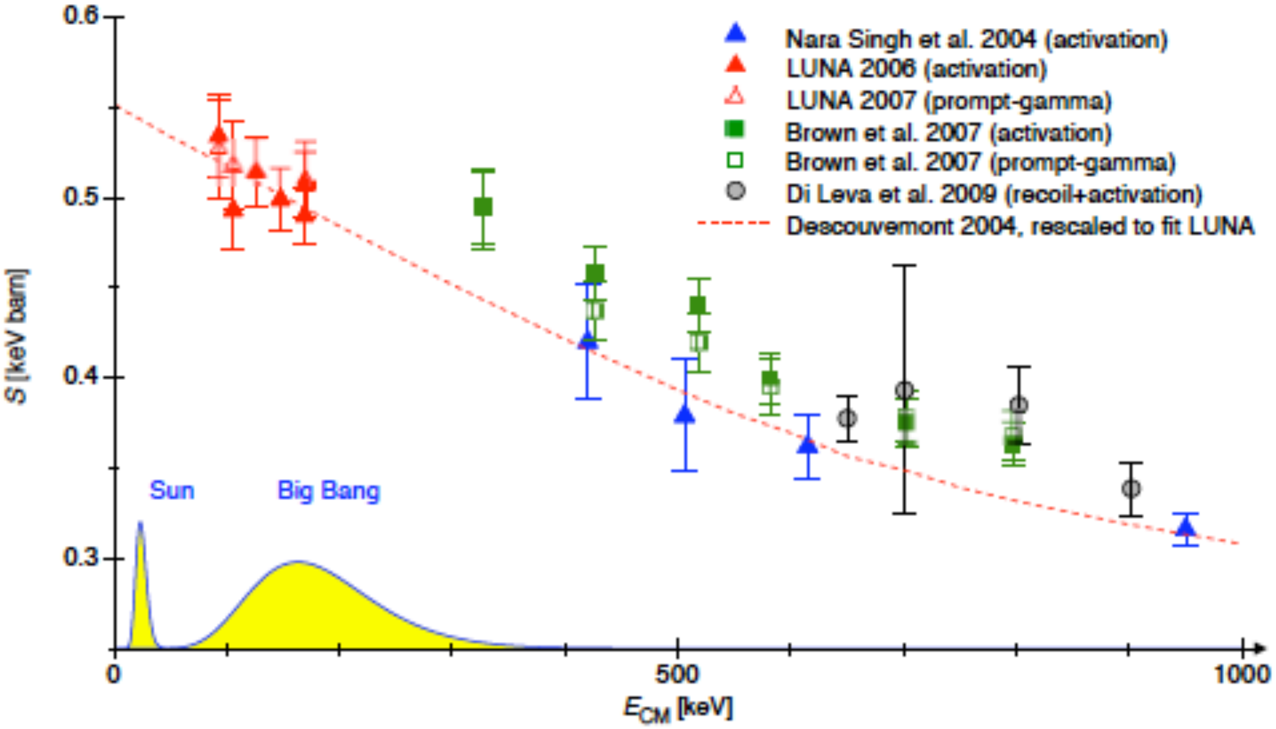


Figure 8: Astrophysical S(E)-factor for ^{3}He(^{4}He,γ)^{7}Be. The results from the modern, high precision experiments are shown with their total error.

Chain of nuclear reaction in the Sun: Again

pp I chain (termination 85%)

$$\begin{aligned} p+p &\rightarrow {}^2H+e^++\nu_e \\ p+p &\rightarrow {}^2H+e^++\nu_e \\ {}^2H+p &\rightarrow {}^3He+\gamma \\ {}^2H+p &\rightarrow {}^3He+\gamma \\ {}^3He+{}^3He &\rightarrow {}^4He+p+p \end{aligned}$$

$$p+p+p+p \rightarrow {}^4He+2e^++2\nu_e(pp)+2\gamma$$

pp II chain (termination 15%)

$$\begin{aligned} p+p &\rightarrow {}^2H+e^++\nu_e \\ {}^2H+p &\rightarrow {}^3He+\gamma \\ {}^3He+{}^4He &\rightarrow {}^7Be+\gamma \\ {}^7Be+e^- &\rightarrow {}^7Li+\nu_e \\ {}^7Be+p &\rightarrow {}^4He+{}^4He \end{aligned}$$

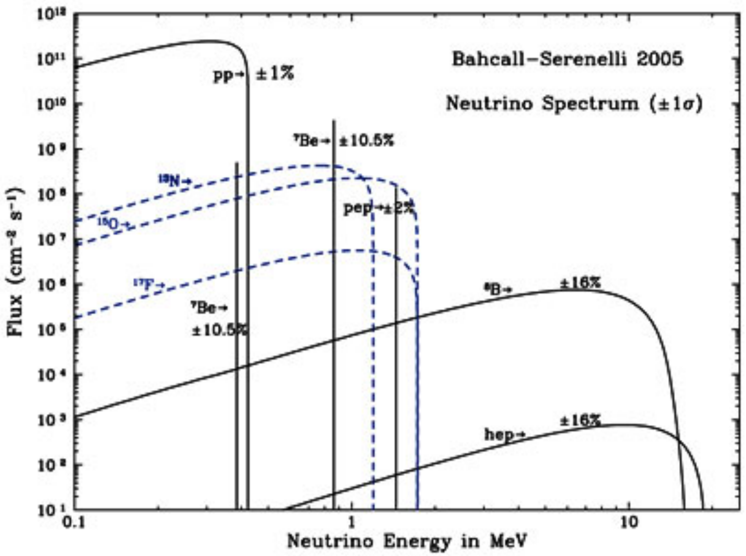
$$p+p+p+p+e^- \rightarrow {}^4He+e^++\nu_e(pp)+\nu_e({}^7Be)+2\gamma$$

Main engine of the Sun

pp III chain (termination 0.02%)

$$\begin{aligned} p+p &\rightarrow {}^2H+e^++\nu_e \\ {}^2H+p &\rightarrow {}^3He+\gamma \\ {}^3He+{}^4He &\rightarrow {}^7Be+\gamma \\ {}^7Be+p &\rightarrow {}^8B+\gamma \\ {}^8B &\rightarrow {}^8Be^*+e^++\nu_e \\ {}^8Be^* &\rightarrow {}^4He+{}^4He \end{aligned}$$

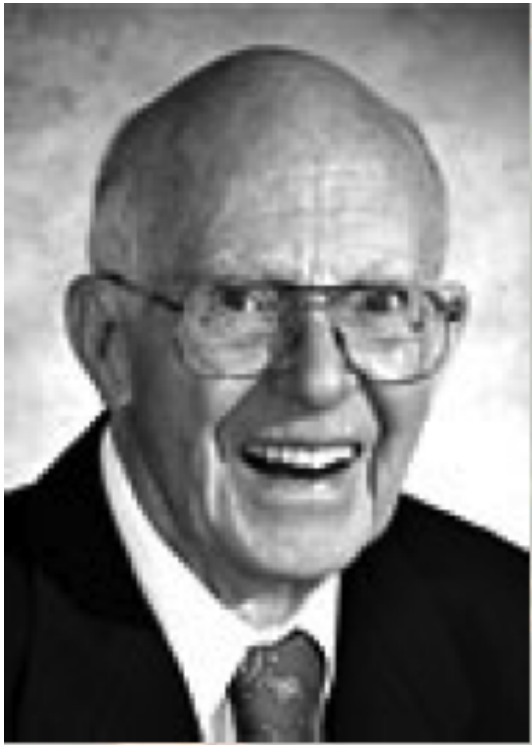
$p+p$



^{37}Cl and ^{71}Ga experiments: Radio-chemical experiments

- Homestake (Ray Davis): ν_e+^{37}Cl -> e^-+^{37}Ar (Pioneer!)
- Ga experiment: ν_e+^{71}Ga -> e^-+^{37}Ge

- Sensitive only to ν_e
- Low energy threshold



Results of ^{37}Cl experiment: production rate of ^{37}Ar

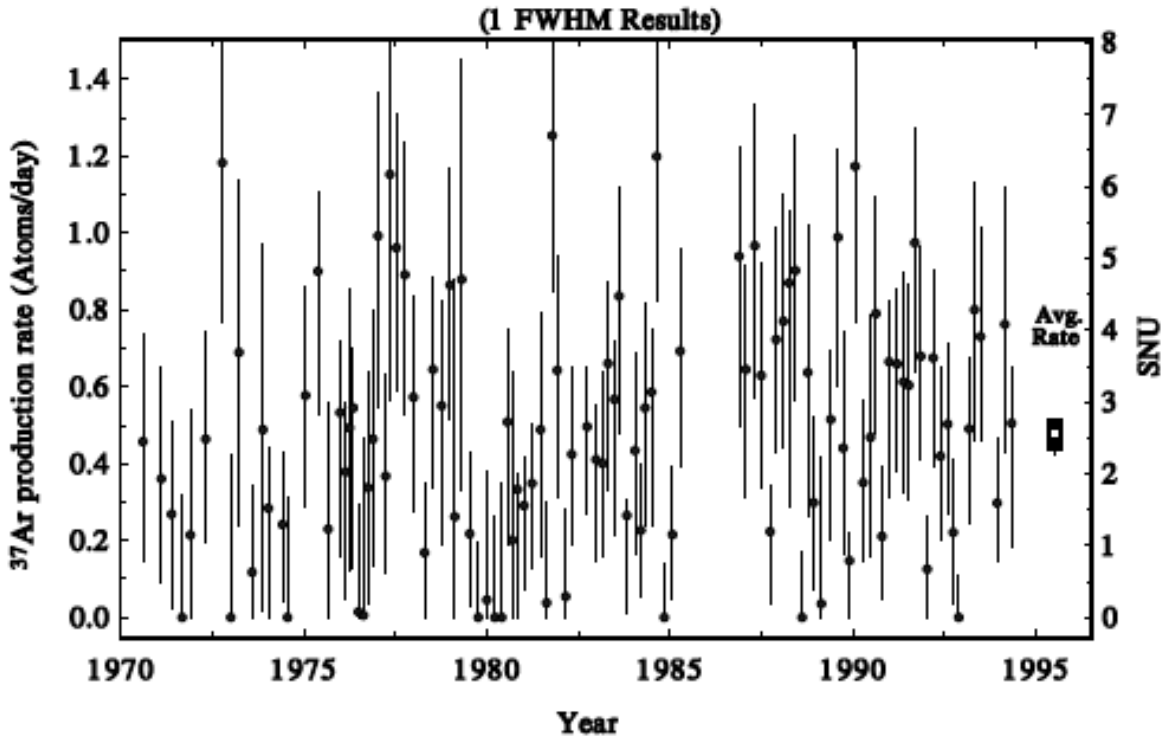


FIG. 13.—Homestake Experiment—one FWHM results. Results for 108 individual solar neutrino observations made with the Homestake chlorine detector. The production rate of ^{37}Ar shown has already had all known sources of nonsolar ^{37}Ar production subtracted from it. The errors shown for individual measurements are statistical errors only and are significantly non-Gaussian for results near zero. The error shown for the cumulative result is the combination of the statistical and systematic errors in quadrature.

1 SNU = 10^{-36} events / (No. of target atom sec)

~1/3 of SSM expectation: solar nu problem started!

Gallium Experiments

SAGE(60 t),
GALLEX/GNO (30 t)

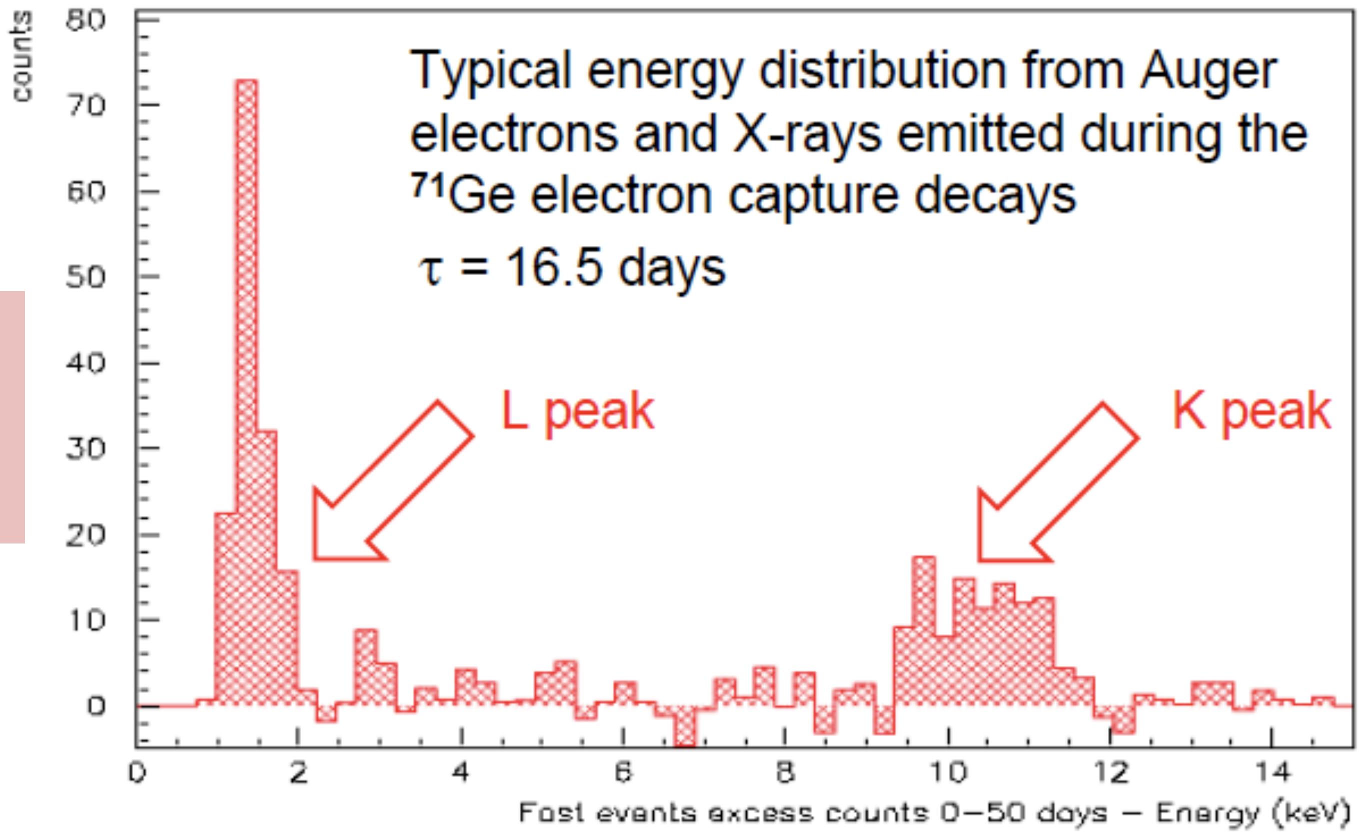
$^{71}Ga + \nu_e \rightarrow {}^{71}Ge + e^-$

Radiochemical Target

Small proportional counters are used to count the Germanium

Energy Threshold: 0.233 MeV

Sensitive to pp, ^7Be, ^8B, CNO, and pep ν's

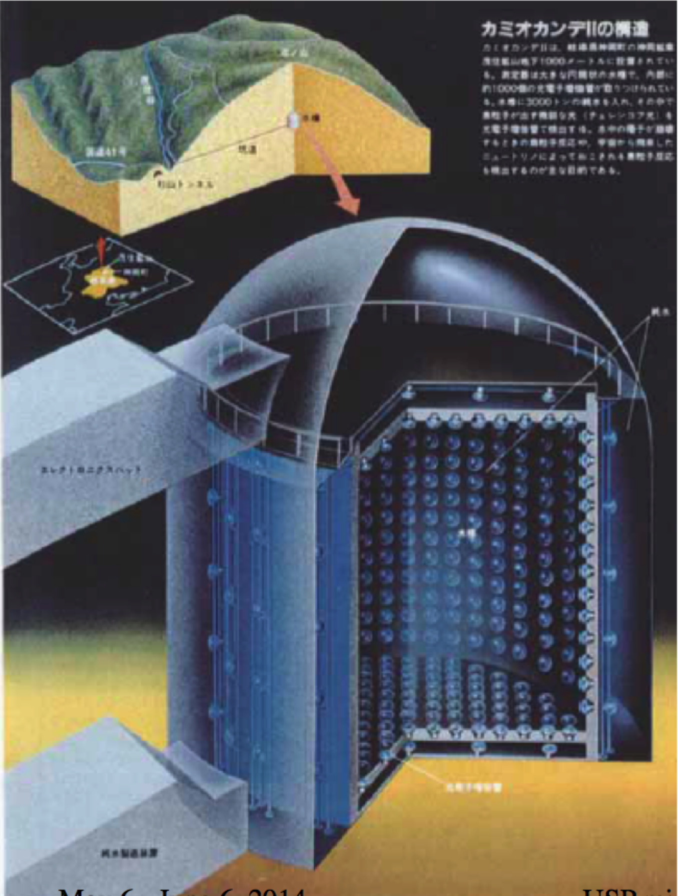

Typical energy distribution from Auger electrons and X-rays emitted during the ^{71}Ge electron capture decays

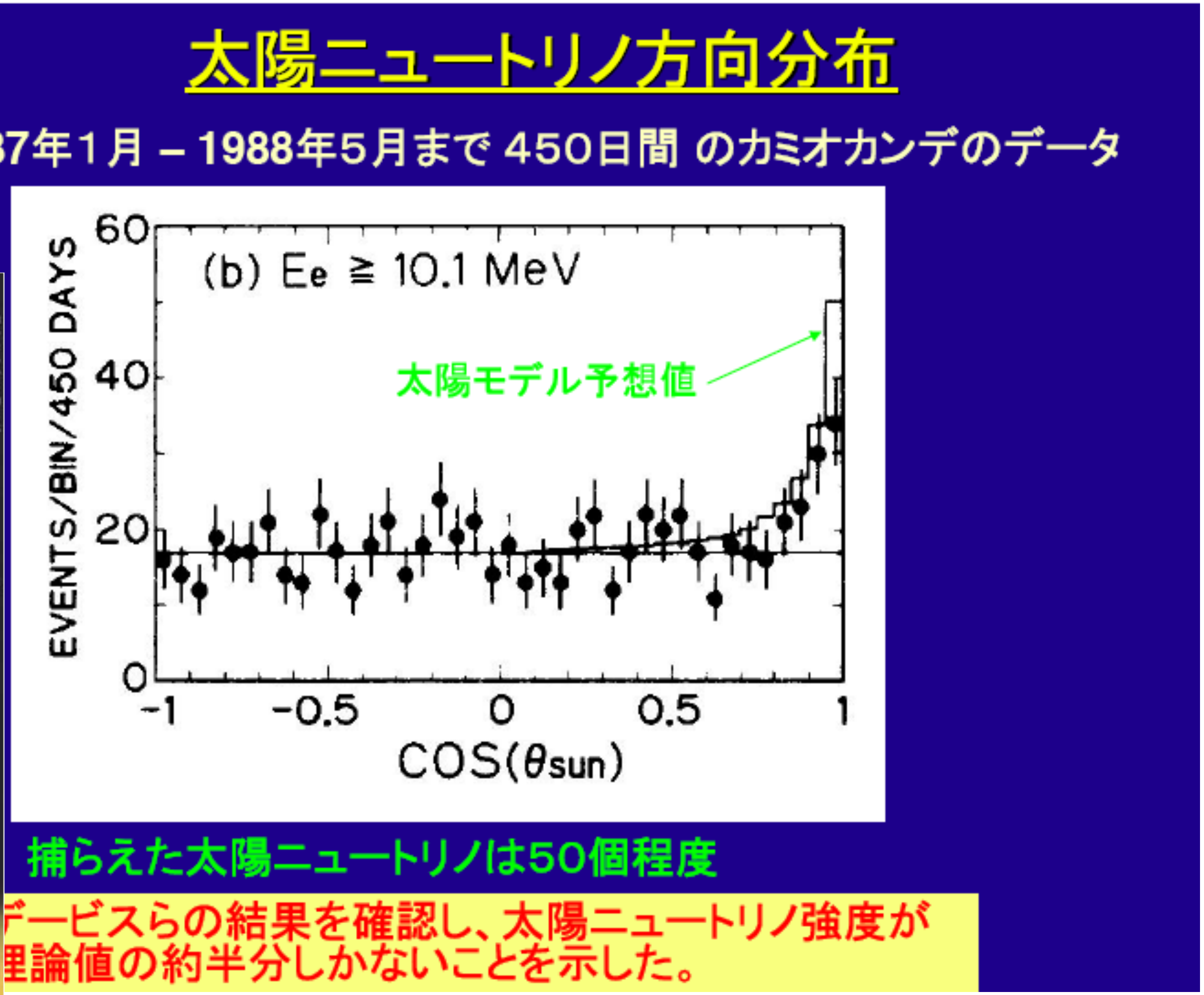
τ = 16.5 days

~60% of SSM prediction

Kamiokande II

1st experiment which proved that neutrinos comes from the Sun





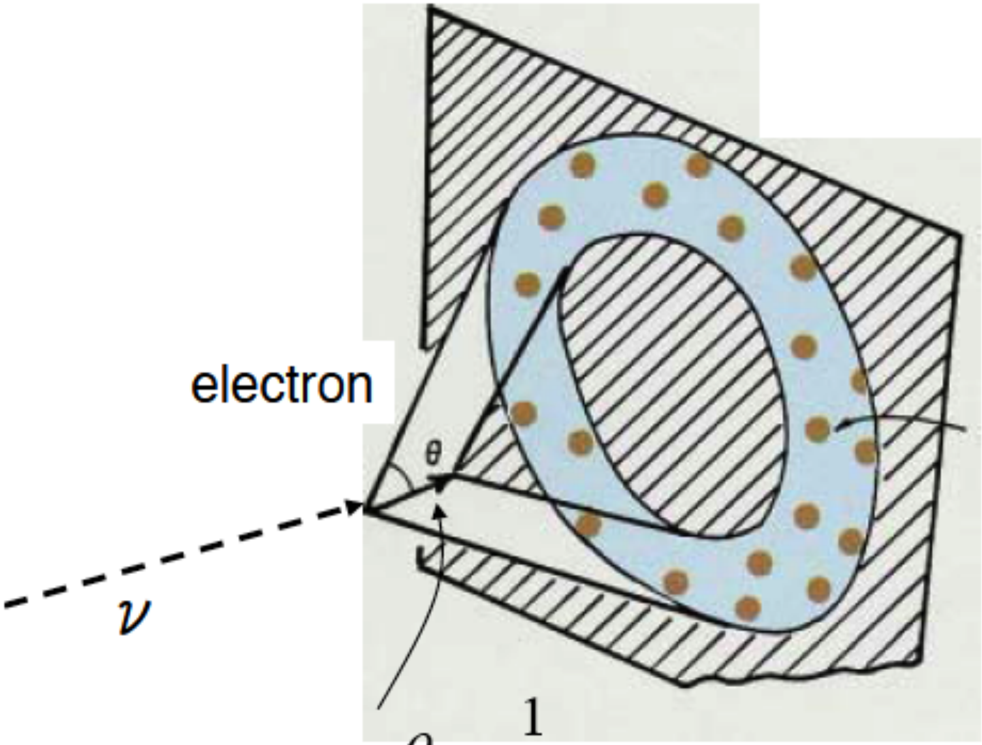
太陽ニュートリノ方向分布

1987年1月 – 1988年5月まで 450日間 のカミオカンデのデータ

捕らえた太陽ニュートリノは50個程度

デービスらの結果を確認し、太陽ニュートリノ強度が
理論値の約半分しかないことを示した。

Detecting Cherenkov photons



Photomultiplier tube (PMT)

$$\cos\theta = \frac{1}{n\beta}$$

n=1.34 in water

⇒ θ =42deg. for β =1

Number of Ch. photons with λ =300-600 nm emitted by a relativistic particle per cm = 340.

