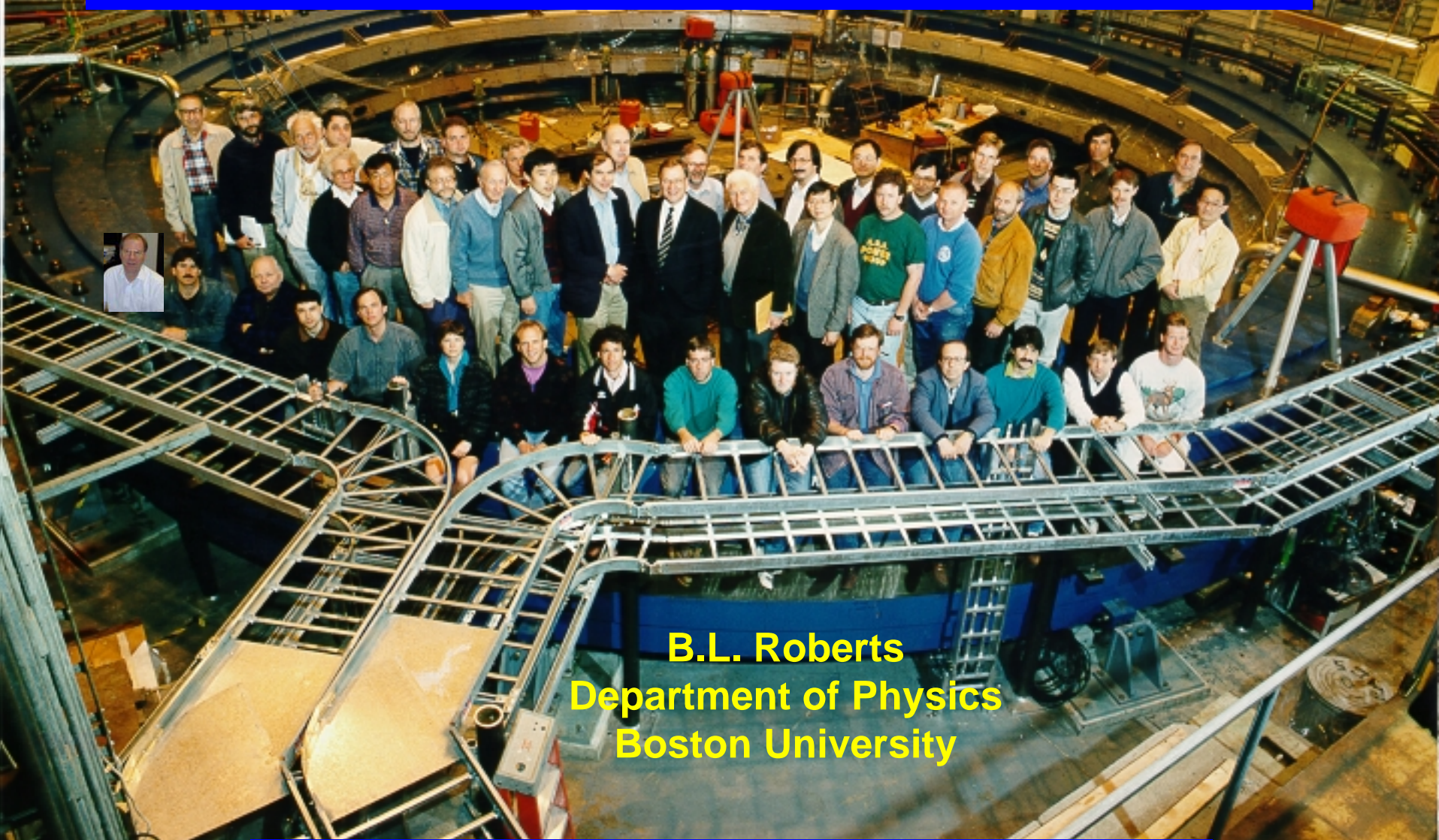


The Muon: A Laboratory for Particle Physics



B.L. Roberts
Department of Physics
Boston University

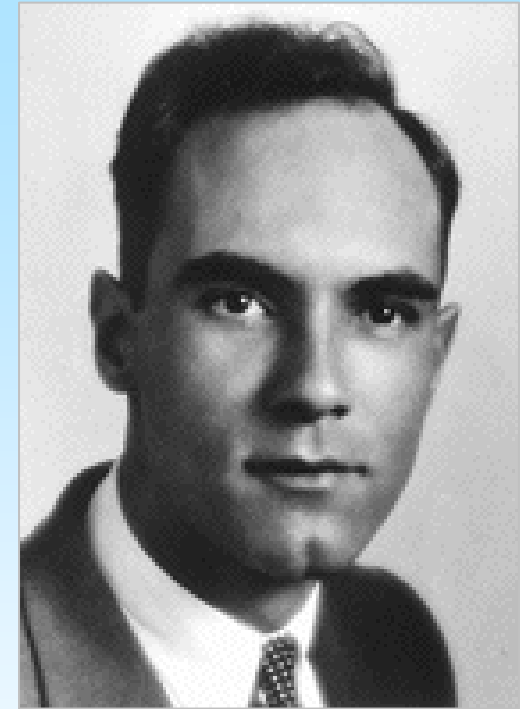
Outline

- Introduction to the muon
- Selected weak interaction parameters
- Magnetic and electric dipole moments
- Lepton Flavor Violation
- Summary and conclusions.

The Muon: Discovered in 1936



Discovered in cosmic rays by
Seth Neddermeyer and Carl
Anderson



MAY 15, 1937

PHYSICAL REVIEW

VOLUME 51

Note on the Nature of Cosmic-Ray Particles

SETH H. NEDDERMEYER AND CARL D. ANDERSON
California Institute of Technology, Pasadena, California
(Received March 30, 1937)

MEASUREMENTS¹ of the energy loss of massive than protons but more penetrating than particles occurring in the cosmic-ray electrons obeying the Bethe-Heitler theory, we showers have shown that this loss is proportional have taken about 6000 counter-tripped photo-

Confirmed by Street and Stevenson

NOVEMBER 1, 1937

PHYSICAL REVIEW

VOLUME 52

LETTERS TO THE EDITOR