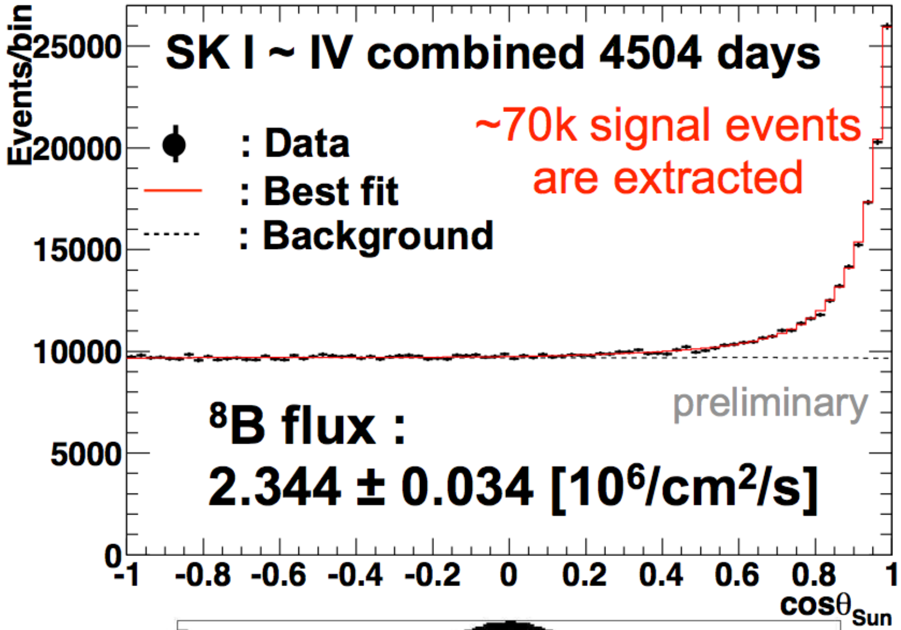
⇓

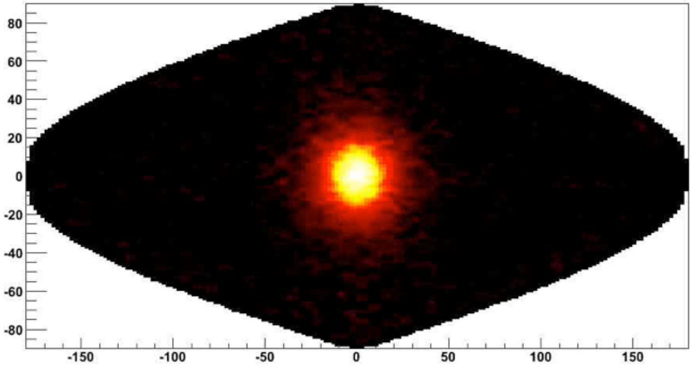
Need an efficient detection of the photons. ⇒ Large PMTs

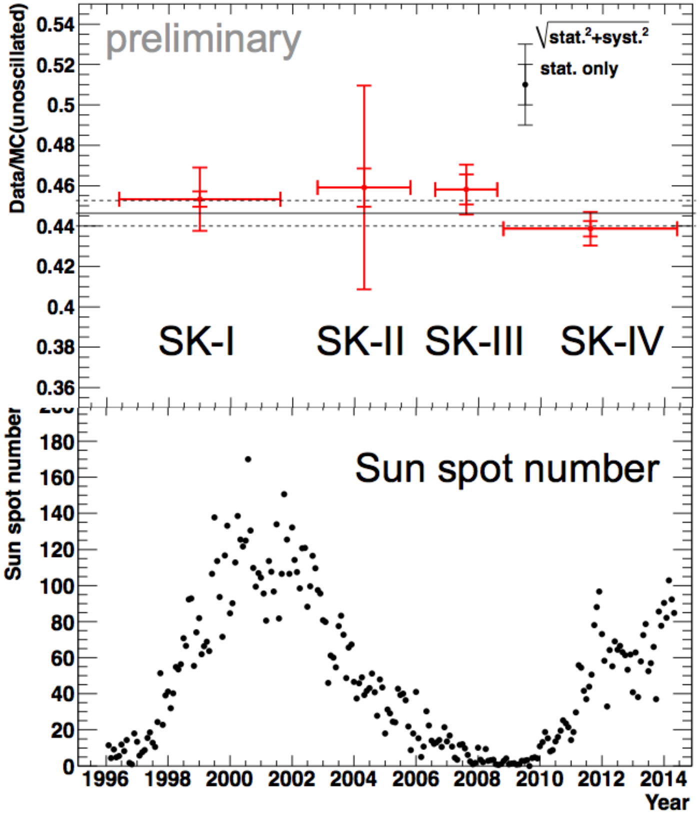
Precision SK measurement Now

Koshio Nu2014

Observed solar neutrino signal



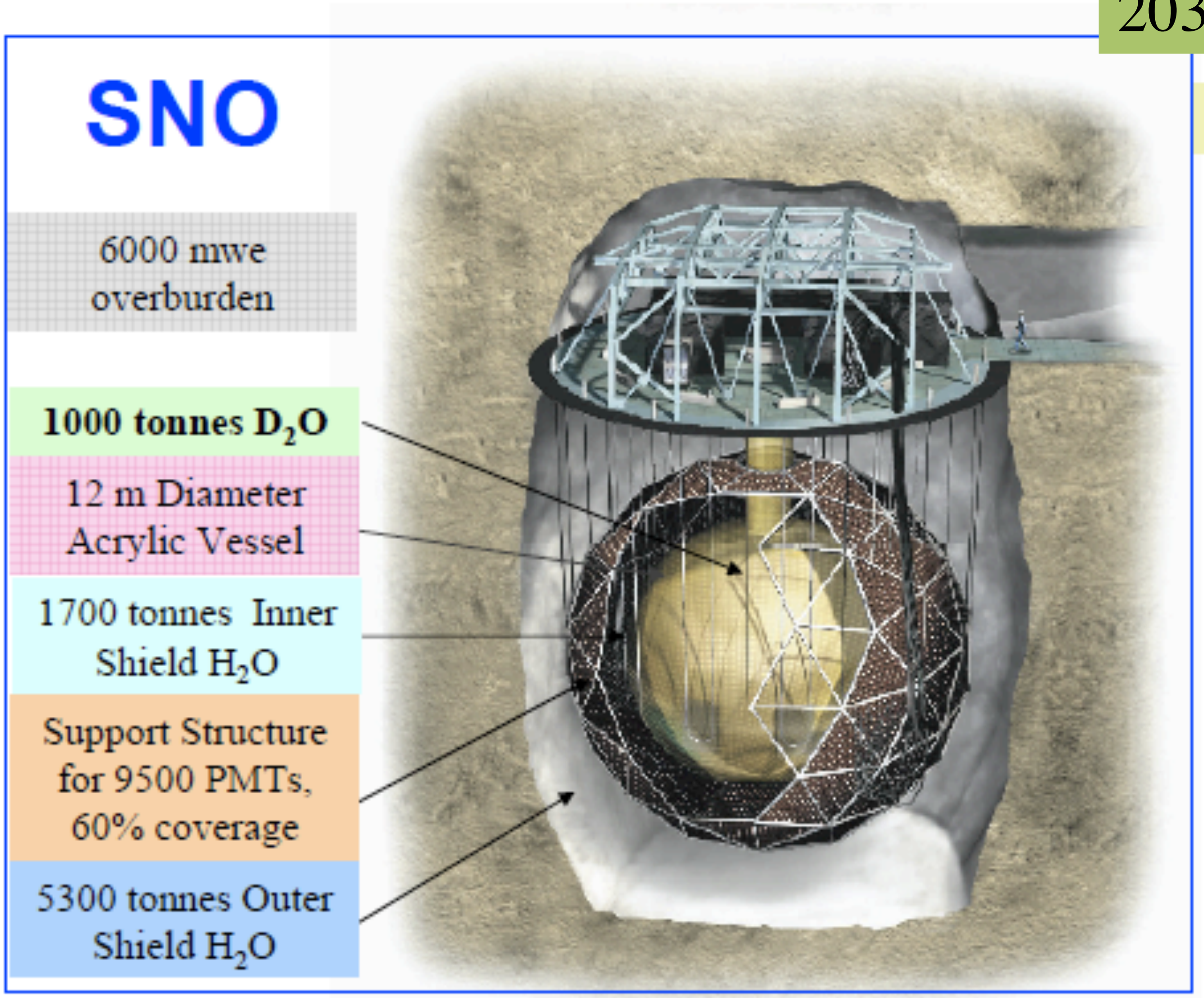




No correlation with solar activity is seen.
More sophisticated analyses such as
yearly flux plot are being prepared.

2039m underground

SNO



6000 mwe overburden

1000 tonnes D_2O

12 m Diameter Acrylic Vessel

1700 tonnes Inner Shield H_2O

Support Structure for 9500 PMTs, 60% coverage

5300 tonnes Outer Shield H_2O

Image courtesy National Geographic

3 Reactions:

$$\nu_x + e^- \rightarrow \nu_x + e^- \quad \text{ES}$$
$$\nu_e + d \rightarrow p + p + e^- \quad \text{CC}$$
$$\nu_x + d \rightarrow p + n + \nu_x \quad \text{NC}$$

3 neutron detection methods:

$$n + d \rightarrow t + \gamma + 6.25 \text{ MeV}$$
$$n + {}^{35}Cl \rightarrow {}^{36}Cl + \gamma + 8.6 \text{ MeV}$$
$$n + {}^{3}He \rightarrow p + t + 0.76 \text{ MeV}$$

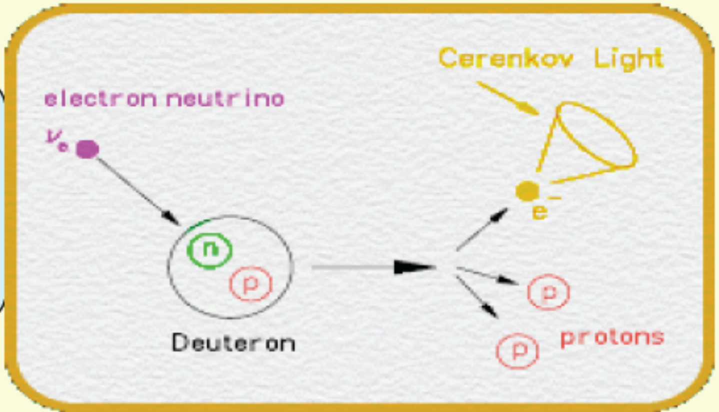
3 Phases:

- Just D_2O
- D_2O + 2 tonnes NaCl
- D_2O + ^{3}He Proportional Counters (“NCDs”)

Reactions in the SNO detector

Charged Current (CC)

$\nu_e + d \rightarrow p + p + e^-$



some directional info

$(1\text{-}1/3\cos\theta_{sun})$

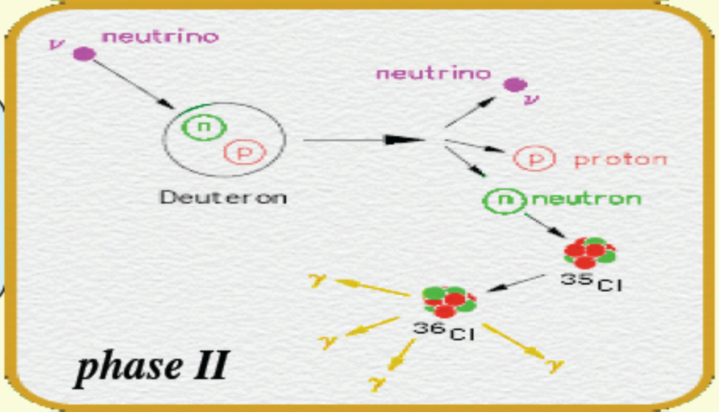
only sensitive to ν_e

good E_ν sensitivity

unique!

Neutral Current (NC)

$\nu_x + d \rightarrow \nu_x + p + n$



phase II

equally sensitive to all

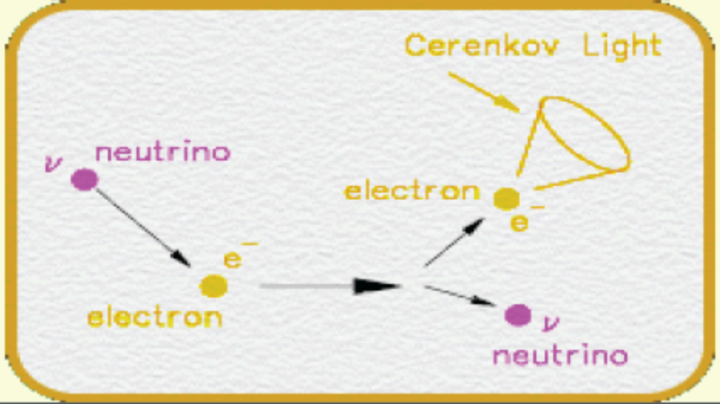
active flavours

detect neutron capture

Elastic Scattering

(ES)

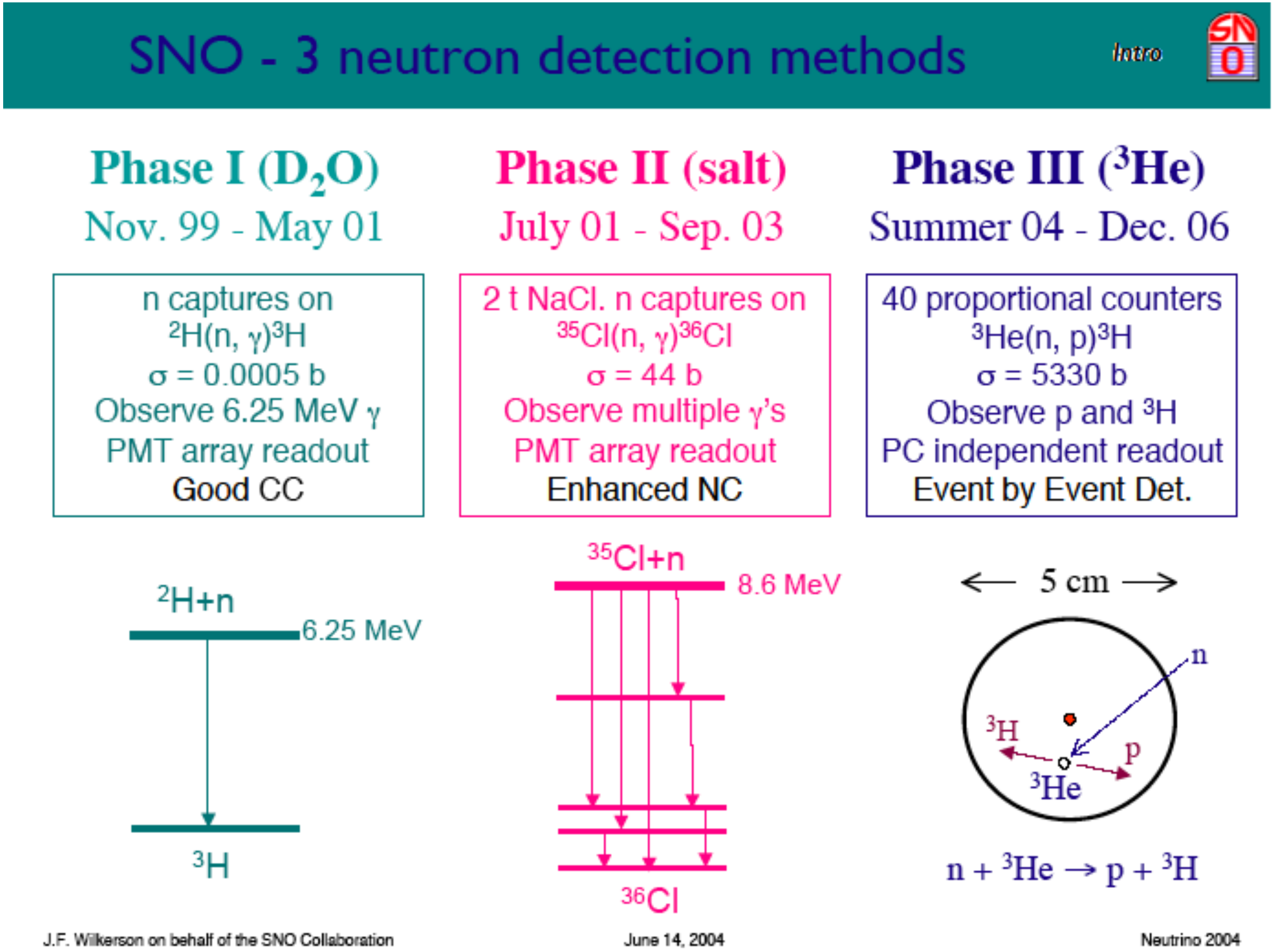
$\nu_x + e^- \rightarrow \nu_x + e^-$



directional sensitivity

mostly ν_e (factor 6.5)

smaller cross section



SNO - 3 neutron detection methods

Intro

Phase I (D_2O)	**Phase II (salt)**	**Phase III (^3He)**
Nov. 99 - May 01	July 01 - Sep. 03	Summer 04 - Dec. 06
n captures on $^2H(n, \gamma)^3H$ $\sigma = 0.0005$ b Observe 6.25 MeV γ PMT array readout Good CC	2 t NaCl. n captures on $^{35}Cl(n, \gamma)^{36}Cl$ $\sigma = 44$ b Observe multiple γ's PMT array readout Enhanced NC	40 proportional counters $^3He(n, p)^3H$ $\sigma = 5330$ b Observe p and ^3H PC independent readout Event by Event Det.

$$n + {}^3He \rightarrow p + {}^3H$$

J.F. Wilkerson on behalf of the SNO Collaboration

June 14, 2004

Neutrino 2004

391 days salt data – in numbers

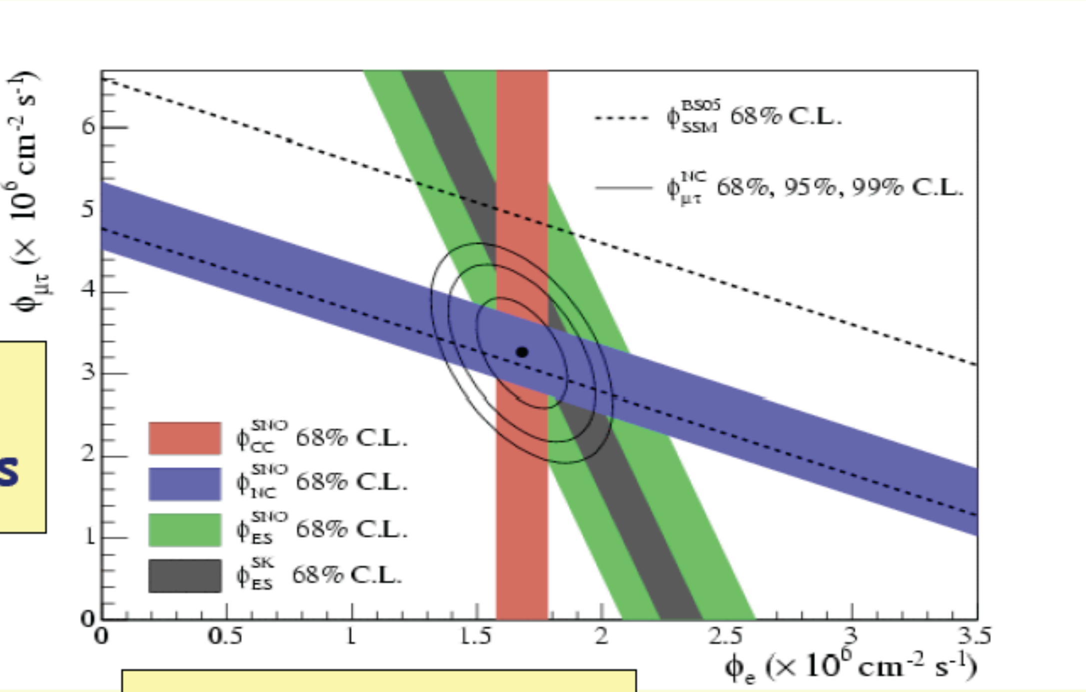
$$\phi_{CC} = 1.68 \; {}^{+0.06}_{-0.06}(\text{stat.})^{+0.08}_{-0.09}(\text{syst.})$$

$$\phi_{NC} = 4.94 \; {}^{+0.21}_{-0.21}(\text{stat.})^{+0.38}_{-0.34}(\text{syst.})$$

$$\phi_{ES} = 2.35 \; {}^{+0.22}_{-0.22}(\text{stat.})^{+0.15}_{-0.15}(\text{syst.})$$

$$\frac{\phi_{CC}}{\phi_{NC}} = 0.340 \pm 0.023(\text{stat.})^{+0.029}_{-0.031}$$

(In units of $10^6\,\text{cm}^{-2}\text{s}^{-1}$)



μ, τ neutrinos

electron ν

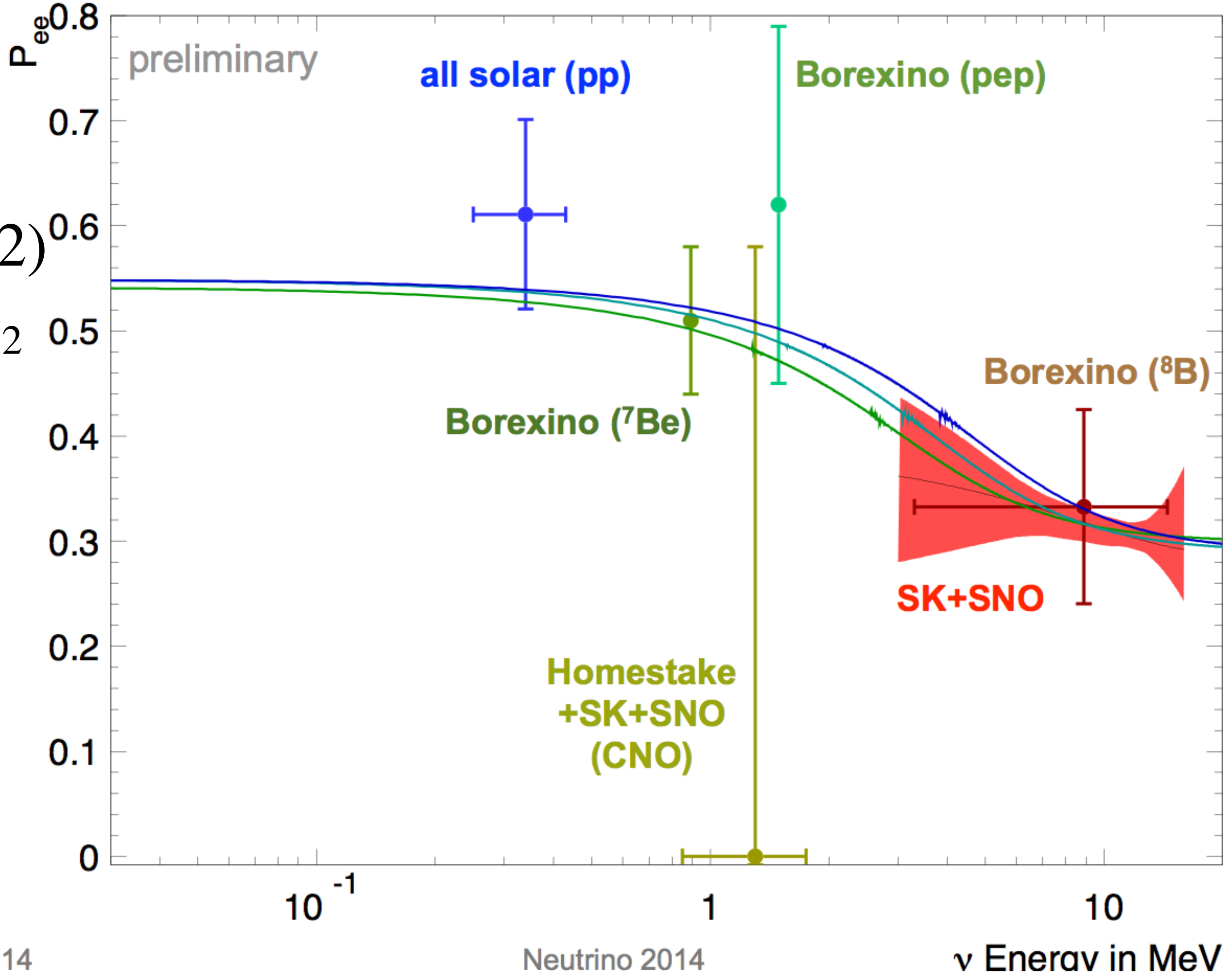
fluxes for all neutrinos

SNO solves the solar neutrino problem

Solar ν spectrum consistent with MSW

Allowed survival probability

Pee = 1-(1/2) $c_{13}^4 \sin^2 2\theta_{12}$

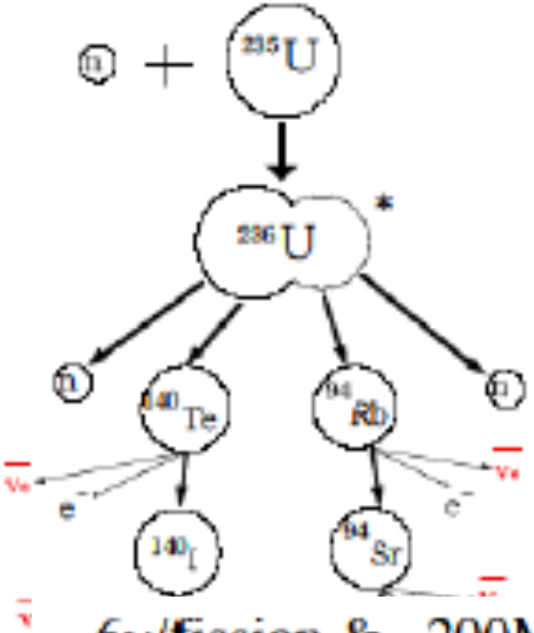


Pee = $\sin^2\theta_{12}$

Reactor is a rich source of neutrinos

Reactor ν and its detection

Reactor $\bar{\nu}_e$



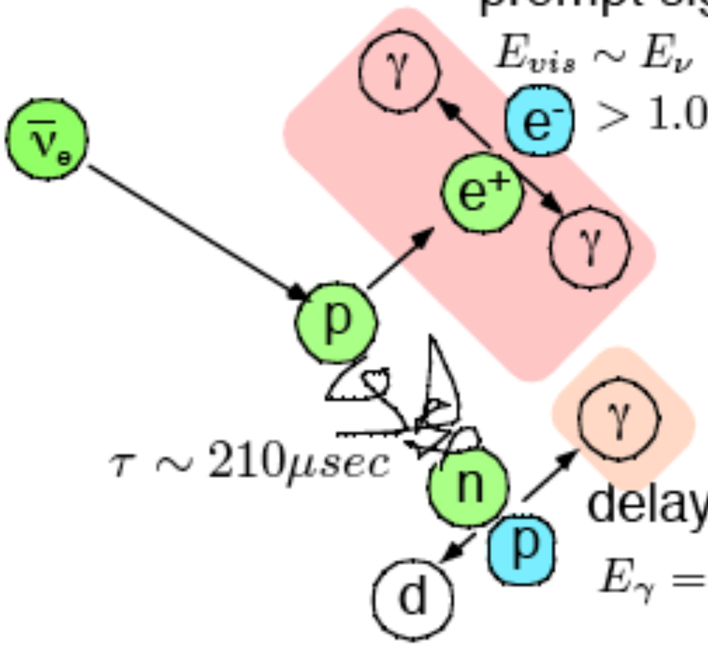
~6ν/fission & ~200MeV/fiss

⇓

$\sim 6\times10^{20}\,\bar{\nu}_e/s/reactor$ (1GWe)

$\bar{\nu}_e$ Detection

ν_e-bar+p->e⁺+n



prompt signal

$E_{vis} \sim E_\nu - 0.78MeV$

$> 1.022\,MeV$

$\tau \sim 210\mu sec$

delayed signal

$E_\gamma = 2.22MeV$

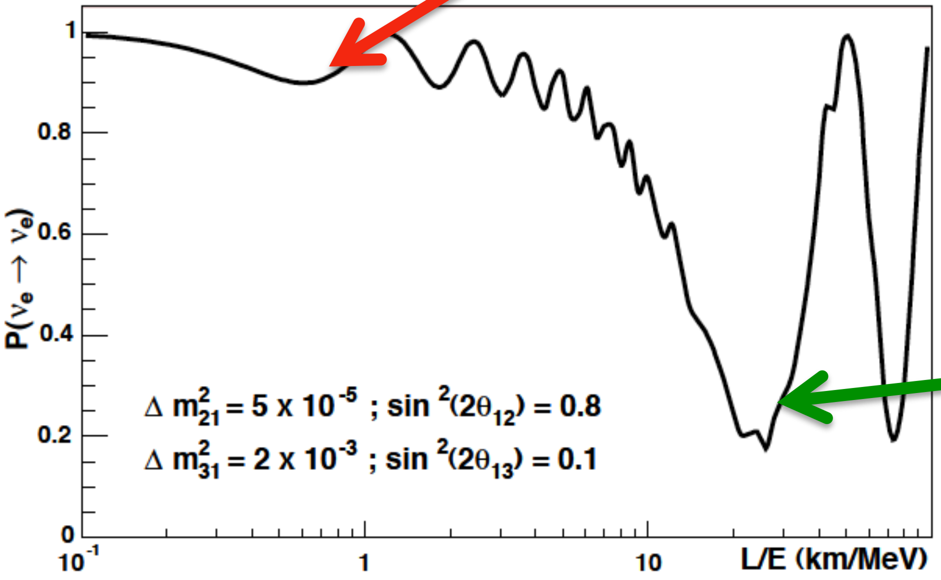
Used by Reines-Cowan to observe ν_e-bar for the first time

2 different regimes of reactor neutrino oscillation

ν_e disappearance probability

$$1 - P(\bar{\nu}_e \to \bar{\nu}_e) = \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right) + O(\epsilon s^2_{13}) + O(\epsilon^2)$$

$$\epsilon \equiv \Delta m^2_{21}/\Delta m^2_{31} \simeq 0.03$$



$$P(\bar{\nu}_e \to \bar{\nu}_e) \approx 1 - \frac{1}{2}\sin^2 2\theta_{13} - c^4_{13} \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right)$$

Figure 3: Probability of ν_e disappearance versus L/E for θ_{13} at its current upper limit

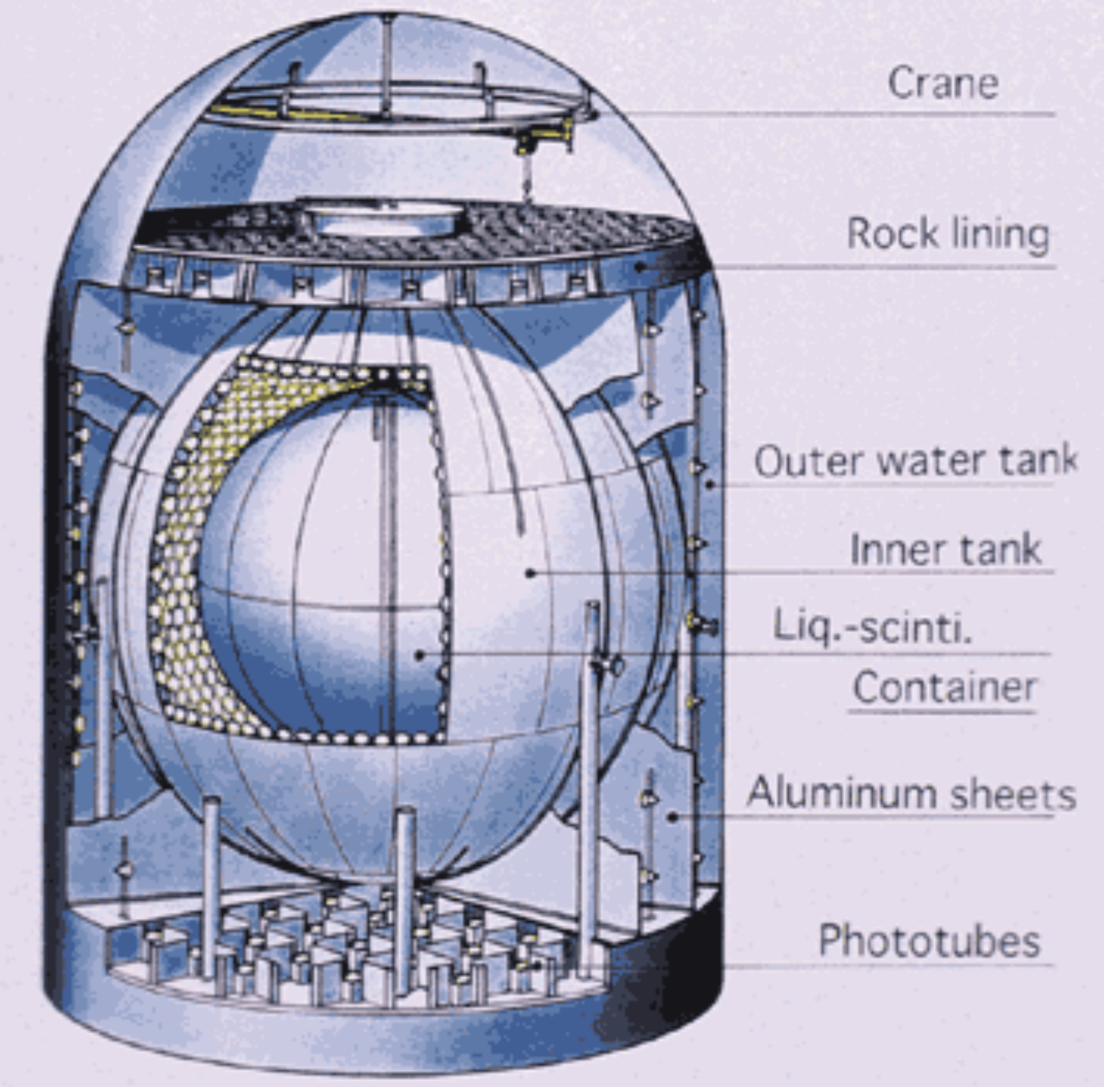
With which baseline L neutrinos oscillate?

$$\frac{\Delta m^2 L}{4E} = 1.27\left(\frac{\Delta m^2}{10^{-3}\mathrm{eV}^2}\right)\left(\frac{L}{1000\mathrm{km}}\right)\left(\frac{E}{1\mathrm{GeV}}\right)^{-1} \quad \text{atmospheric, accelerator } \nu$$

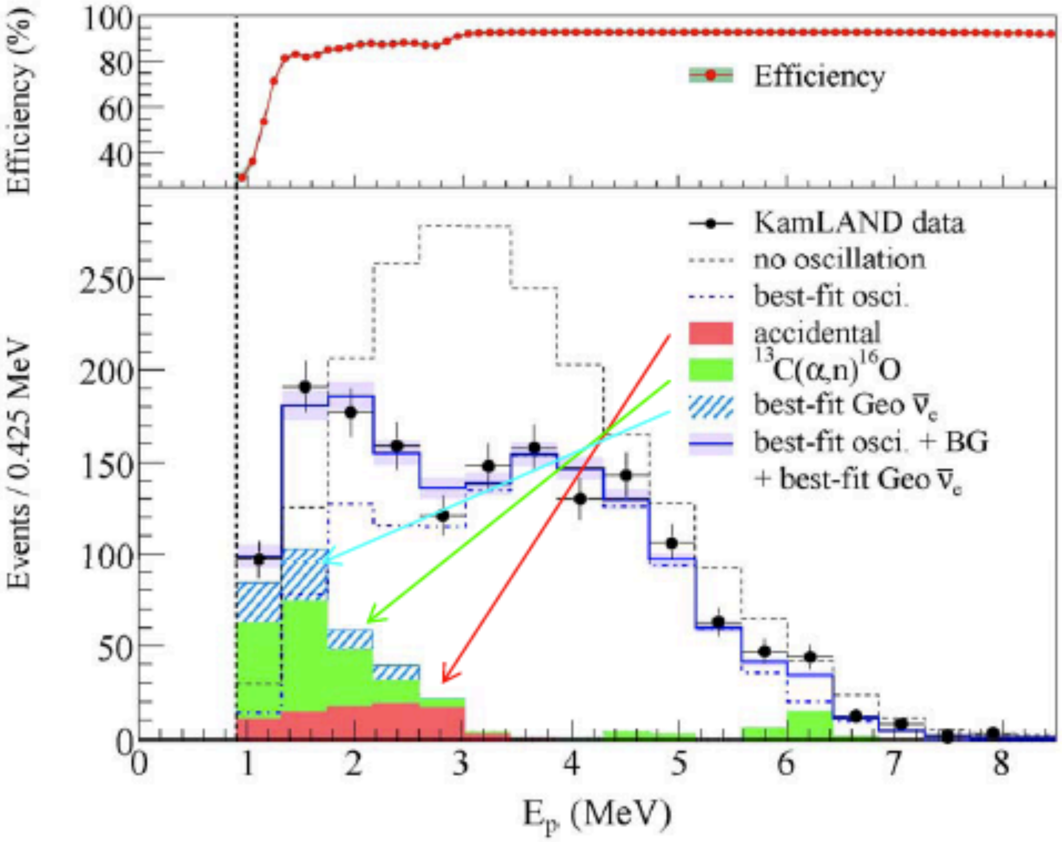
$$= 1.27\left(\frac{\Delta m^2}{10^{-3}\mathrm{eV}^2}\right)\left(\frac{L}{1\mathrm{km}}\right)\left(\frac{E}{1\mathrm{MeV}}\right)^{-1} \quad \text{reactor neutrino: short baseline}$$

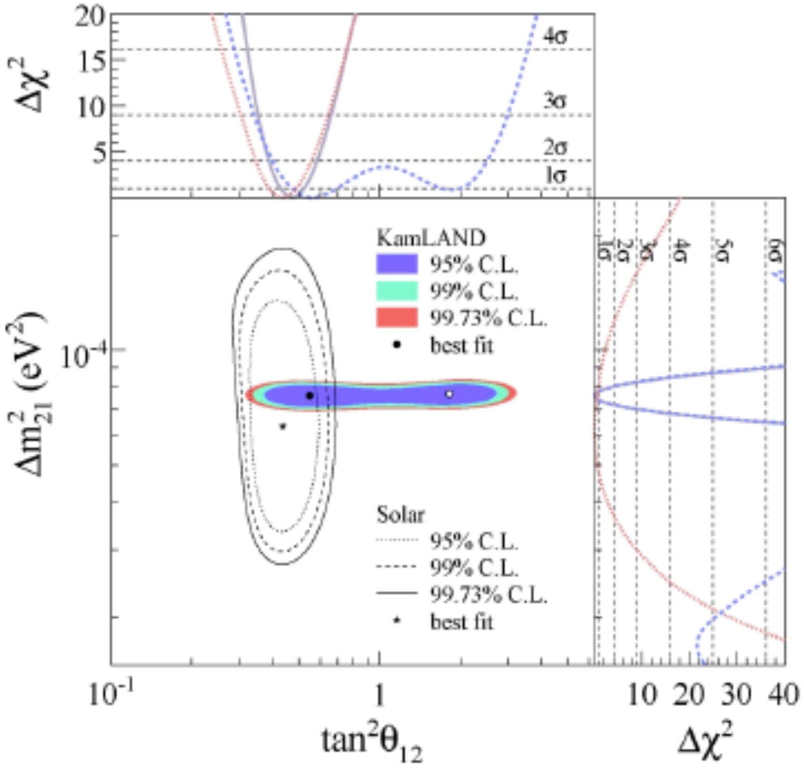
$$= 1.27\left(\frac{\Delta m^2}{10^{-5}\mathrm{eV}^2}\right)\left(\frac{L}{100\mathrm{km}}\right)\left(\frac{E}{1\mathrm{MeV}}\right)^{-1} \quad \text{reactor neutrino: long baseline}$$

KamLAND @Kamioka mine



KamLAND: Modulation of energy spectrum



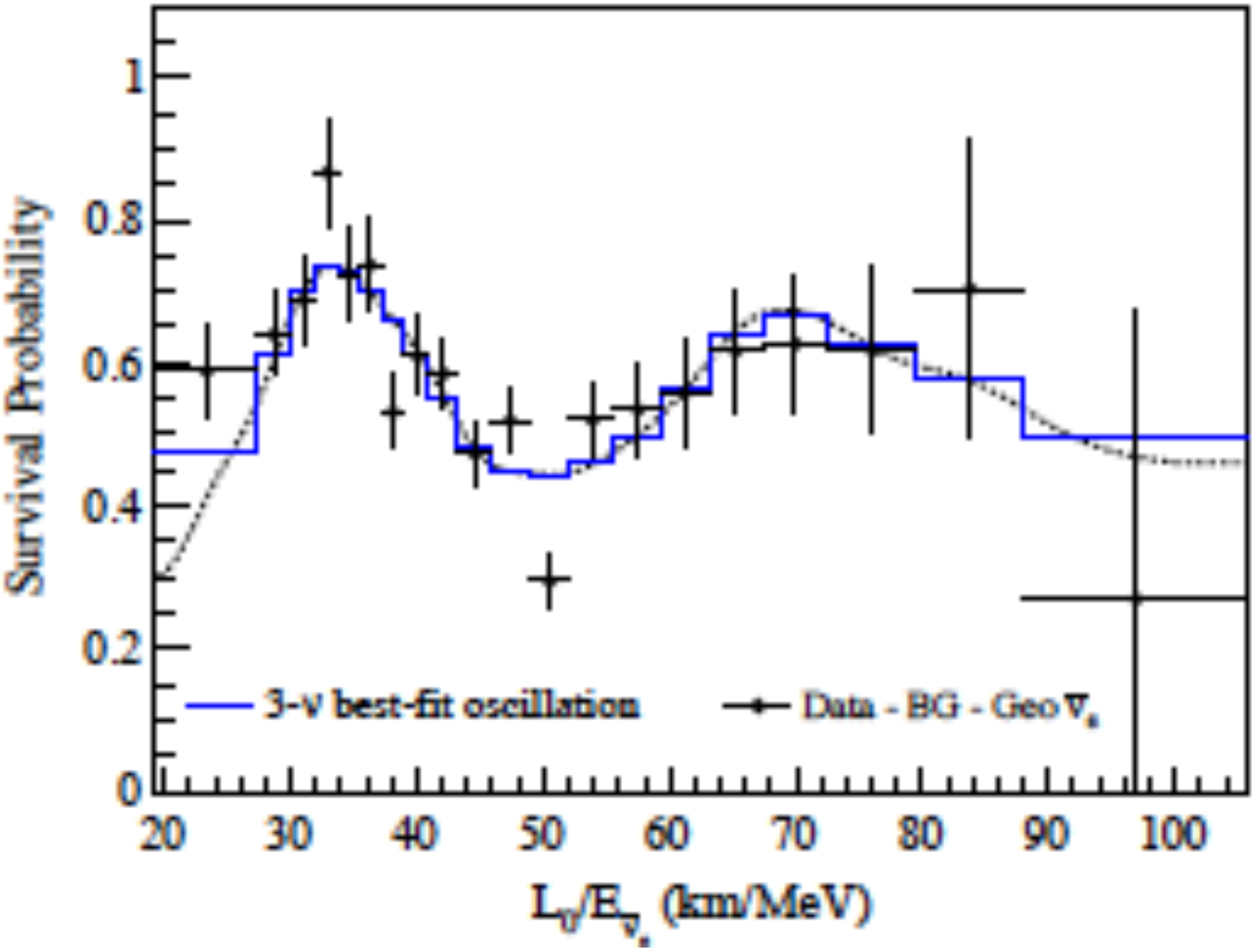


Fit to scaled no-oscillation spectrum : exclude at 5.1 σ

$$\Delta m^2 = 7.58^{+0.21}_{-0.20} \times 10^{-5}\text{eV}^2$$

$$\tan^2\theta = 0.56^{+0.14}_{-0.09}$$

The cleanest evidence for neutrino OSCILLATION

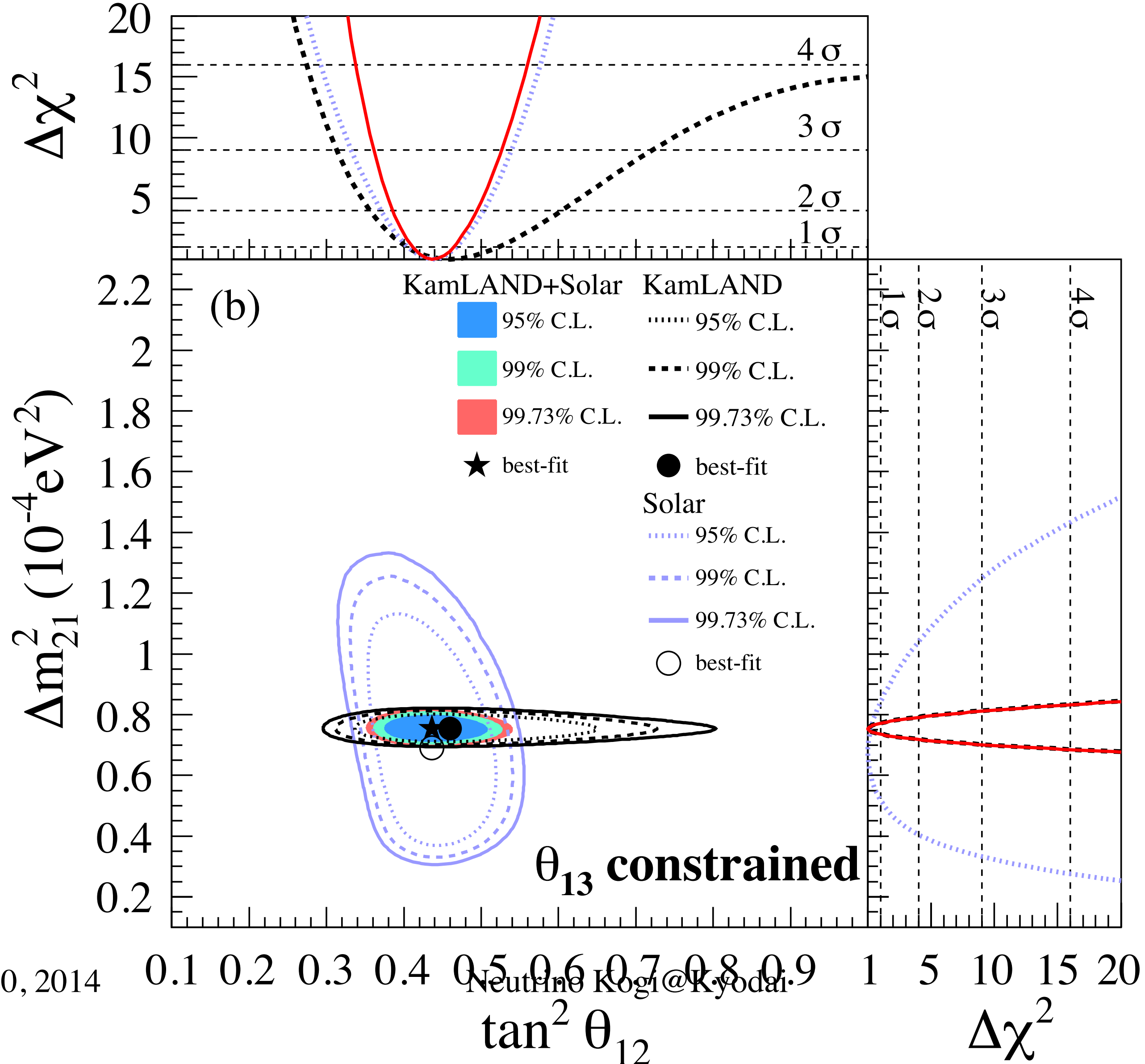


Ju

SNO(+SK) and KamLAND solved the solar neutrino problem

Determination of θ_{12}, Δm^2_{21}

$$\tan^2\theta_{12} = 0.436^{+0.029}_{-0.025},\ \Delta m^2_{21} = 7.53^{+0.18}_{-0.18} \times 10^{-5}\,\mathrm{eV}^2,$$



θ_{12} and θ_{13}

$$\tan^2\theta_{12} = 0.436^{+0.029}_{-0.025},\ \Delta m^2_{21} = 7.53^{+0.18}_{-0.18} \times 10^{-5}\,\mathrm{eV}^2,$$