Prompt publication of brief reports of important discoveries in physics may be secured by addressing them to this department. Closing dates for this department are, for the first issue of the month, the eighteenth of the preceding month, for the second issue, the third of the month. Because of the late closing dates for the section no proof can be shown to authors. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents.

Communications should not in general exceed 600 words in length.

New Evidence for the Existence of a Particle of Mass Intermediate Between the Proton and Electron

Anderson and Neddermeyer¹ have shown that for energies tracks of high energy particles.

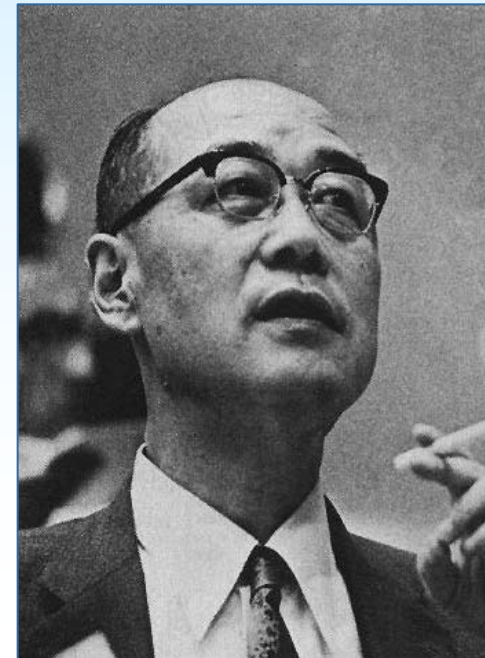
between those of the proton and electron. If this is true, it should be possible to distinguish clearly such a particle from an electron or proton by observing its track density

Research Laboratory of Physics,
Harvard University,
Cambridge, Massachusetts,
October 6, 1937.

J. C. STREET
E. C. STEVENSON

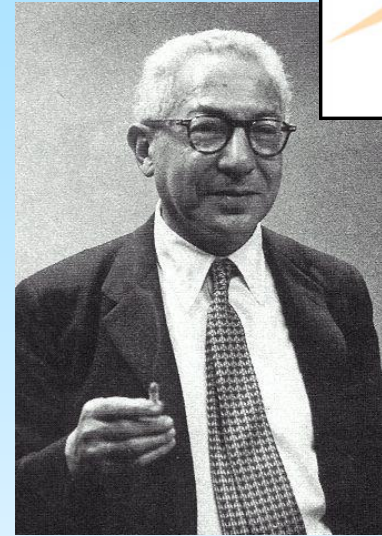
- ¹ Anderson and Neddermeyer, Phys. Rev. **50**, 263 (1936).
² Street and Stevenson, Phys. Rev. **51**, 1005 (1937).
³ Neddermeyer and Anderson, Phys. Rev. **51**, 885 (1937).