KamLAND+solar Mar.2013

- $\sin^2\theta_{12} = 0.304$ +/- 0.013
- Error of $\sin^2\theta_{12} = 4.3\%$
- Error of $\Delta m^2_{21} = 2.4\%$

Daya Bay Nu2014

$$\sin^2 2\theta_{13} = 0.084^{+0.005}_{-0.005}$$

$$|\Delta m^2_{ee}| = 2.44^{+0.10}_{-0.11} \times 10^{-3}\mathrm{eV}^2$$

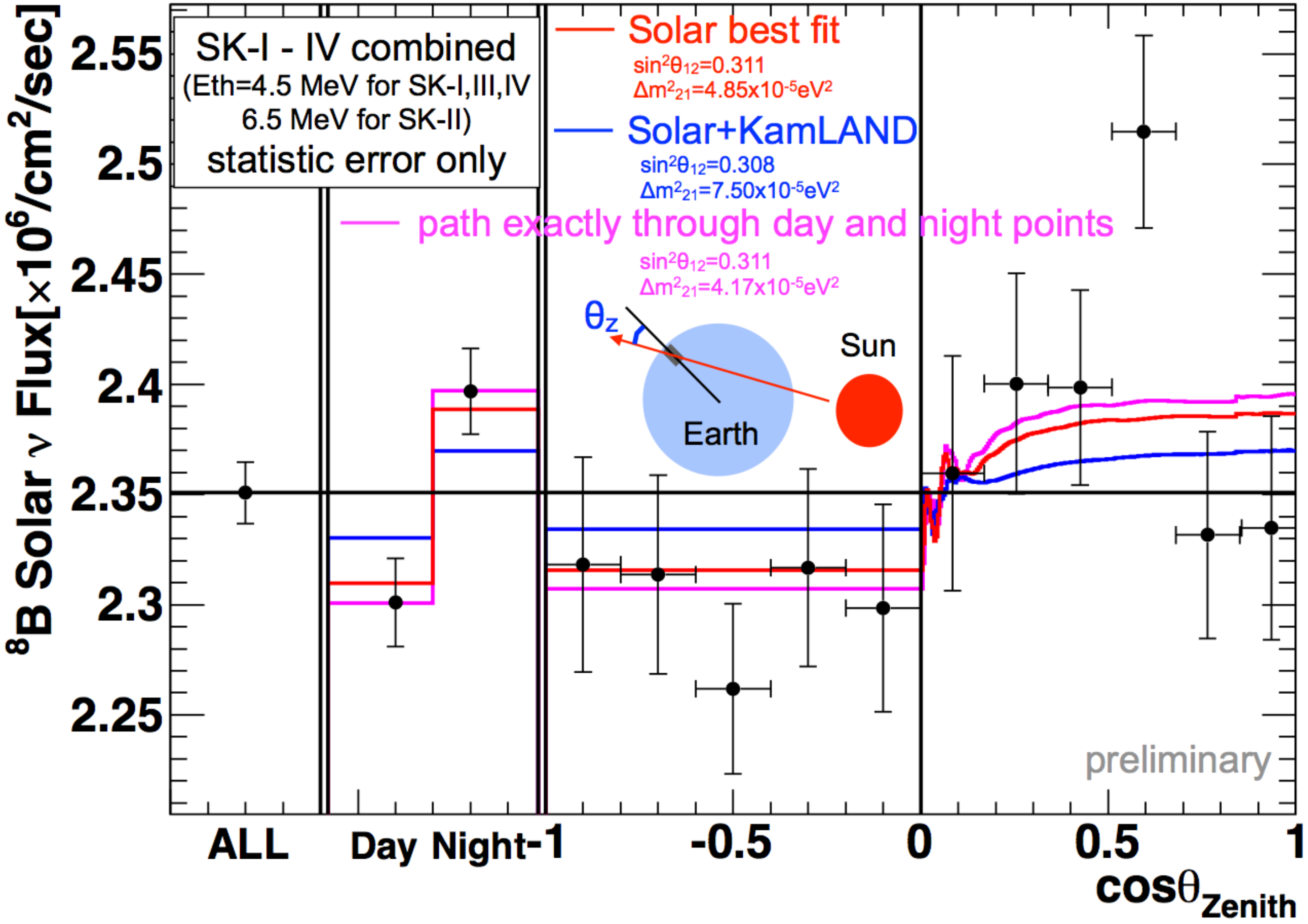
- Error of $\sin^2\theta_{13}$ (Daya Bay) = 6.1%
- Error of Δm^2_{31} (Daya Bay) ~= 4% !!
- Error of Δm^2_{31} (MINOS) ~= 4%

What's new in 12 sector?

Day-night variation seen! (SK)

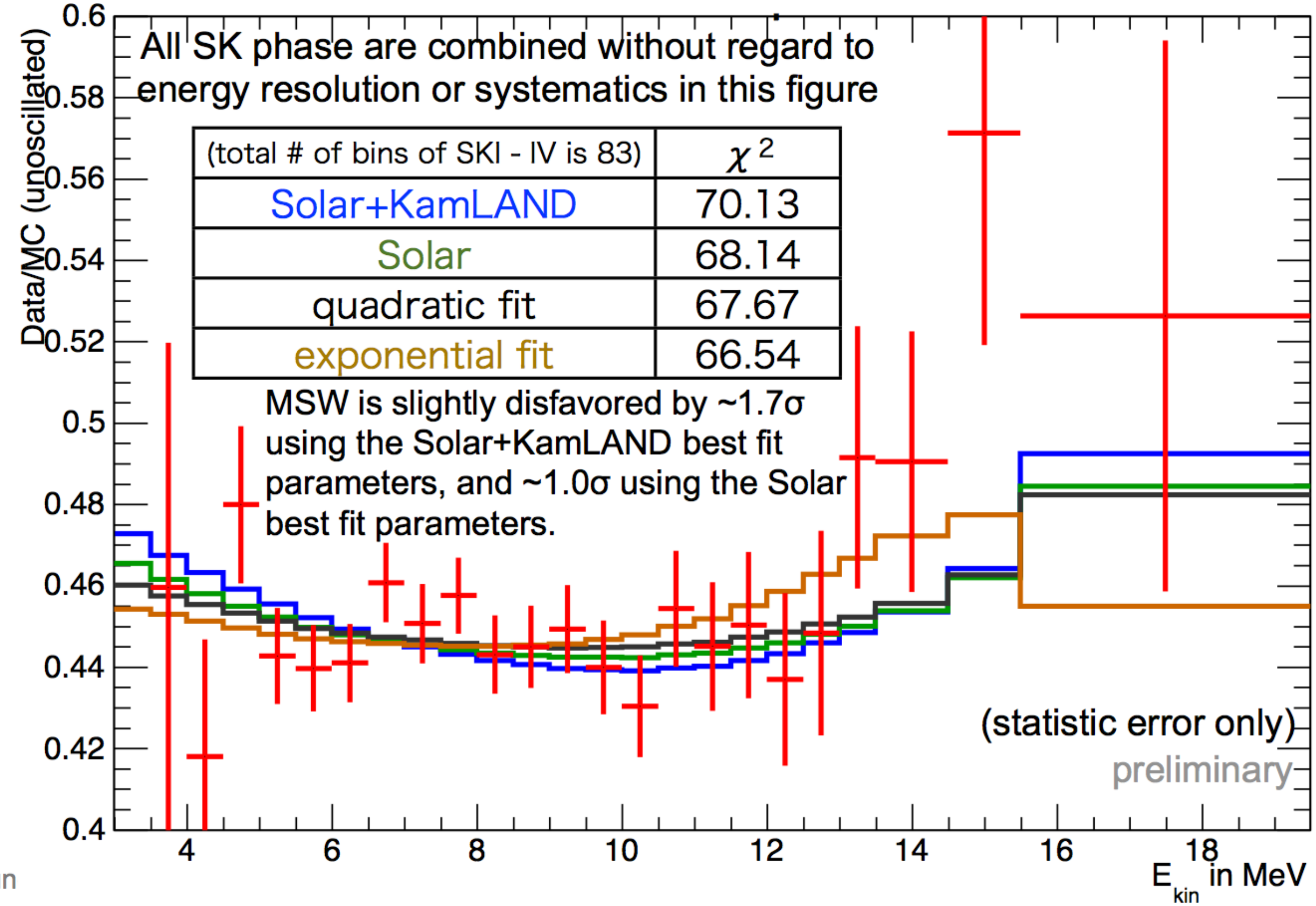
Koshio Nu2014

Zenith angle distribution



Spectrum upturn seen though still at 1σ level (SK)

Recoil electron spectrum

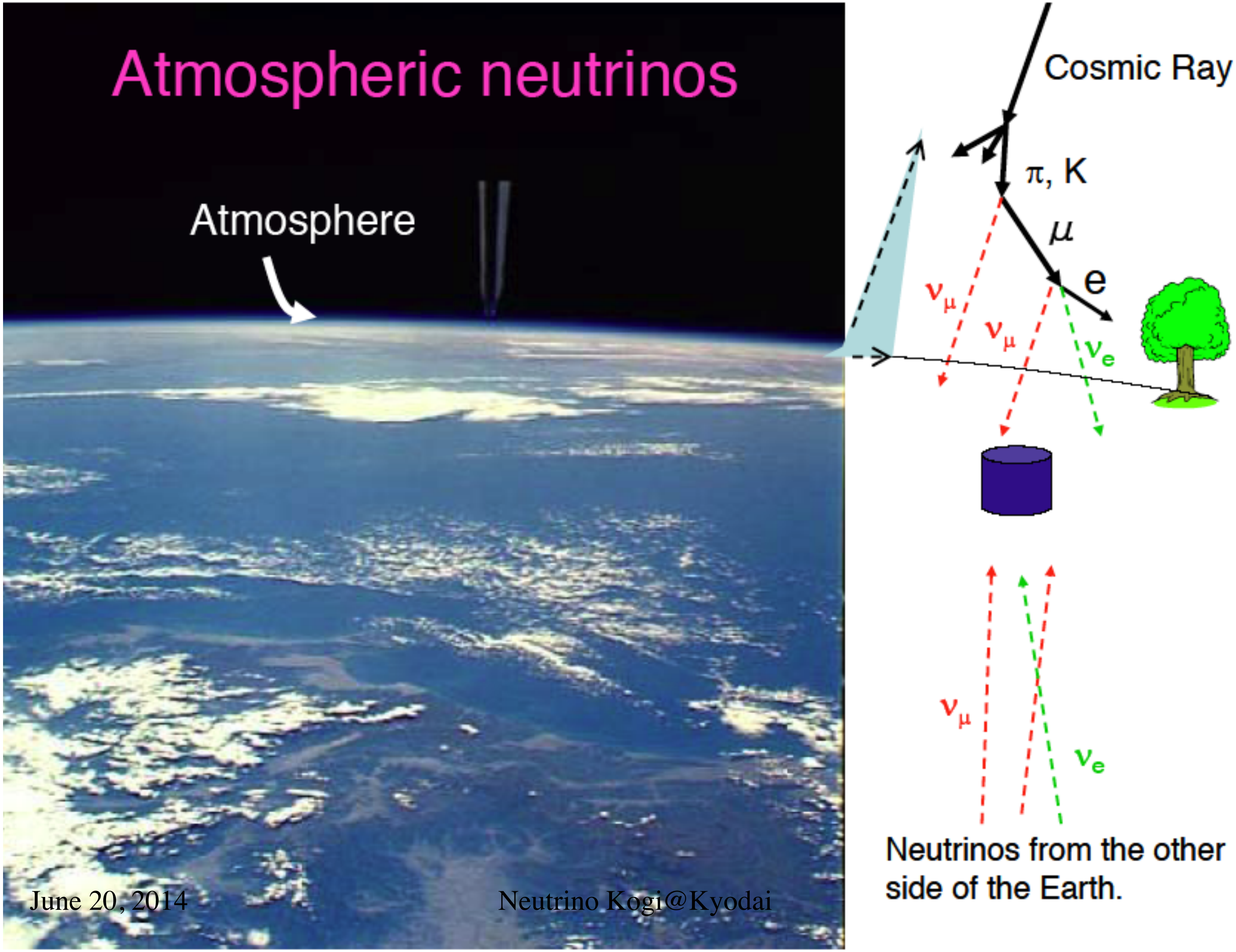


All SK phase are combined without regard to energy resolution or systematics in this figure

(total # of bins of SKI - IV is 83)	χ^2
Solar+KamLAND	70.13
Solar	68.14
quadratic fit	67.67
exponential fit	66.54

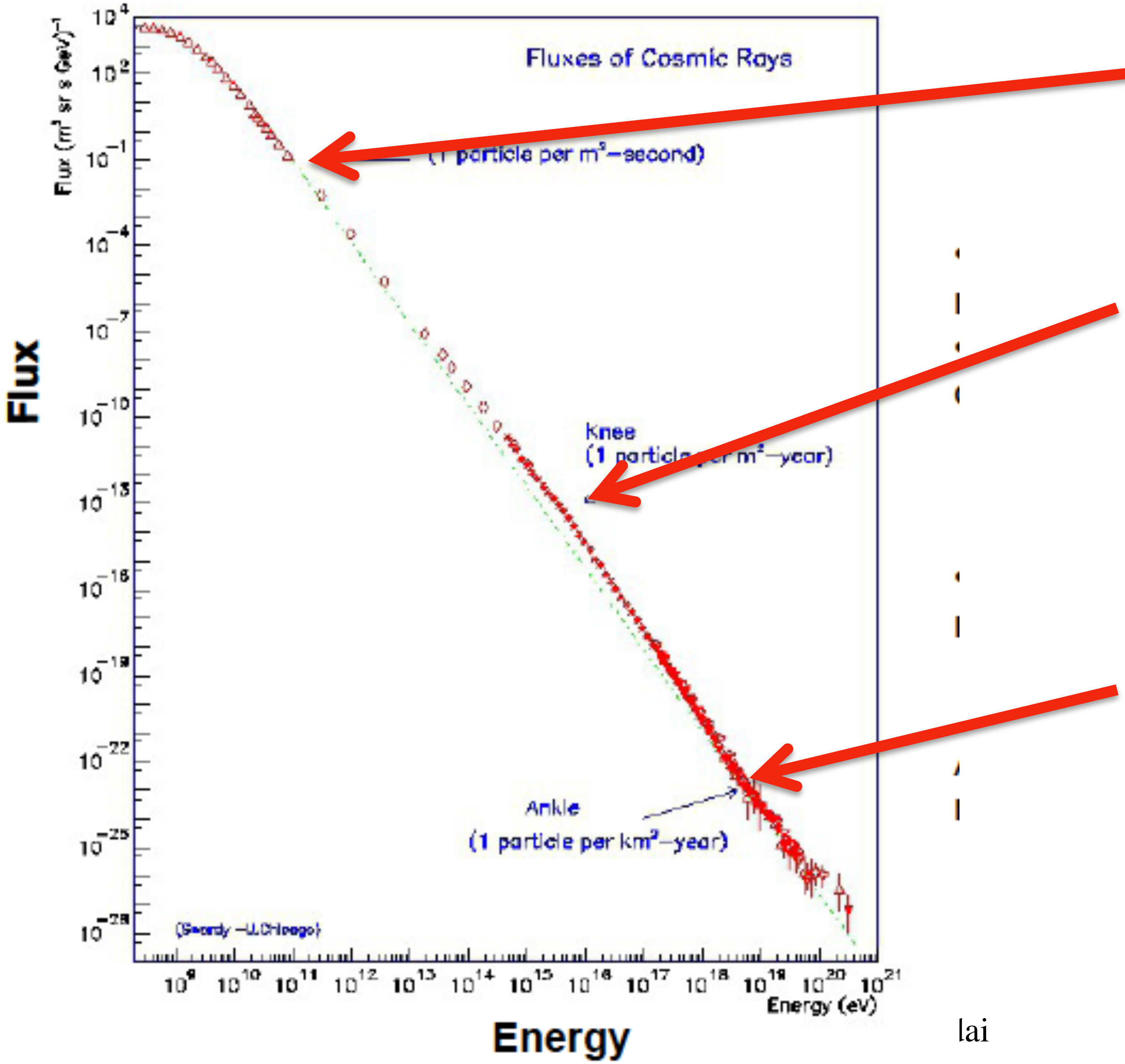
MSW is slightly disfavored by ~1.7σ using the Solar+KamLAND best fit parameters, and ~1.0σ using the Solar best fit parameters.

Cosmic rays are arriving at the Earth



Neutrinos from the other side of the Earth.

Cosmic ray energy spectrum



1 particle / m^2 sec

1 particle / m^2 year

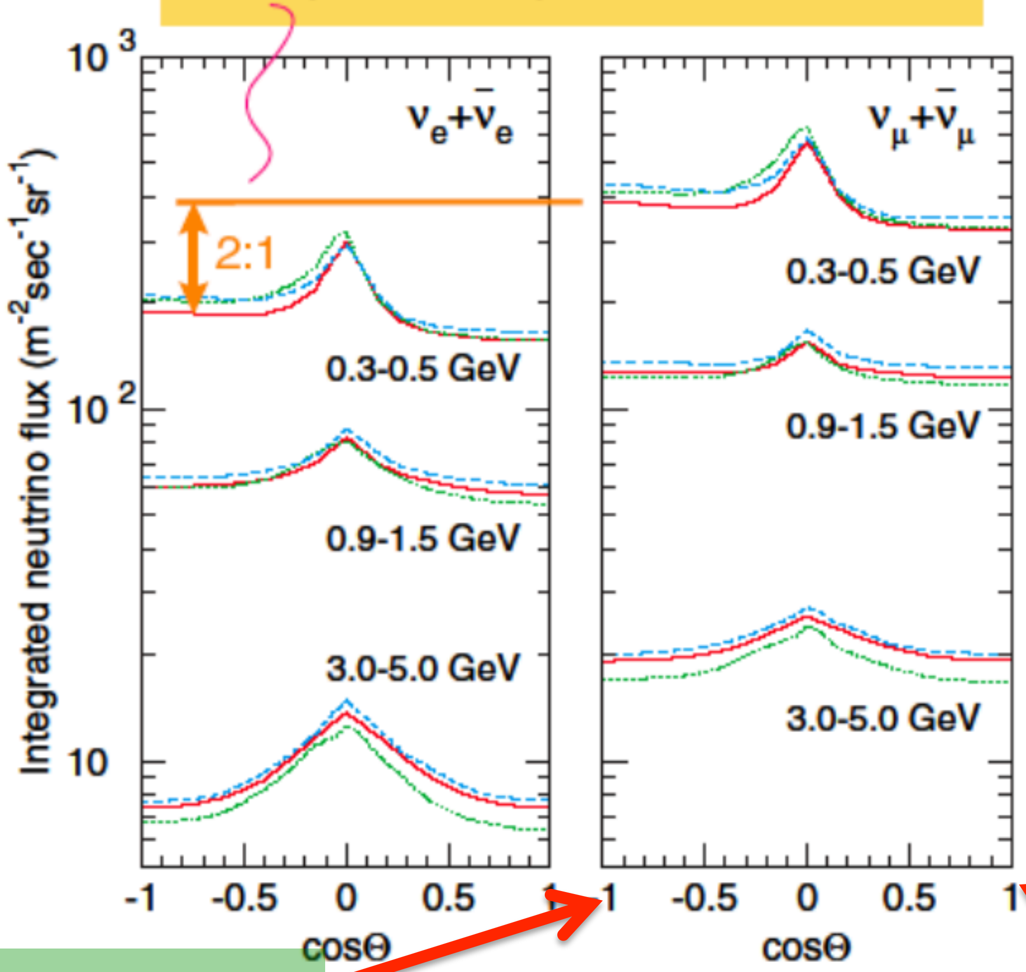
1 particle / km^2 year

lai

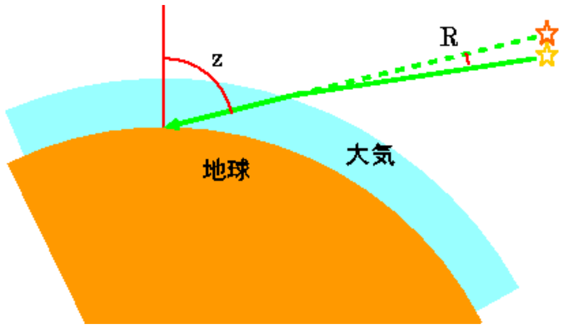
J. Cronin, T.K. Gaisser, and S.P. Swordy, Sci. Amer. v276, p44 (1997)

Zenith angle dependence of atmospheric neutrinos

Mixed beam of ν_e, $\bar{\nu}_e$, ν_μ, and $\bar{\nu}_\mu$
Well predicted flux ratio



What is zenith angle?



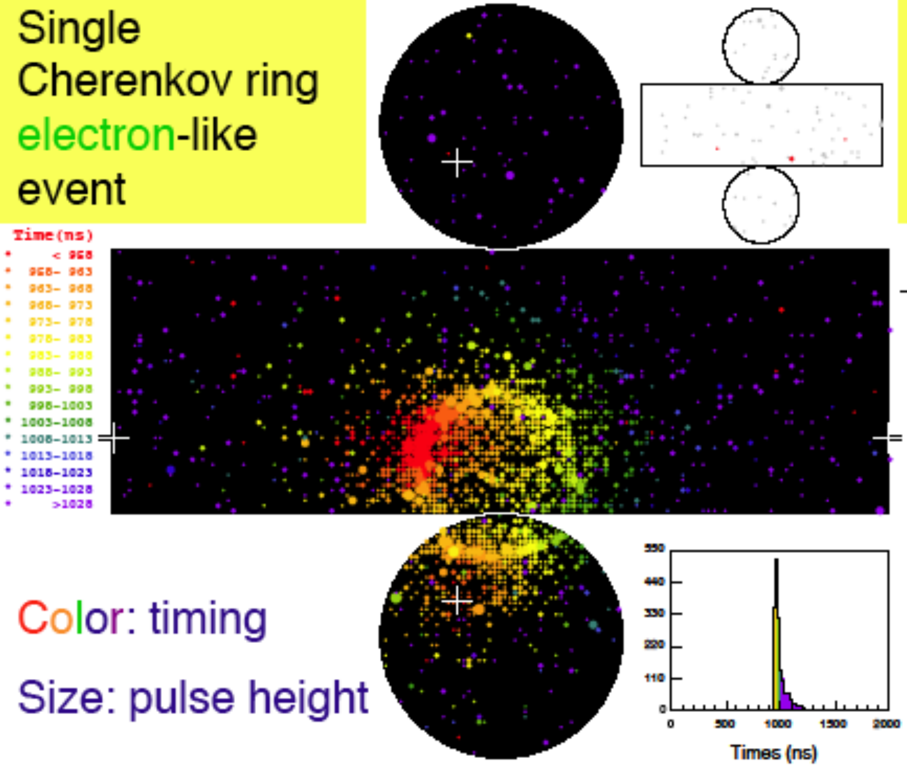
Upward going

Downward going

up-down symmetry at high energy

Particle identification

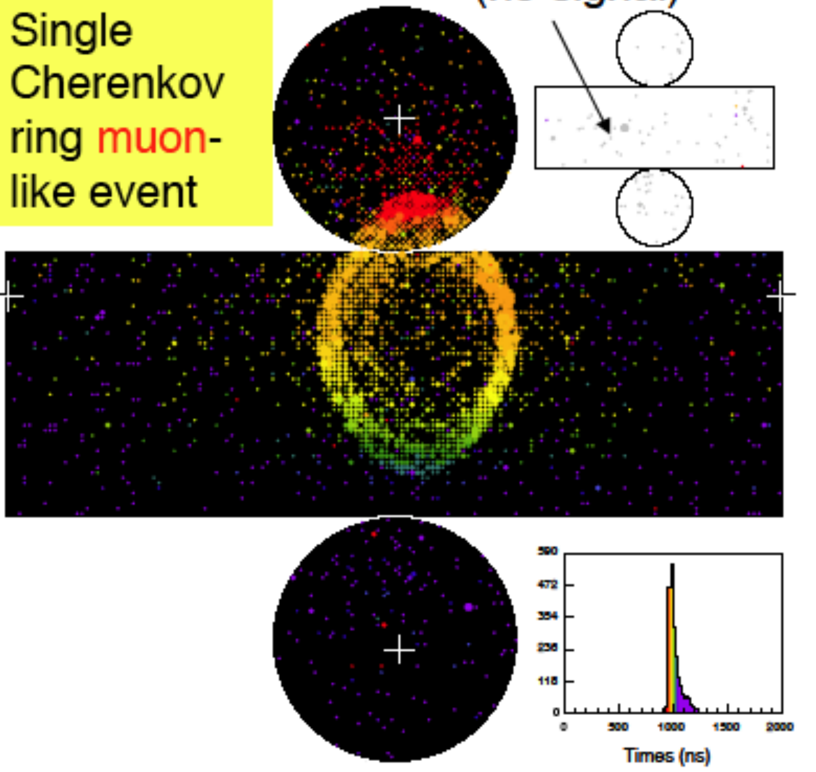
Single Cherenkov ring electron-like event



Color: timing

Size: pulse height

Single Cherenkov ring muon-like event

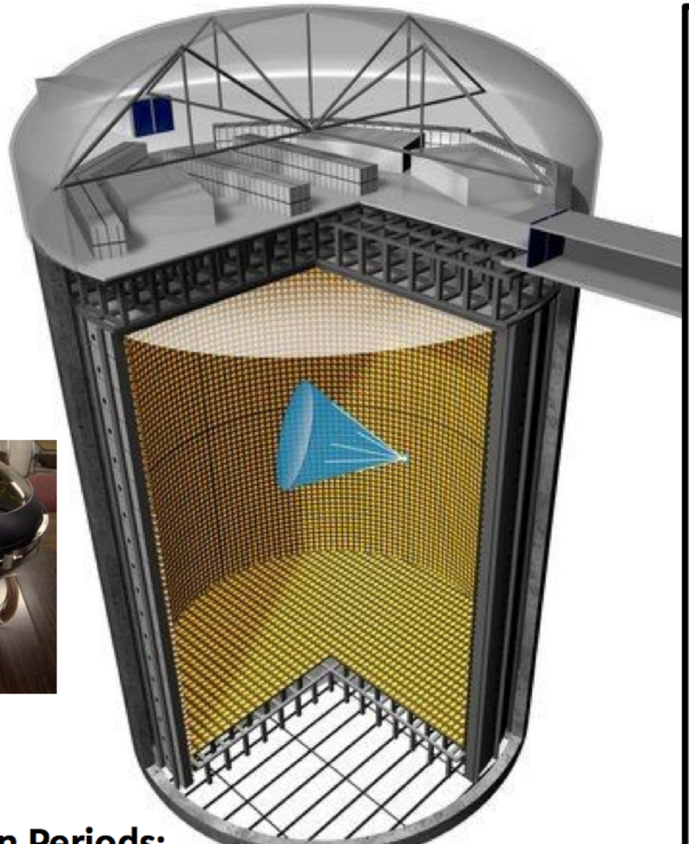


Outer detector (no signal)

Particle ID

$$\log(L) = \sum_{\theta<70\,\mathrm{deg}} \left(\frac{p.e.(obs'd) - p.e._{e\ or\ \mu}(expected)}{\sigma_{p.e.}} \right)^2$$

Super-Kamiokande: Introduction



Four Run Periods:
SK-I (1996-2001) SK-II (2003-2005)
SK-III (2005-2008) SK-IV (2008-Present)

Dinucleon Decay Search
Poster#157 J. Gustafson
Trilepton Decay Search
Poster #216, V.Takhistov

- 22.5 kton fiducial volume
- Optically separated into
  - Inner Detector 11,146 20” PMTs
  - Outer Detector 1885 8” PMTs

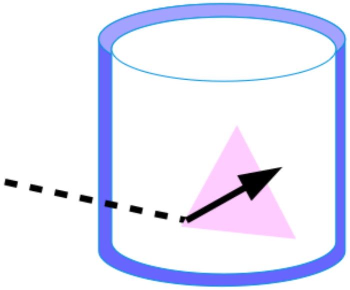
- No net electric or magnetic fields
- Excellent PID between showering (e-like) and non-showering (m-like)
  - < 1% MIS ID at 1 GeV

- Today: 4581 days of atmospheric neutrino data
  - 40,000 Events
  - Statistics limited

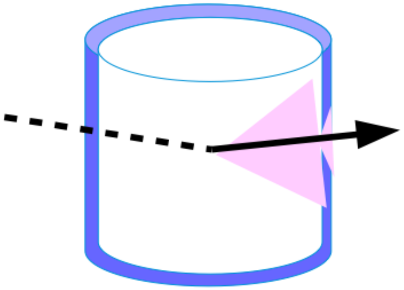
- Multipurpose machine
  - Solar and Supernova Neutrinos
  - **Atmospheric Neutrinos (this talk)**
  - Nucleon Decay
  - Far detector for T2K

Super-K Atmospheric ν Event Topologies

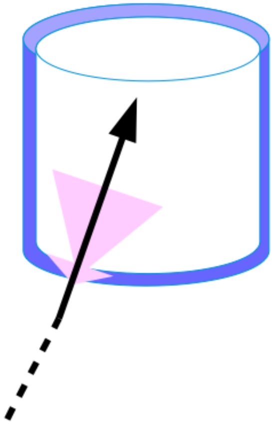
Fully Contained (FC)

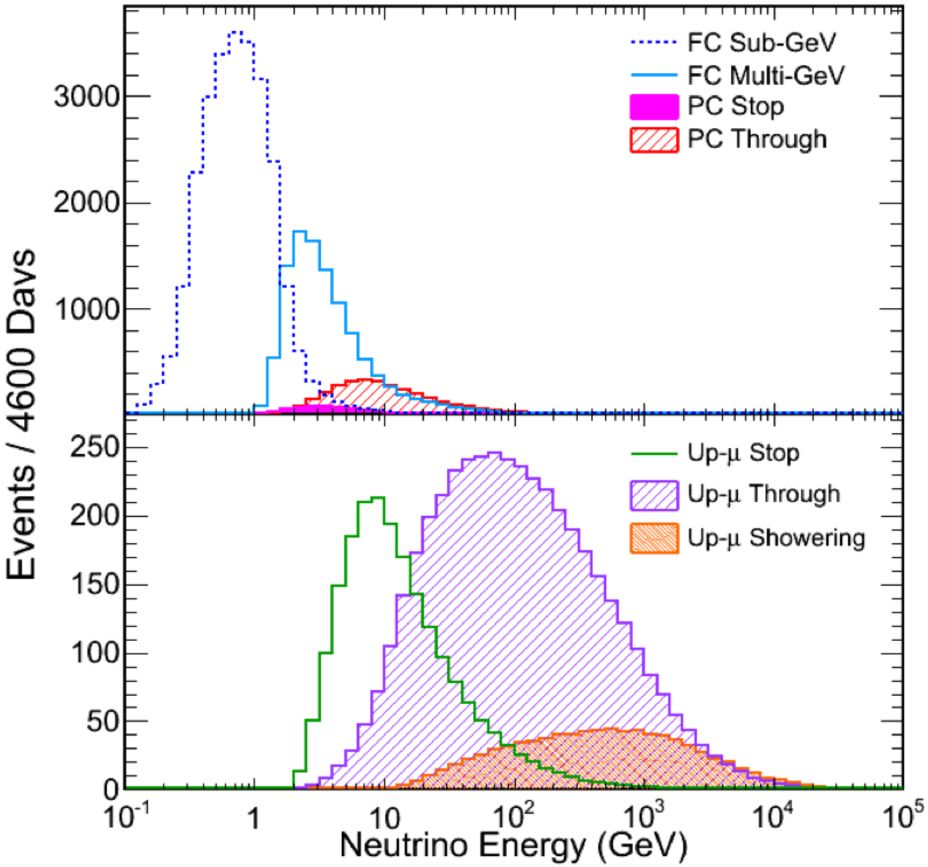


Partially Contained (PC)



Upward-going Muons (Up-μ)



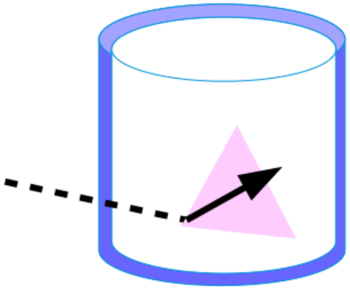


- Average energies
  - FC: ~1 GeV , PC: ~10 GeV, UpMu:~ 100 GeV

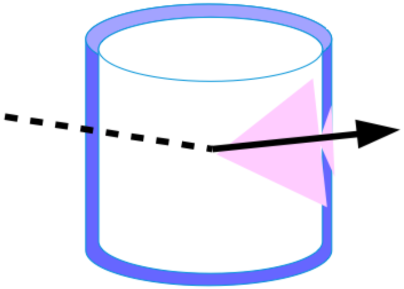
R.Wendell Nu2014

Super-K Atmospheric ν Analysis Samples

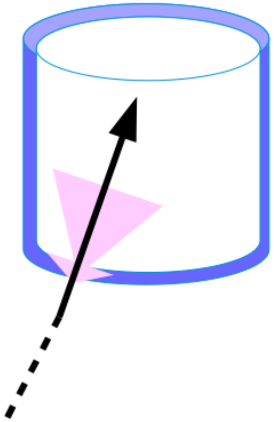
Fully Contained (FC)

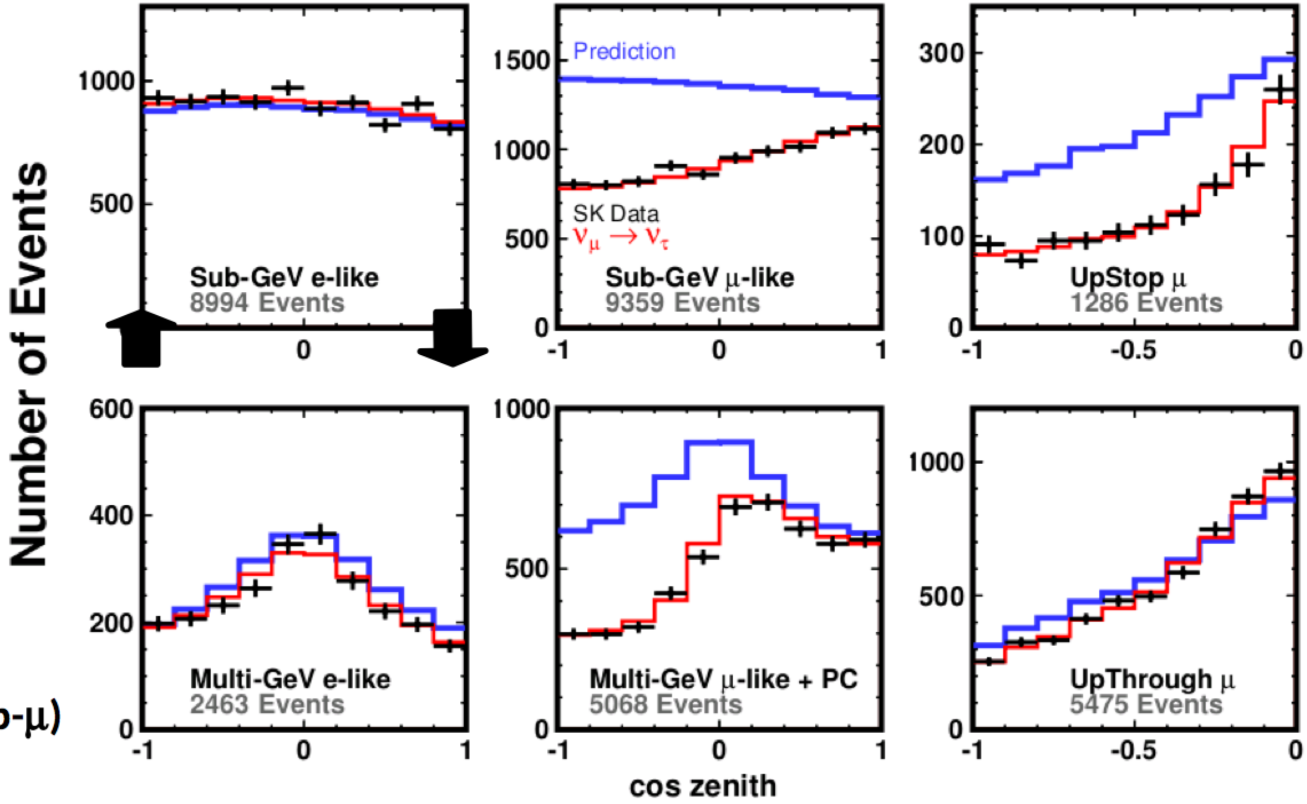


Partially Contained (PC)



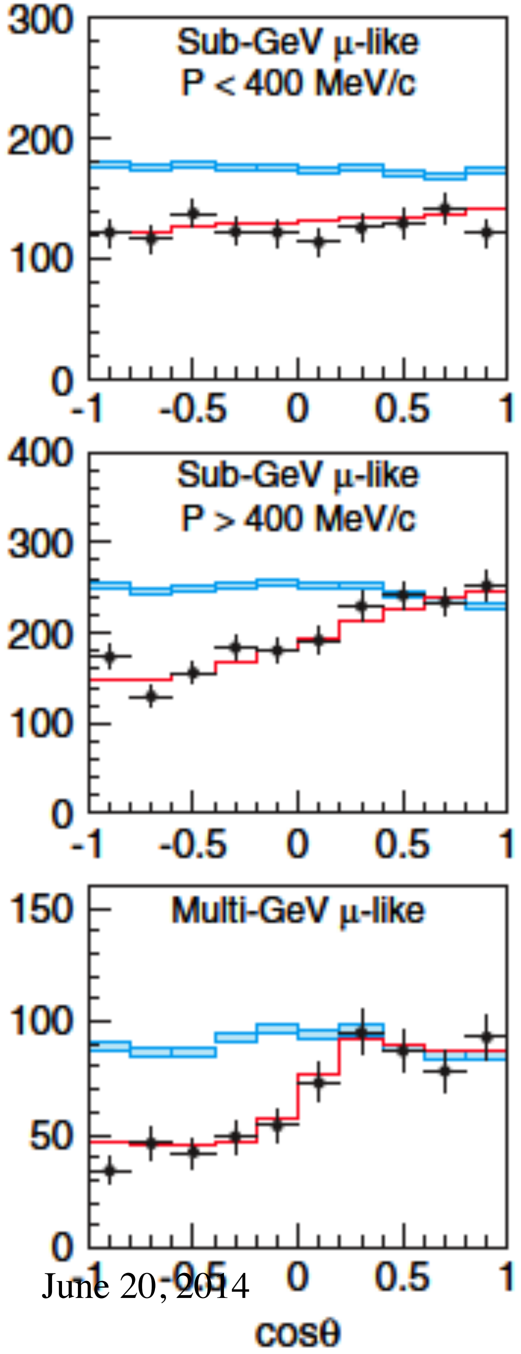
Upward-going Muons (Up-μ)





- In total **19** analysis samples: multi-GeV e-like samples are divided into ν-like and $\bar{\nu}$-like subsamples
- Dominated by ν_μ->ν_τ oscillations
- Interested in subdominant contributions to this picture
  - Ie three-flavor effects, Sterile Neutrinos, LIV, etc.

In a nutshell



Focus in on μ-like events

- Pay attention to the high energy (p > 400 MeV) events
- Downward-going events agrees with Monte Carlo
- But, a large discrepancy exists in upward-going events = neutrinos from the other side of the earth (from Brazil!)
- Neutrino oscillation takes place with oscillation length of ~1000 km

With which baseline L neutrinos oscillate?

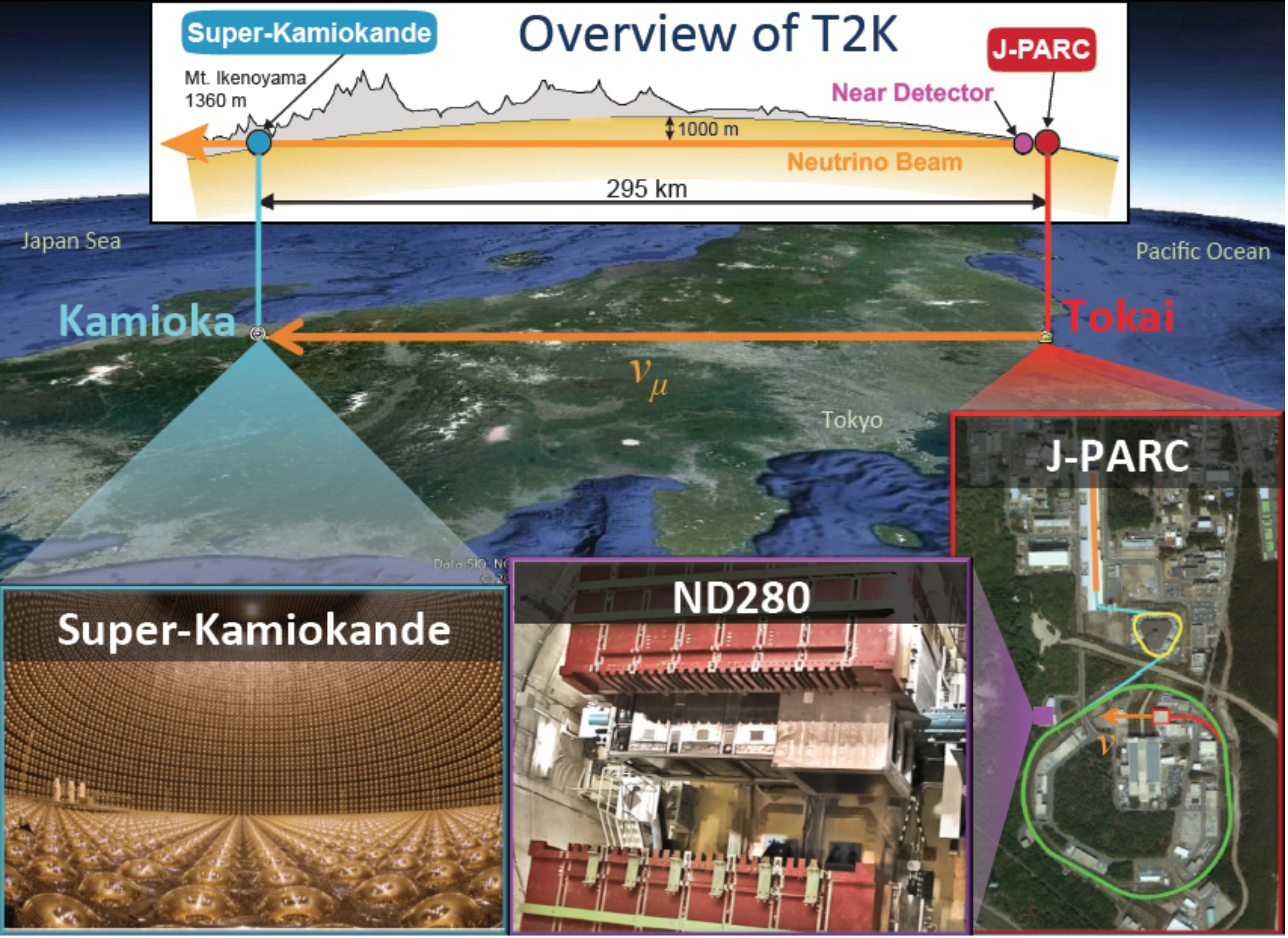
$$\frac{\Delta m^2 L}{4E} = 1.27\left(\frac{\Delta m^2}{10^{-3}\mathrm{eV}^2}\right)\left(\frac{L}{1000\mathrm{km}}\right)\left(\frac{E}{1\mathrm{GeV}}\right)^{-1} \quad \text{atmospheric, accelerator } \nu$$

$$= 1.27\left(\frac{\Delta m^2}{10^{-3}\mathrm{eV}^2}\right)\left(\frac{L}{1\mathrm{km}}\right)\left(\frac{E}{1\mathrm{MeV}}\right)^{-1} \quad \text{reactor neutrino: short baseline}$$

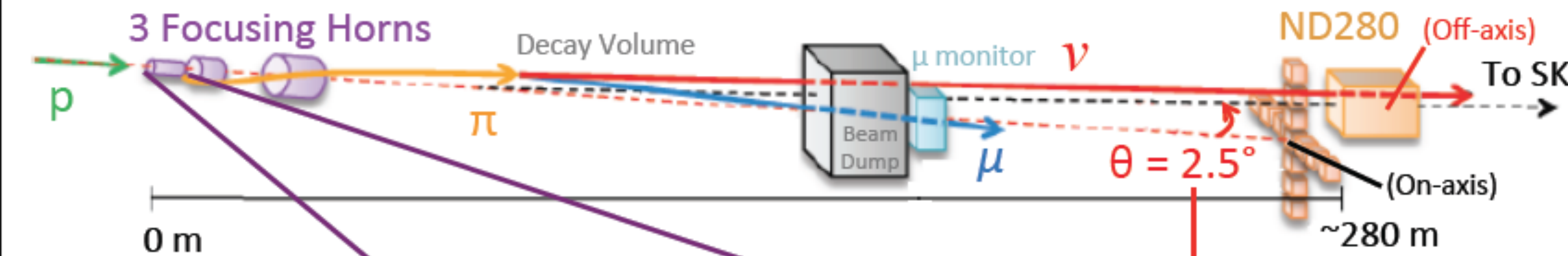
$$= 1.27\left(\frac{\Delta m^2}{10^{-5}\mathrm{eV}^2}\right)\left(\frac{L}{100\mathrm{km}}\right)\left(\frac{E}{1\mathrm{MeV}}\right)^{-1} \quad \text{reactor neutrino: long baseline}$$

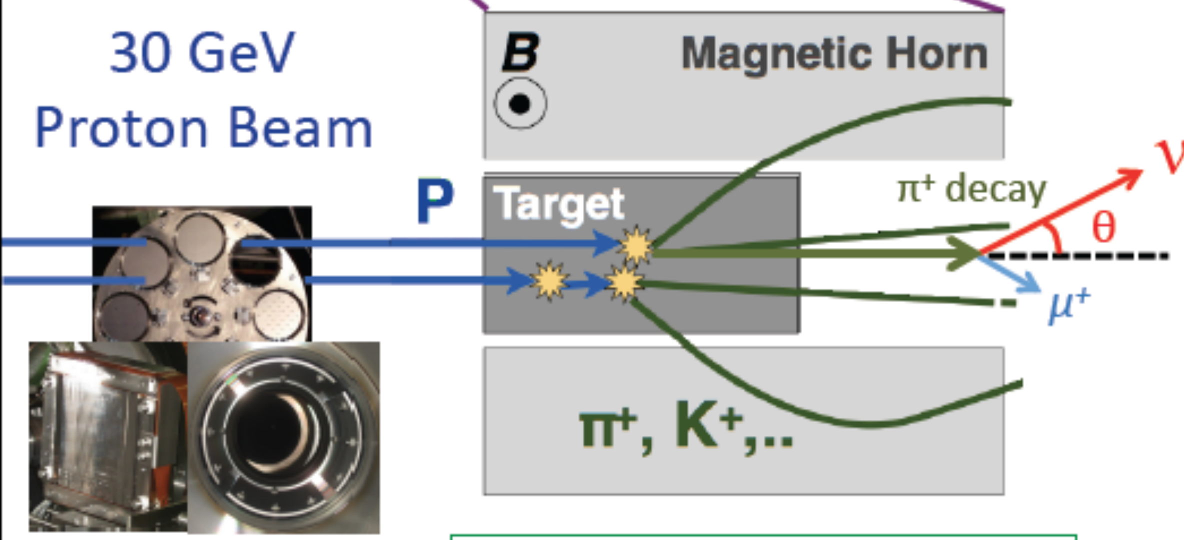
Accelerator neutrinos: Tokai-to-Kamioka (T2K)