It interacted too weakly with matter to be the “Yukawa” particle which was postulated to carry the nuclear force

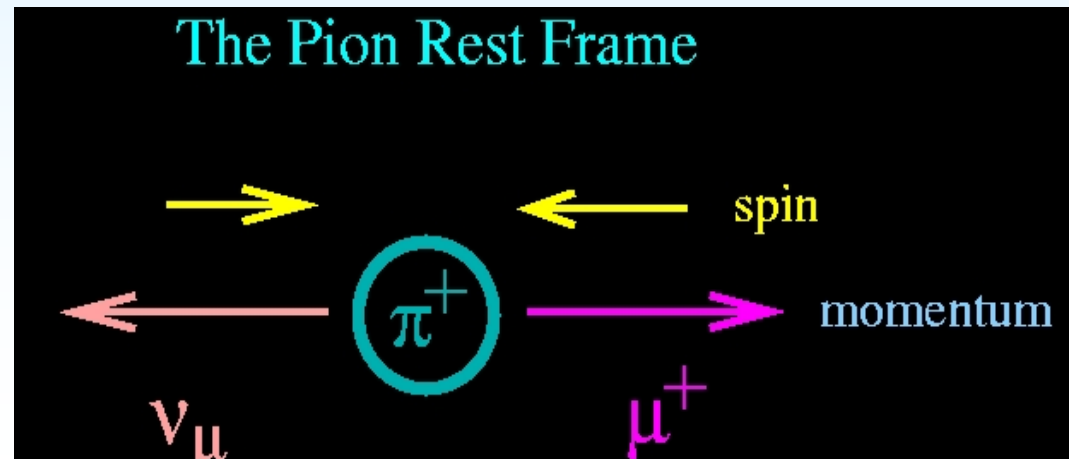


The discovery of the muon was a big surprise...

Who ordered THAT!?!?

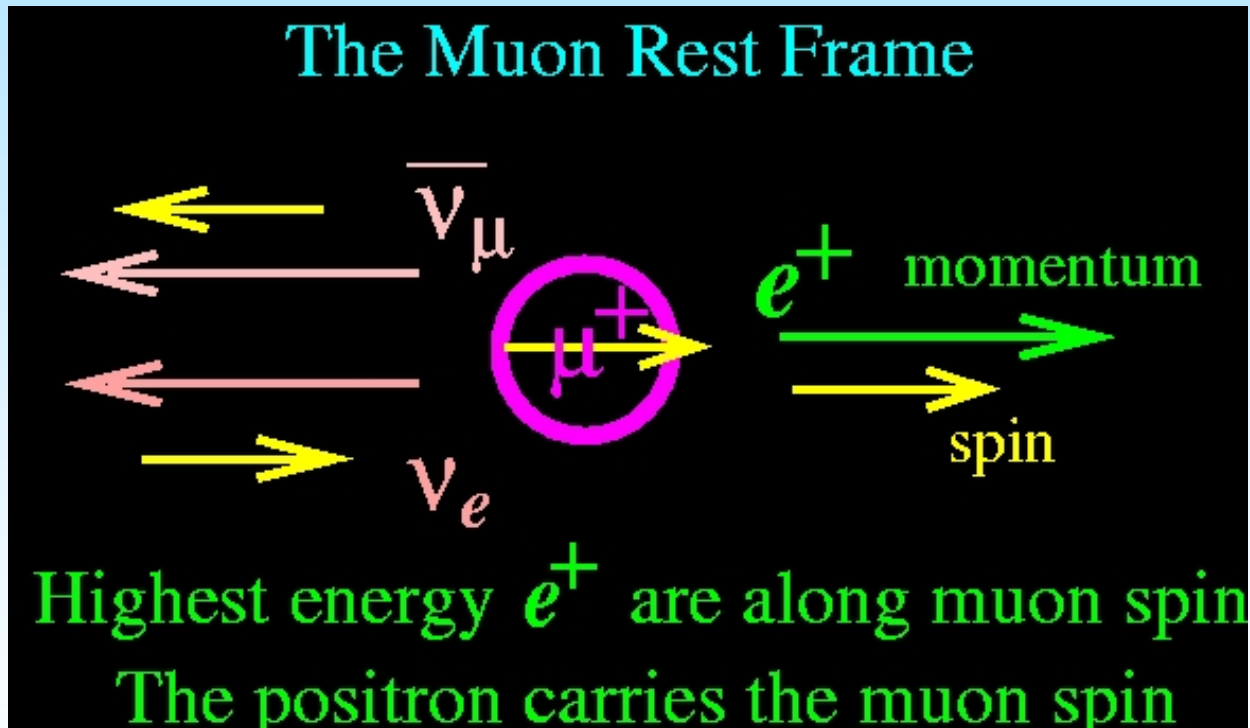


- Lifetime $\sim 2.2 \mu\text{s}$, practically forever
- 2nd generation lepton
- $m_{\mu}/m_e = 206.768\,277(24)$
- produced polarized
 - in-flight decay: both "forward" and "backward" muons are highly polarized
- Paul Scherrer Institut has $10^8 \mu/\text{s}$ in a beam