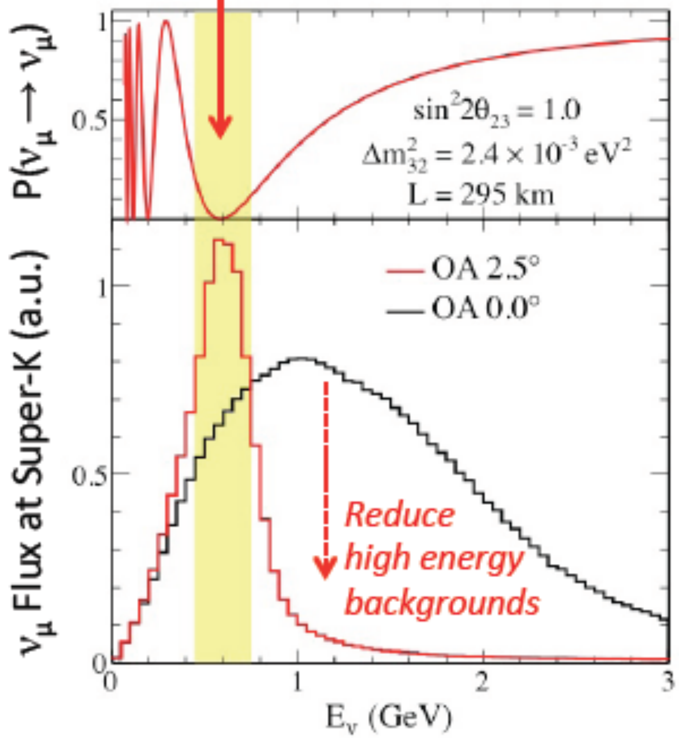
Overview of T2K



Neutrino Beamline







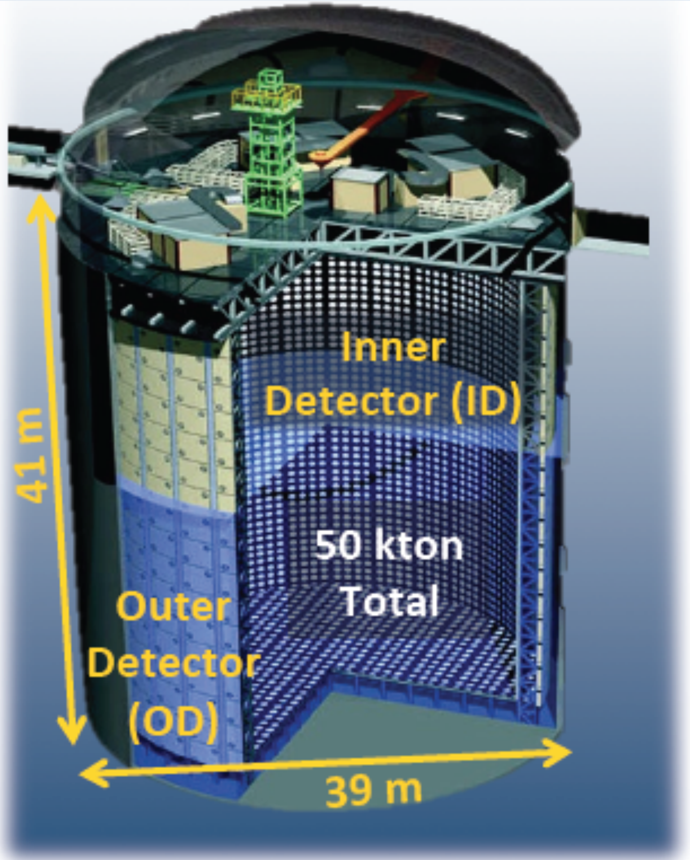
Monitoring of beam profile and intensity

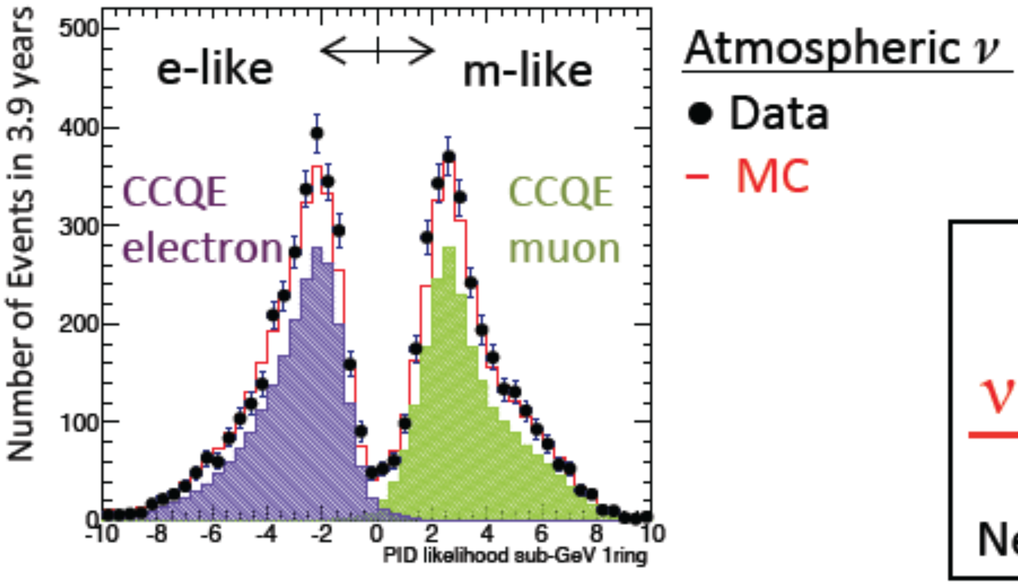
Hadron production measured by external experiments (NA61 @ CERN)

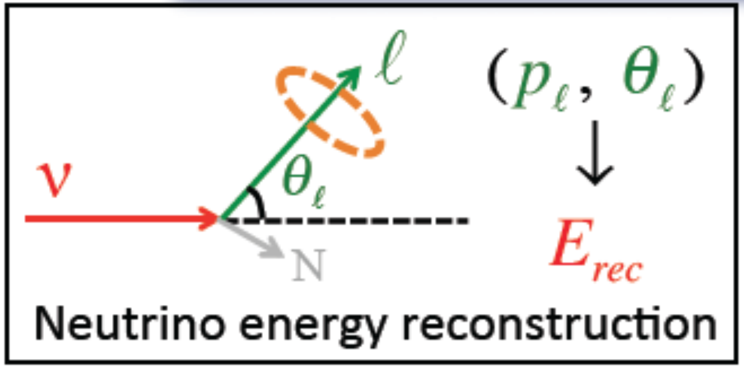
Off-axis neutrino beam
Narrow band @ osc. maximum

Super-Kamiokande

- 22.5 kton fiducial volume water Cherenkov detector
- Inner detector with ~11k PMTs
- Outer detector determines fully contained events
- Very good *e*/*μ* separation



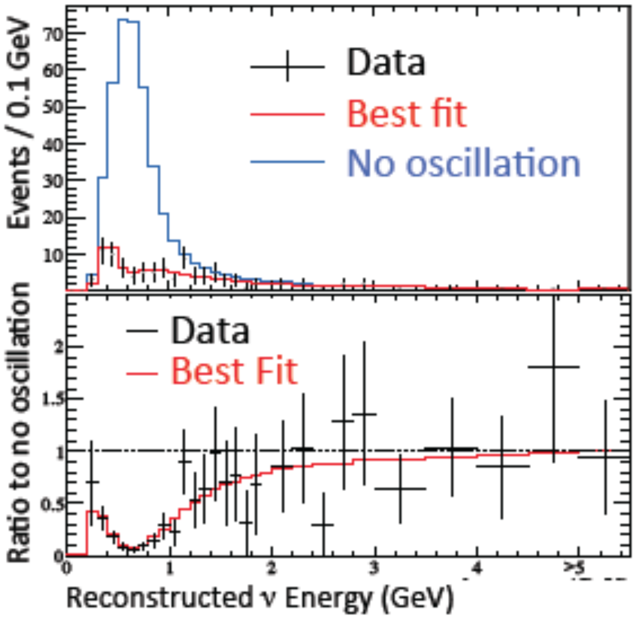


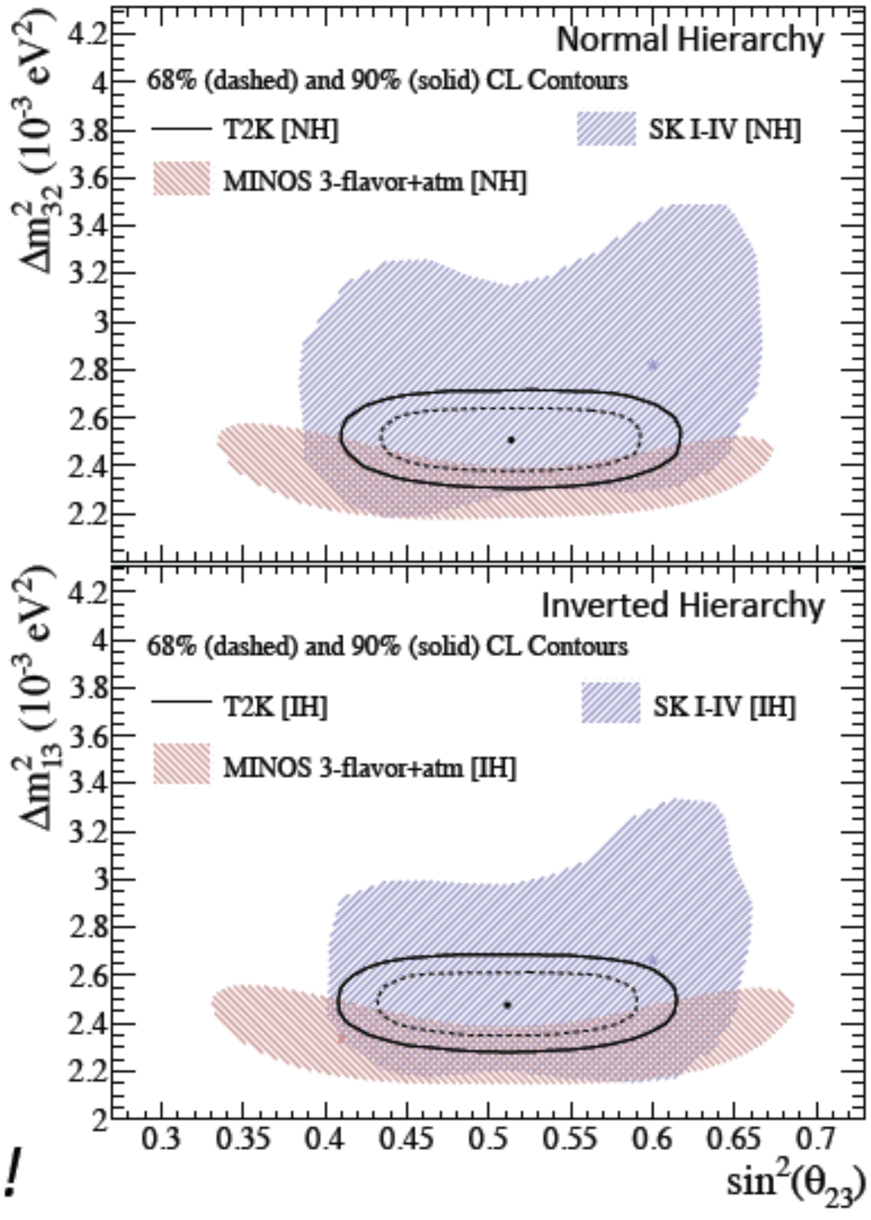


Neutrino energy reconstruction

Precision ν_μ Disappearance Measurement

Maximum likelihood fit in E_{rec}





arXiv:1403.1532 (2014)

		Best-fit ± FC 68% CL
NH	$\sin^2\theta_{23}$	$0.514^{+0.055}_{-0.056}$
	Δm^2_{32} (10^{-3} eV^2)	2.51 ± 0.10
IH	$\sin^2\theta_{23}$	0.511 ± 0.055
	Δm^2_{32} (10^{-3} eV^2)	2.48 ± 0.10

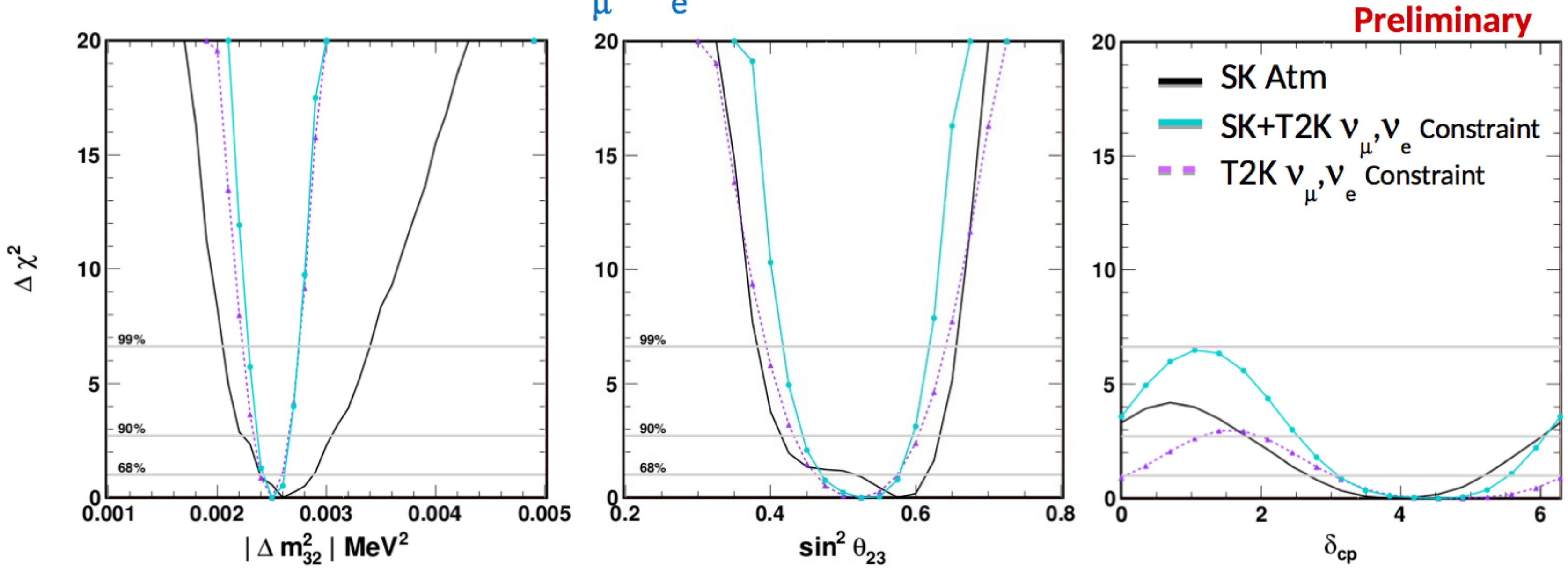
Most precise measurement of θ_{23}!

What’s new in 23 sector?

θ_{23} 2nd octant, $\delta \sim 3\pi/2$ preferred ?

R.Wendell Nu2014

Theta13 **Fixed** SK + T2K ν_μ, ν_e (External Constraint) NH

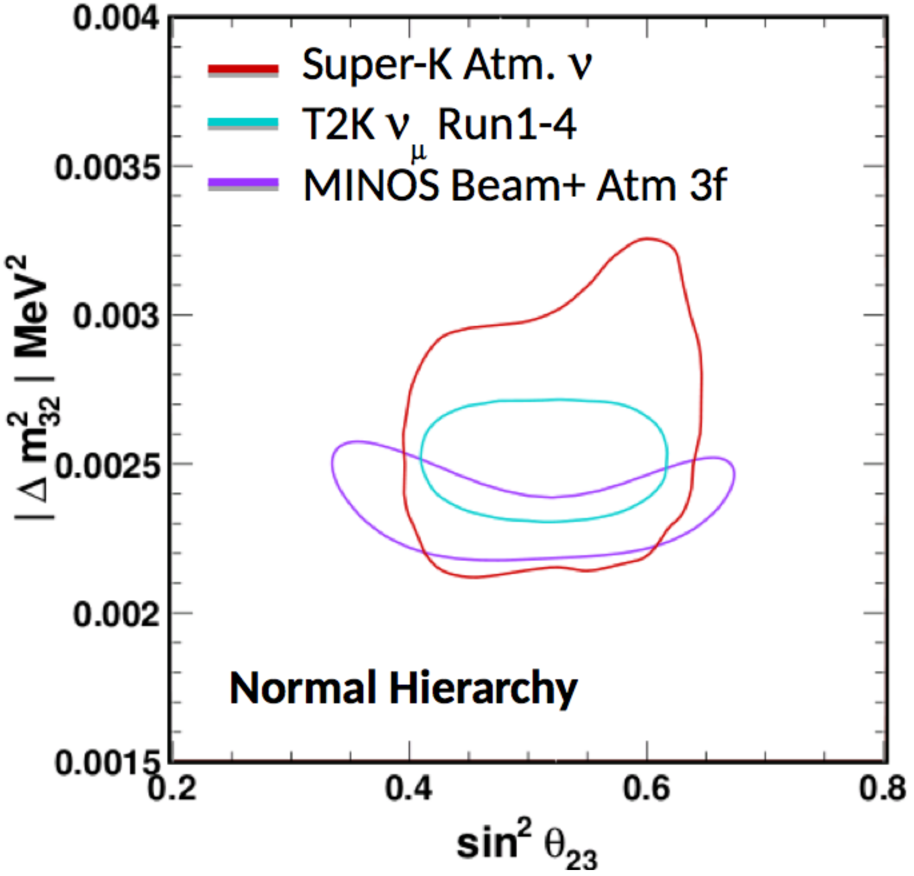


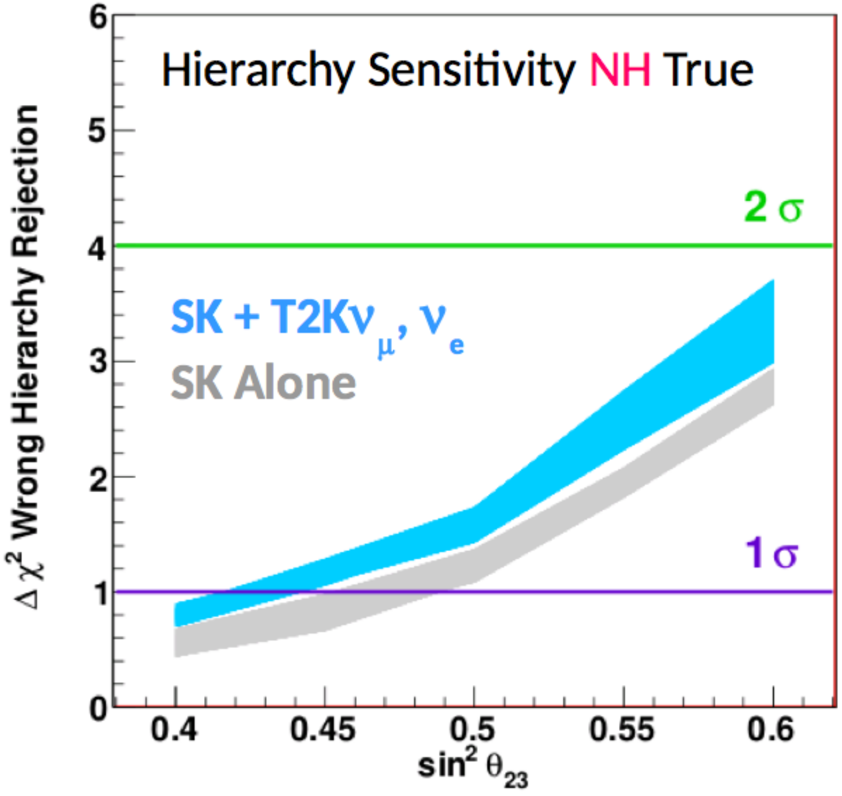
Fit (543 dof)	χ^2	θ_{13}	δ_{cp}	θ_{23}	Δm_{23} ($\times 10^{-3}$)
SK + T2K (NH)	578.2	0.025	4.19	0.55	2.5
SK + T2K (IH)	579.4	0.025	4.19	0.55	2.5

- $\chi^2_{IH} - \chi^2_{NH}$ = **-1.2** (-0.9 SK only)
- CP Conservation ($\sin\delta_{cp}$ = 0) allowed at (at least) 90% C.L. for both hierarchies

Accelerator θ_{23} sensitivity better than SK atm's

Mild preference of normal hierarchy





How to measure θ_{13} ?

To measure θ_{13} one needs ν_e

- $P(\nu_e \to \nu_e)$ is the interference between

e->1 -------> 1->e and e->2 -------> 2->e

e->3 -------> 3->e

Obviously involve $|U_{e3}|^2 = s^2_{13}$

- $P(\nu_\mu \to \nu_e)$ is the interference between

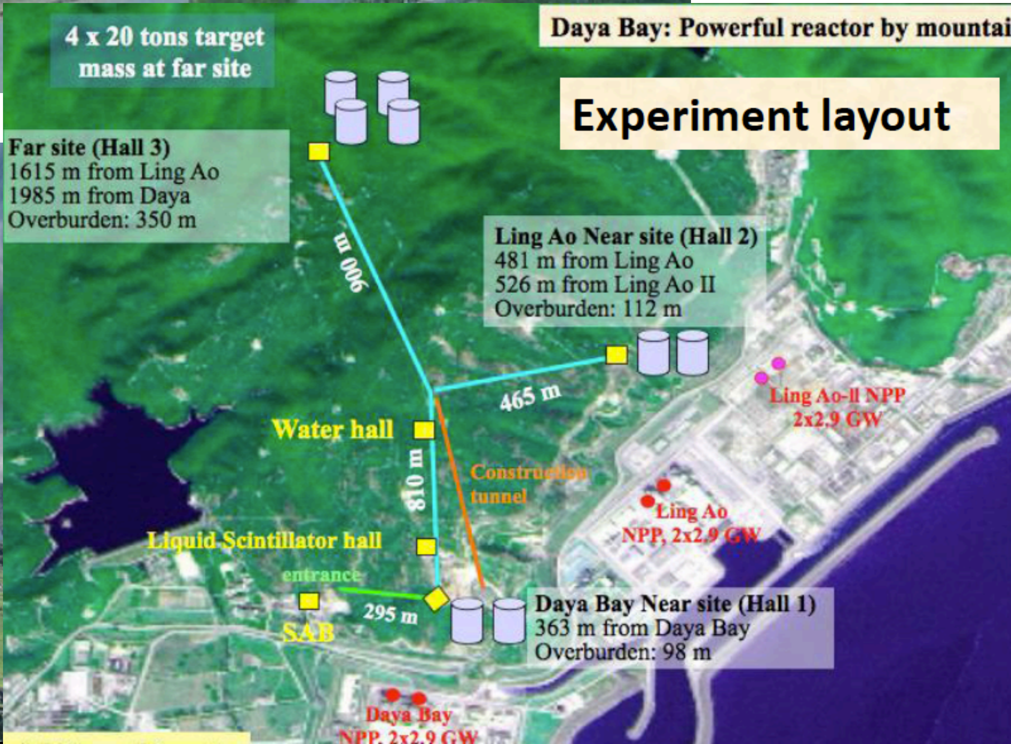
μ->1 -------> 1->e and μ->2 -------> 2->e

μ->3 -------> 3->e

Involve $|U_{e3}| = s_{13}$ but in fact s^2_{13}

Reactor and accelerator are equally good probe for θ_{13}

Reactor measurement of θ_{13}



2 different regimes of reactor neutrino oscillation

need to measure small ν_e deficit !

$$1 - P(\bar{\nu}_e \to \bar{\nu}_e) = \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right) + O(\epsilon s^2_{13}) + O(\epsilon^2)$$

$$\epsilon \equiv \Delta m^2_{21}/\Delta m^2_{31} \simeq 0.03$$

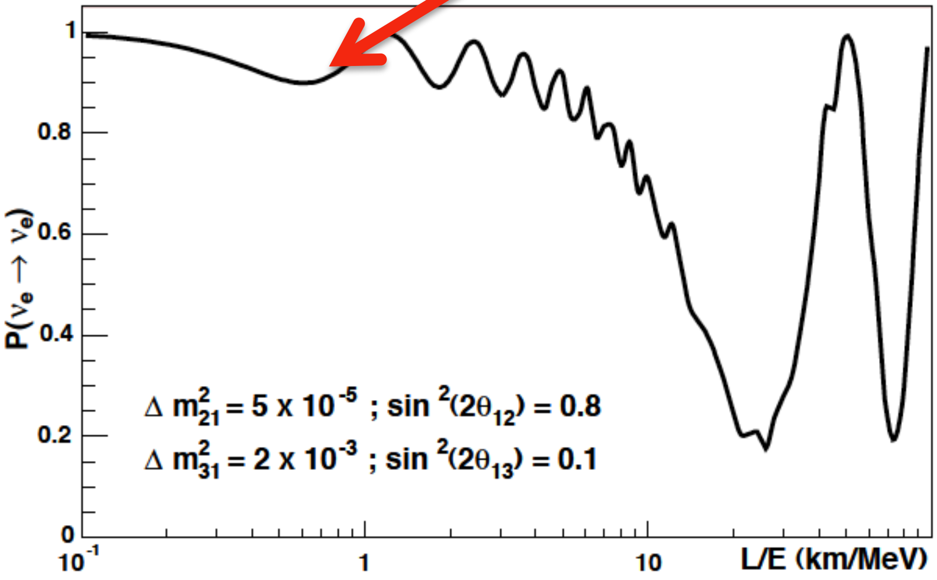
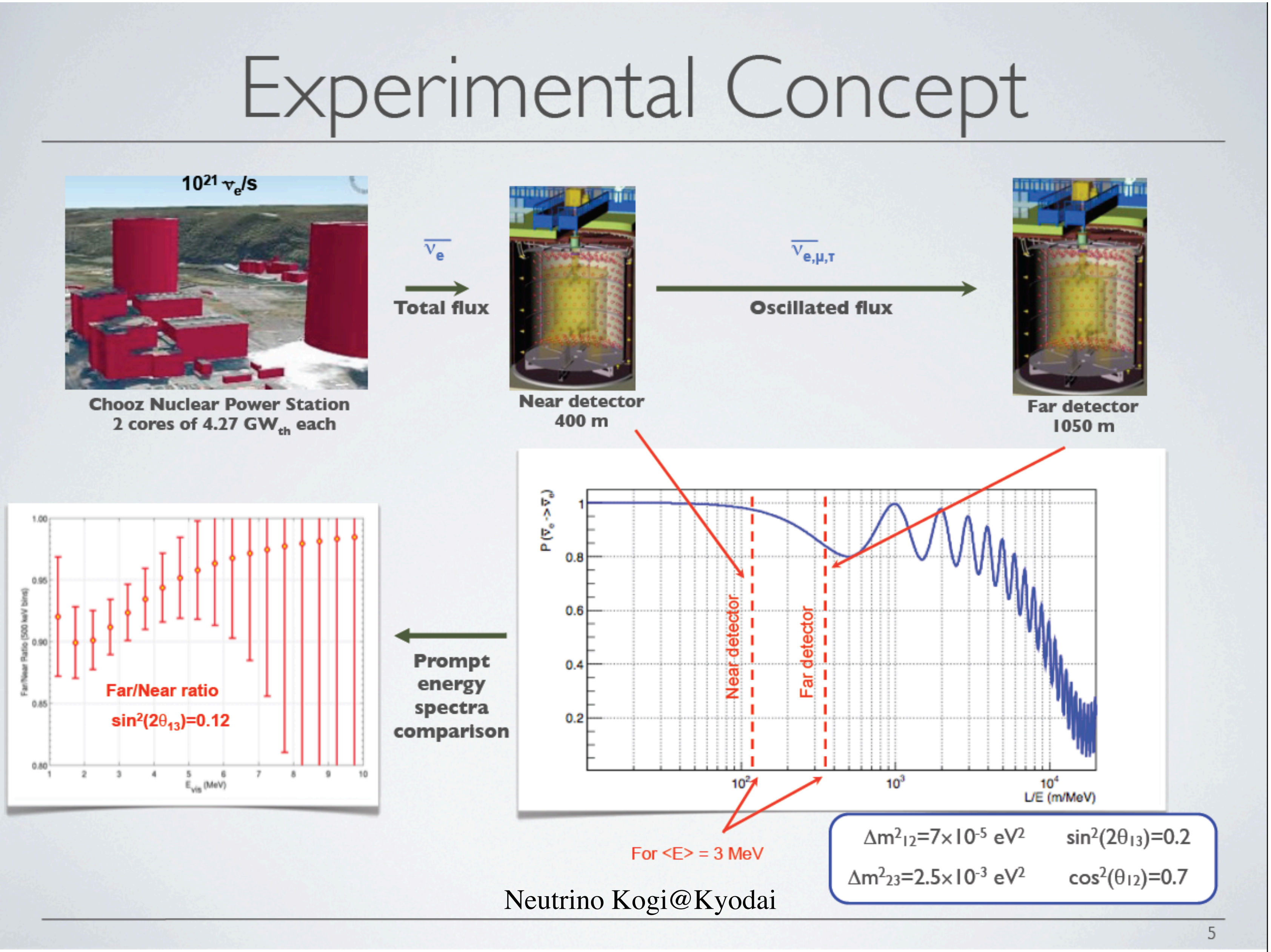


Figure 3: Probability of ν_e disappearance versus L/E for θ_{13} at its current upper limit

Independent of δ, θ_{23}, matter effect, θ_{12}, solar Δm^2 → pure measurement of θ_{13}

Near-far "identical" 2 detectors



Experimental Concept

10^{21} $\bar{\nu}_e$/s

Chooz Nuclear Power Station
2 cores of 4.27 GW$_{th}$ each

$\bar{\nu}_e$

Total flux

Near detector
400 m

$\bar{\nu}_{e,\mu,\tau}$

Oscillated flux

Far detector
1050 m

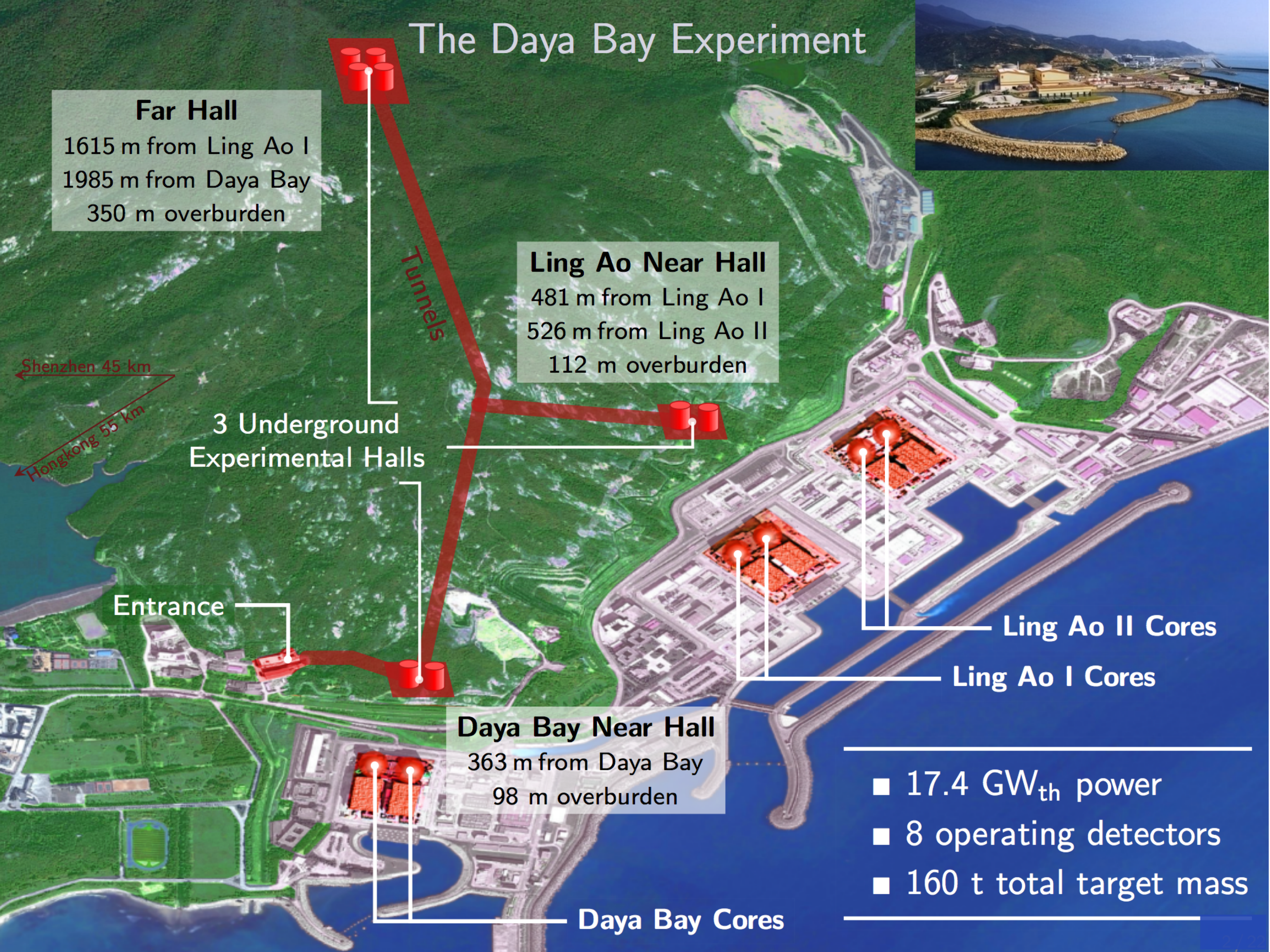
Prompt energy spectra comparison

Far/Near ratio
$\sin^2(2\theta_{13})=0.12$

For <E> = 3 MeV

$\Delta m^2_{12}=7\times10^{-5}$ eV² $\sin^2(2\theta_{13})=0.2$
$\Delta m^2_{23}=2.5\times10^{-3}$ eV² $\cos^2(\theta_{12})=0.7$

The Daya Bay Experiment



Far Hall
1615 m from Ling Ao I
1985 m from Daya Bay
350 m overburden

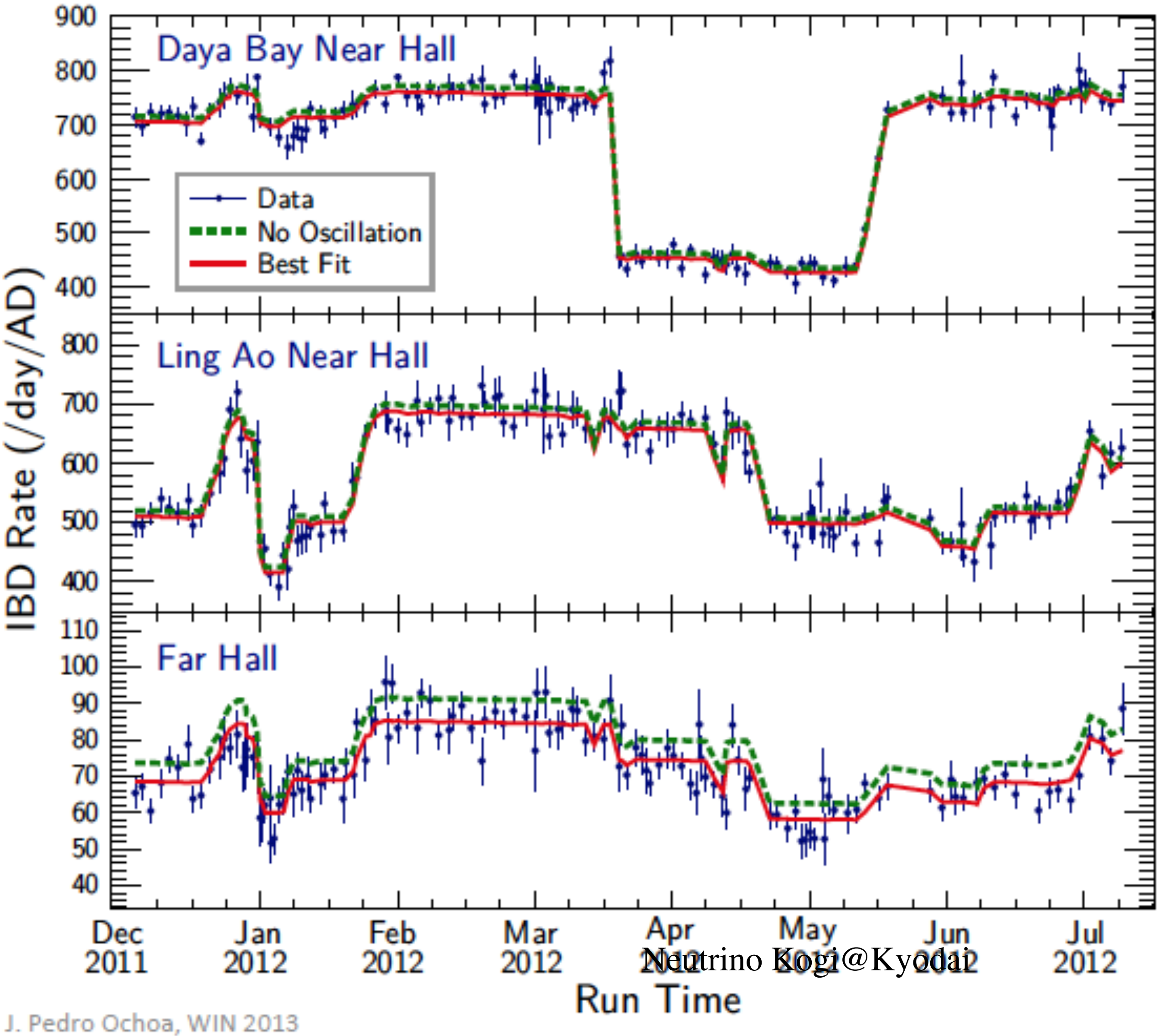
Ling Ao Near Hall
481 m from Ling Ao I
526 m from Ling Ao II
112 m overburden

Daya Bay Near Hall
363 m from Daya Bay
98 m overburden

- 17.4 $\mathrm{GW_{th}}$ power
- 8 operating detectors
- 160 t total target mass

Antineutrino Rates vs. Time

- For main analysis we simultaneously fit all detectors using reactor model, with the absolute normalization as a free parameter:



Note:

- Normalization is determined by fit to data. It is within a few percent of expectations.
- Paper on absolute reactor neutrino flux and shape is in preparation

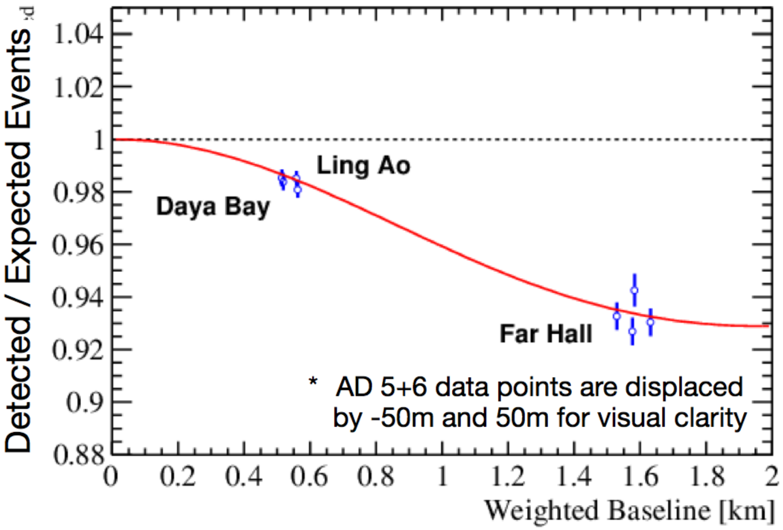
Detected rate strongly correlated with reactor flux expectations

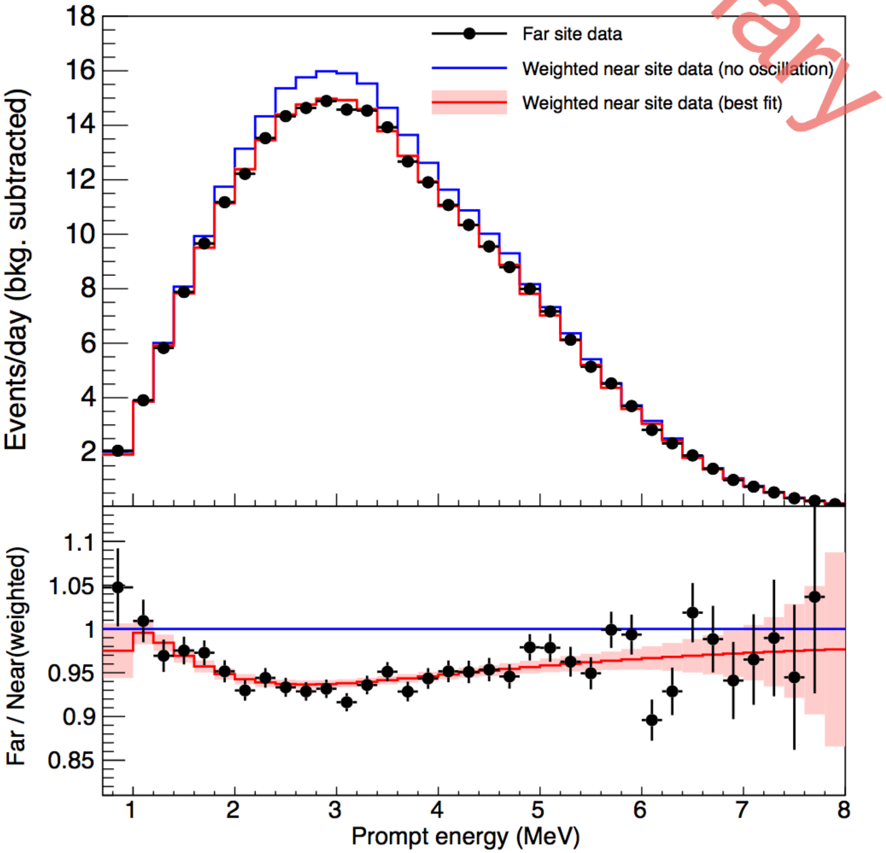
13 sector: beautiful result from Daya Bay

Chao Zhang Nu2014

Far v.s. Near Comparison

$$P_{\bar{\nu}_e \to \bar{\nu}_e} = 1 - \sin^2 2\theta_{13} \sin^2\left(\Delta m^2_{ee} \frac{L}{4E}\right) - \sin^2 2\theta_{12} \cos^4\theta_{13} \sin^2\left(\Delta m^2_{21} \frac{L}{4E}\right)$$



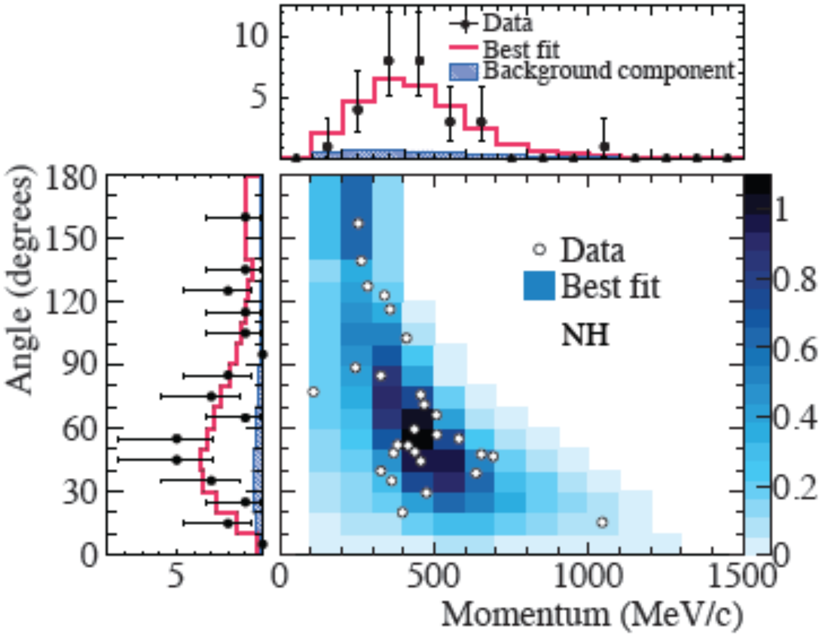


The observed relative rate deficit and relative spectrum distortion are highly consistent with oscillation interpretation

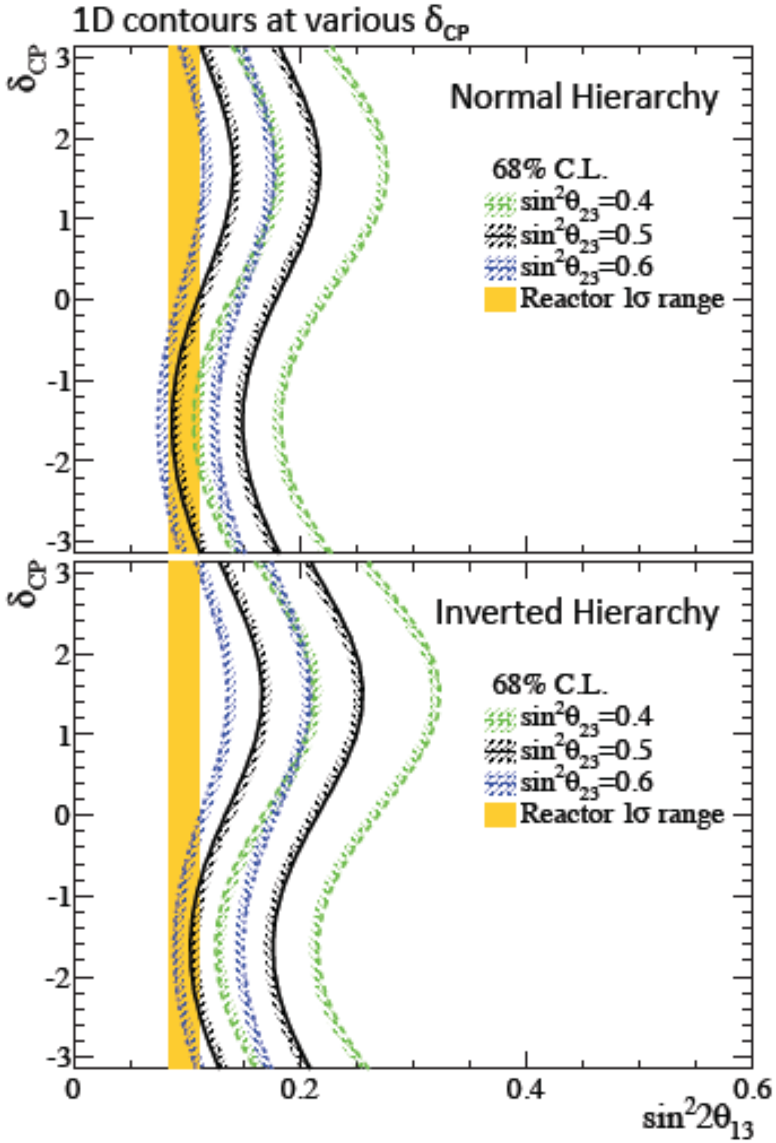
Accelerator measurement of θ_{13}

Observation of $\nu_\mu \rightarrow \nu_e$ Appearance

Maximum likelihood fit in (p_e, θ_e)



1D contours at various δ_{CP}



PRL 112, 061802 (2014)

		Best-fit ± 68% CL
$\sin^2 2\theta_{13}$	NH	$0.140^{+0.038}_{-0.032}$
	IH	$0.170^{+0.045}_{-0.037}$

$@\ \sin^2\theta_{23} = 0.5, \left|\Delta m^2_{32}\right| = 2.4\times10^{-3}\ \text{eV}^2, \delta_{CP} = 0$

Discovery of ν_e Appearance at 7.3σ!

Dependence on θ_{23} motivates a joint $\nu_\mu + \nu_e$ fit...

θ_{12} and θ_{13}

$$\tan^2\theta_{12} = 0.436^{+0.029}_{-0.025},\ \Delta m^2_{21} = 7.53^{+0.18}_{-0.18} \times 10^{-5}\,\mathrm{eV}^2,$$

KamLAND+solar Mar.2013

- $\sin^2\theta_{12}$ = 0.304 +/- 0.013
- Error of $\sin^2\theta_{12}$ = 4.3%
- Error of Δm^2_{21} = 2.4%

Daya Bay Nu2014

$$\sin^2 2\theta_{13} = 0.084^{+0.005}_{-0.005}$$

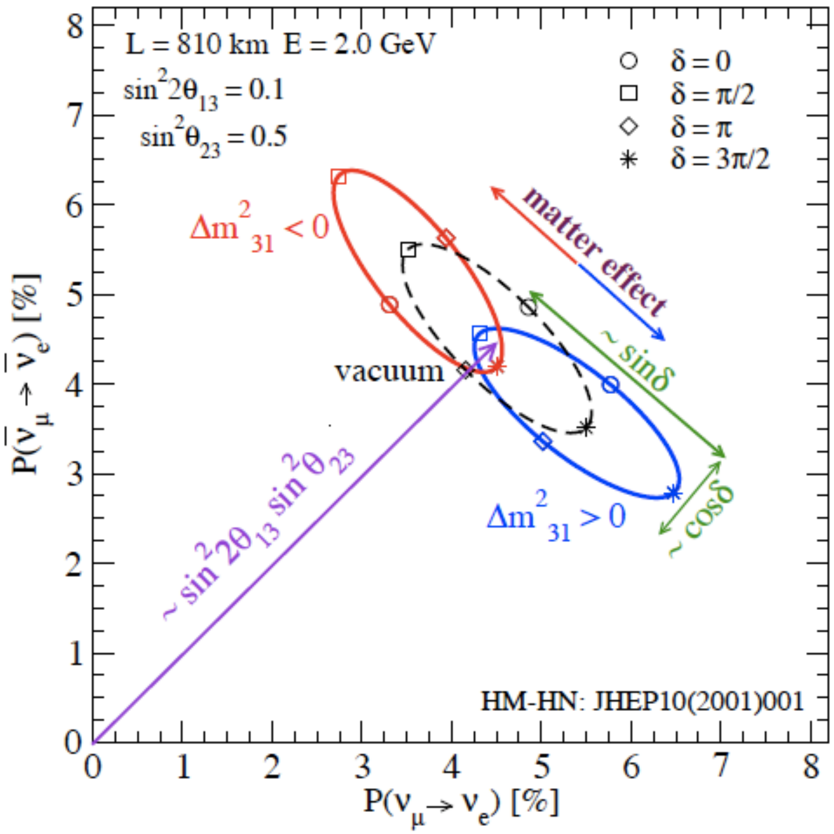
$$|\Delta m^2_{ee}| = 2.44^{+0.10}_{-0.11} \times 10^{-3}\mathrm{eV}^2$$

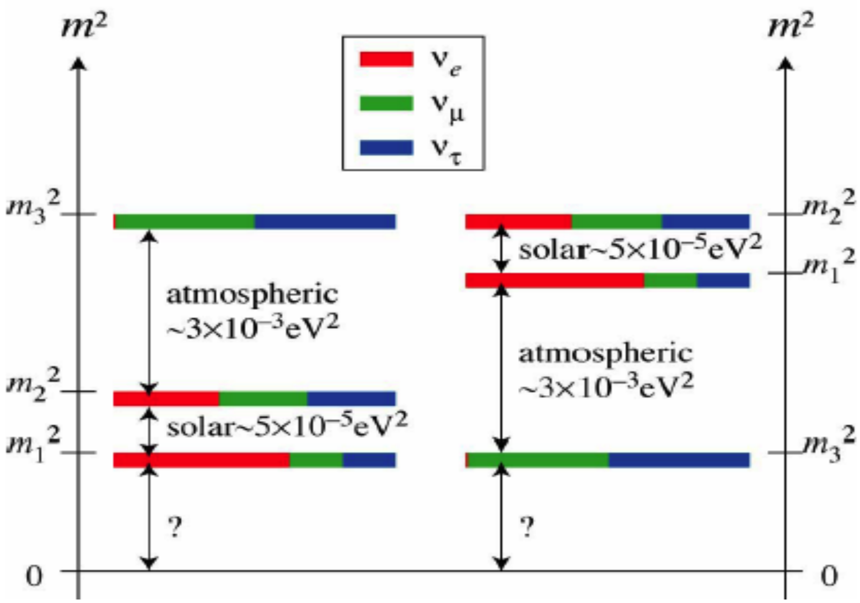
- Error of $\sin^2\theta_{13}$ (Daya Bay) = 6.1%
- Error of Δm^2_{31} (Daya Bay) ~= 4% !!
- Error of Δm^2_{31} (MINOS/T2K) ~= 4%

Then what's next? ν Mass pattern: "mass hierarchy" and CP

Mass hierarchy resolution and CP: understanding the principle

Use matter effect to determine the ν mass hierarchy

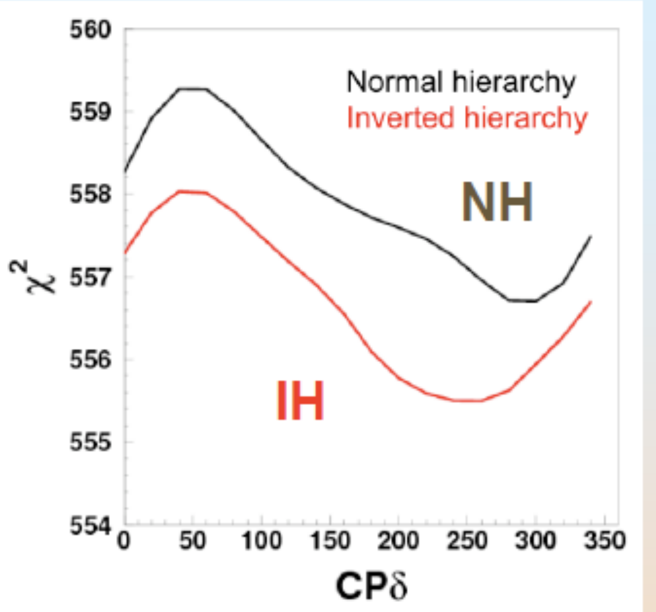




Use P – P-bar asymmetry + spectral modulation to measure CP phase δ

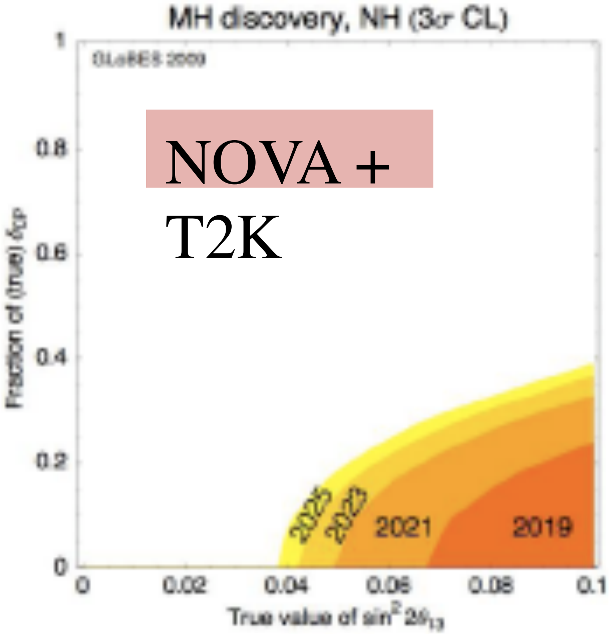
Sensitivity to mass hierarchy by LBL: limited for the time being

SK-atm



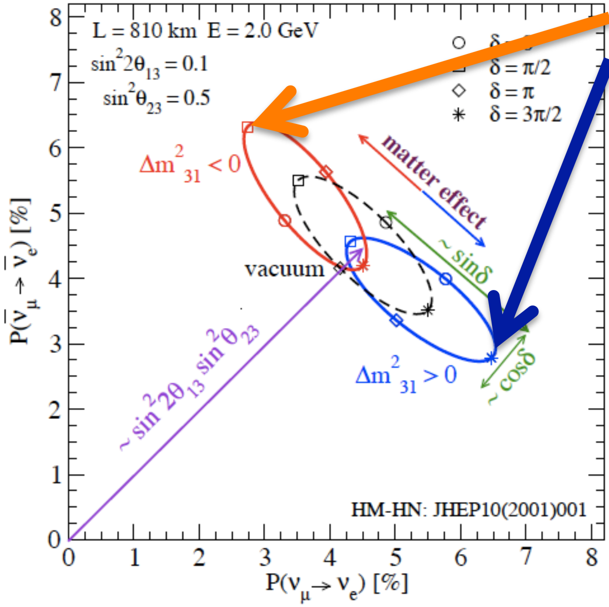
NH χ^2_{min} = 556.7 / 477 dof

IH χ^2_{min} = 555.5 / 477 dof



NOVA + T2K

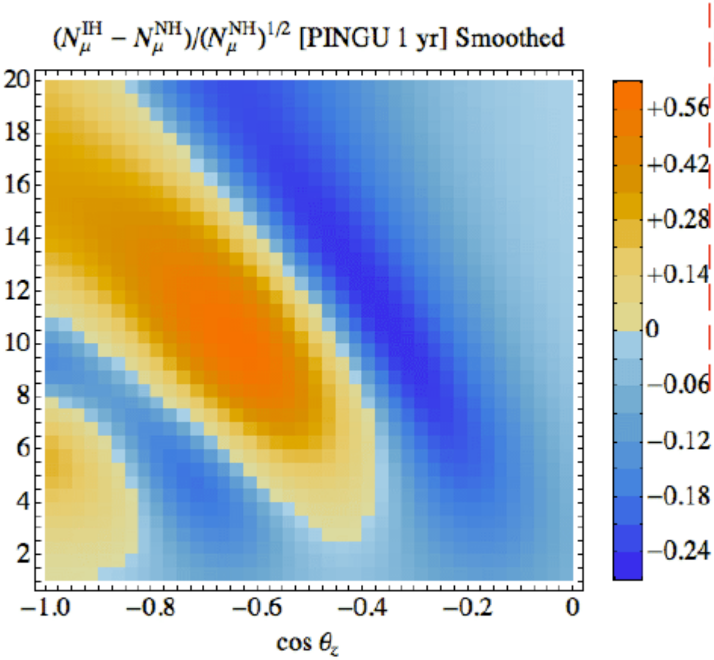
Lucky regions



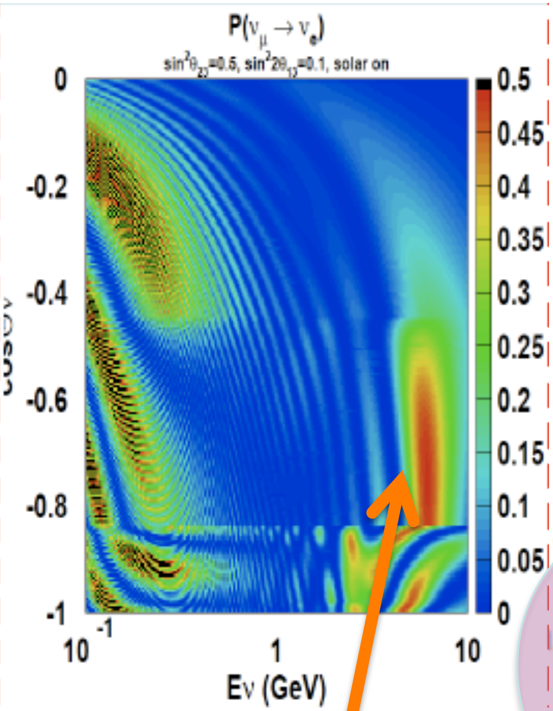
Neutrino Ko

Use of atmospheric ν for mass hierarchy: (PINGU, Hyper-K, LBNE)

PINGU
σ_E=2 GeV
σ_θ=11.25°

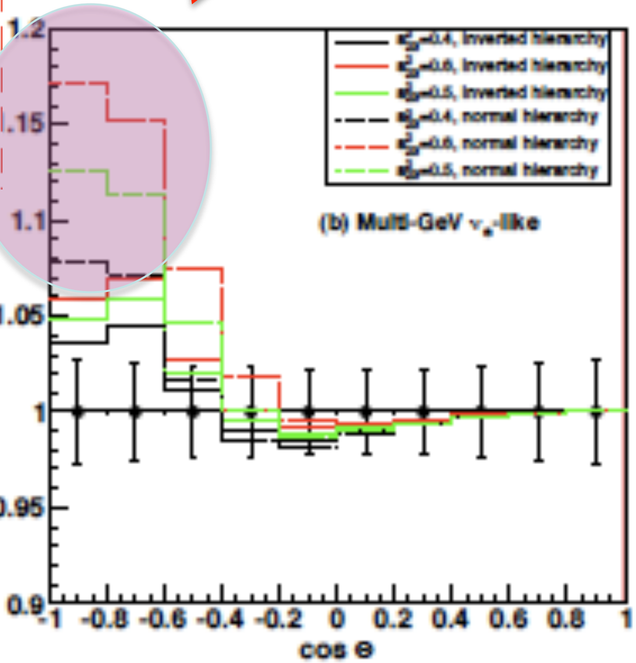


Akhmedov-Razzaque-Smirnov
June 12



MSW resonance

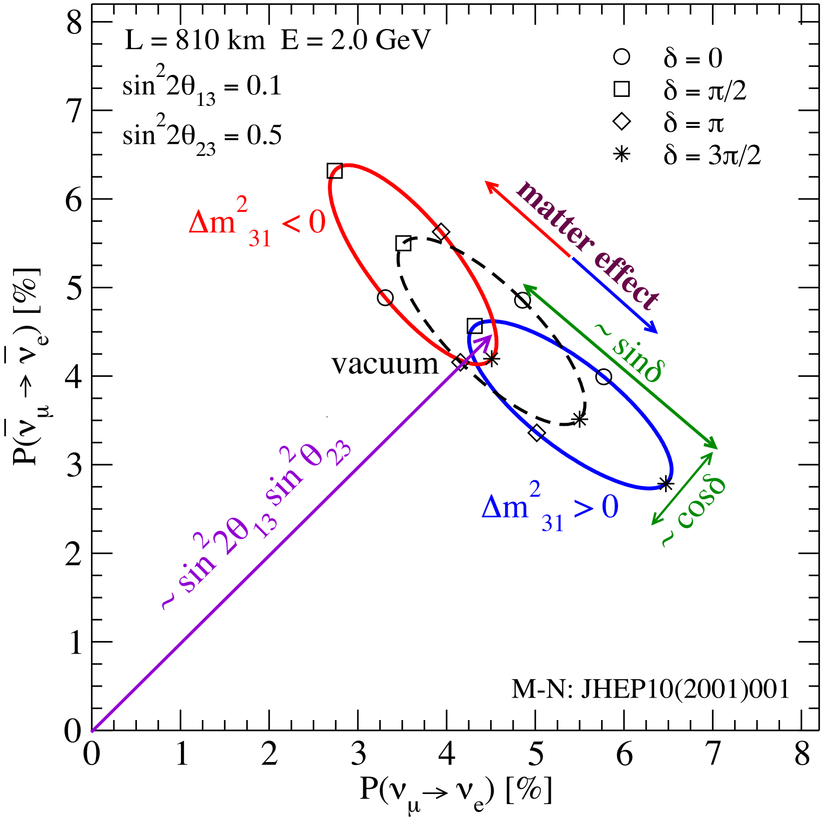
e-like larger for normal hierarchy

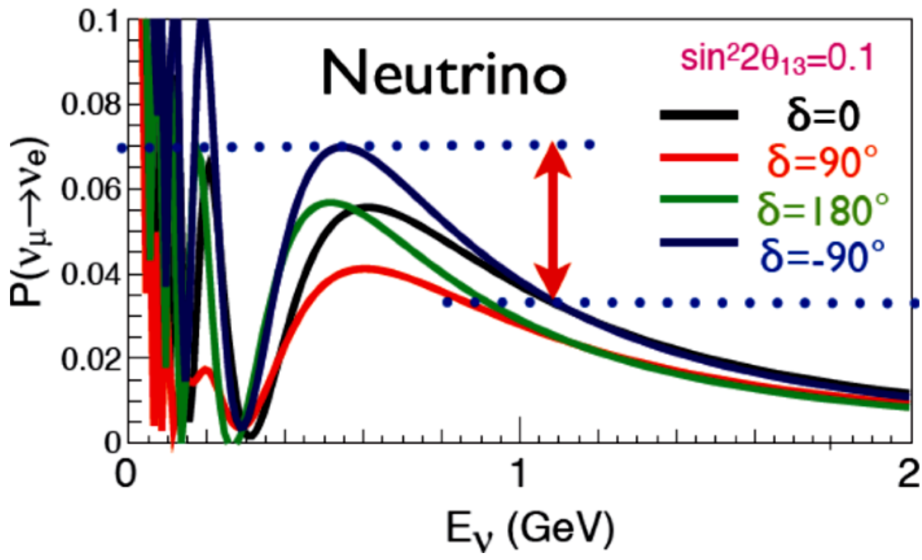


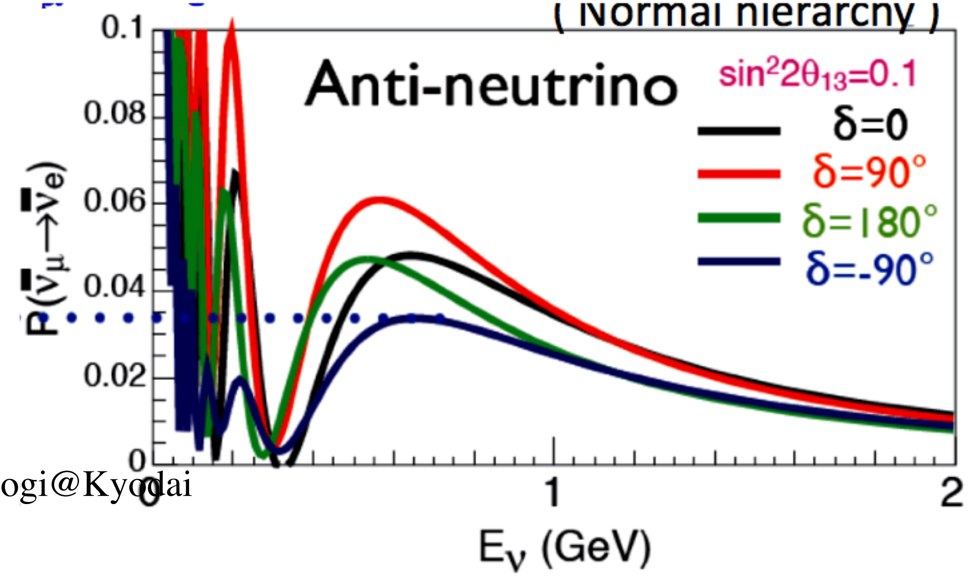
How to measure δ?

Not easy because of double suppression: J_r and $\Delta m^2_{21}/\Delta m^2_{31}$

How oscillation probability in matter depend on $\theta_{13}–\theta_{23}–\delta$?

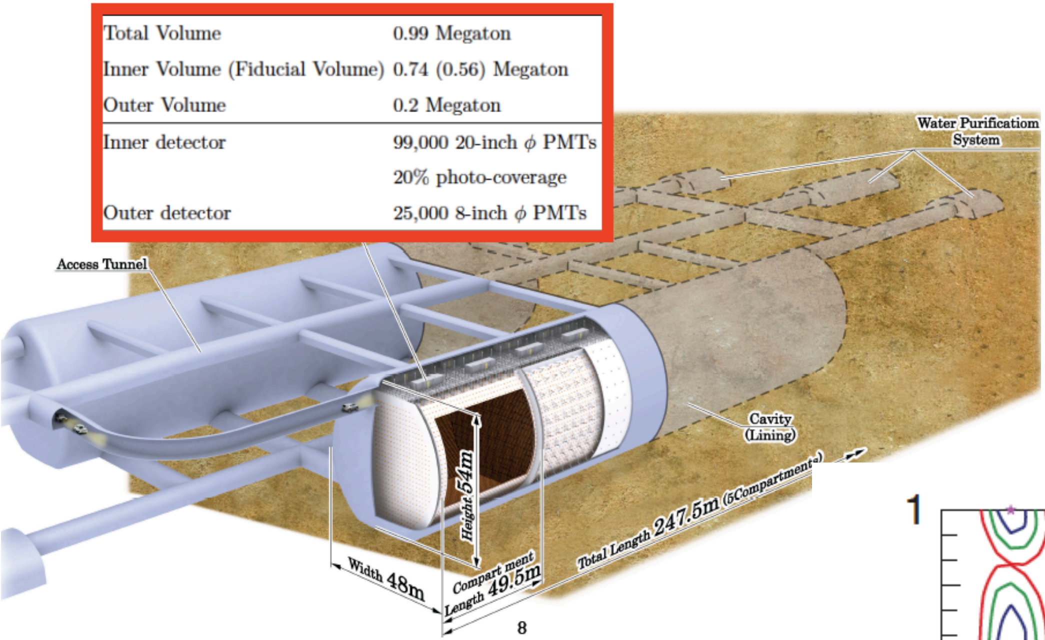






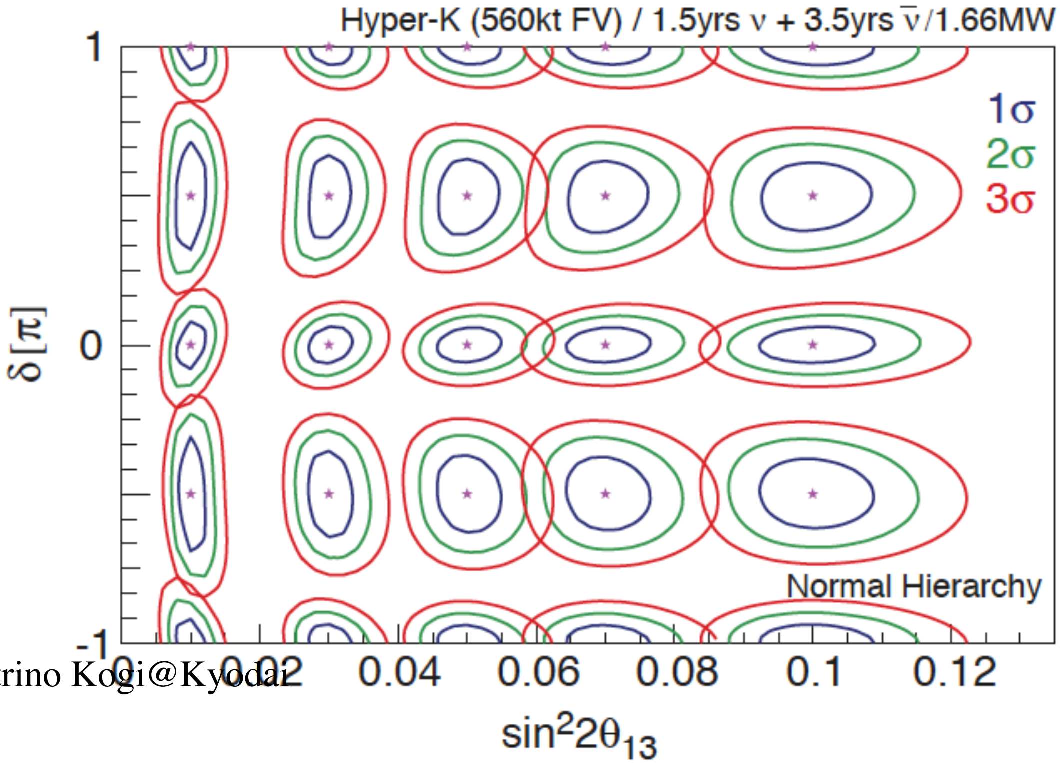
Bi-P plot: dependence on θ_{13}, θ_{23}, and δ displayed pictorially

Hyper-Kamiokande: CP sensitivity



Total Volume	0.99 Megaton
Inner Volume (Fiducial Volume)	0.74 (0.56) Megaton
Outer Volume	0.2 Megaton
Inner detector	99,000 20-inch ϕ PMTs 20% photo-coverage
Outer detector	25,000 8-inch ϕ PMTs

Normal hierarchy



Summary

- First hint for nonzero ν mass came from solar ν, but first evidence was from SK atmospheric ν (23 oscillation)
- KamLAND and solar ν experiments jointly found another channel oscillation/flavor conversion (12 oscillation)
- All the mixing angles are now measured (still consistent with 3 ν scheme)
- ν mass hierarchy and CP δ left
- I failed to cover Majorana vs Dirac, Majorana phase, absolute neutrino mass scale etc.