Death of the Muon

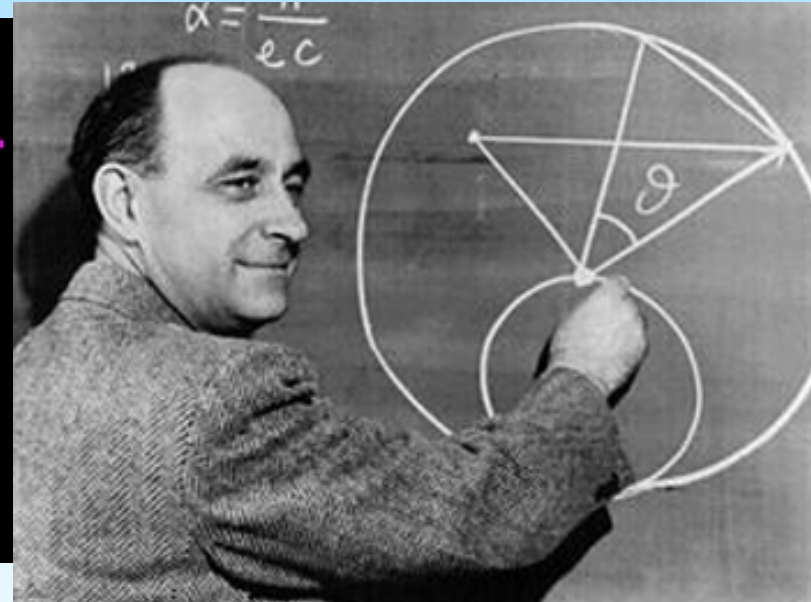
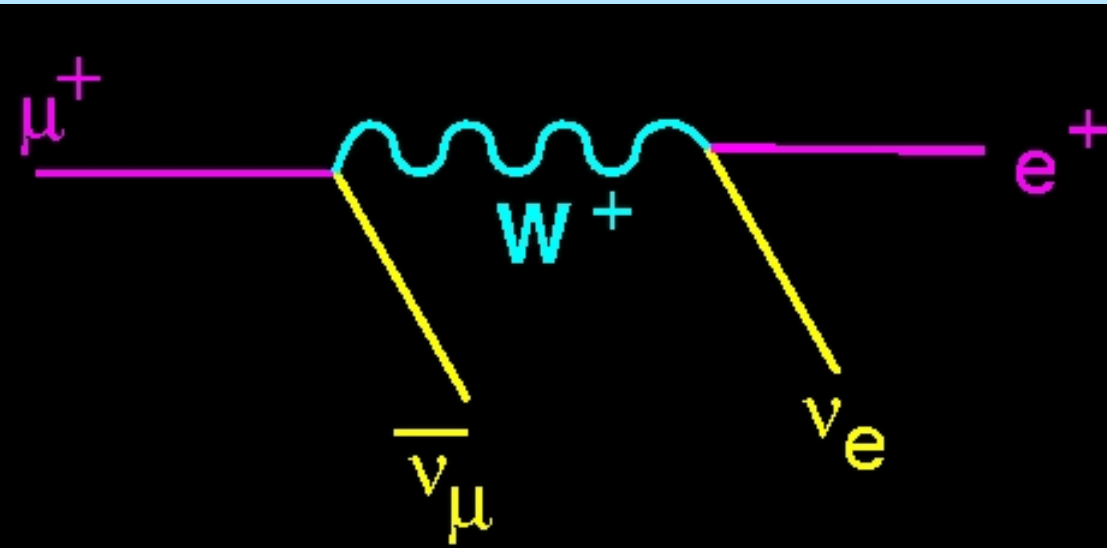
- Decay is self analyzing



What have we learn from the μ 's death?

- The strength of the weak interaction
 - i.e. the Fermi constant G_F (more properly G_μ)
- The fundamental nature of the weak interaction
 - i.e. is it scalar, vector, tensor, pseudo-scalar, pseudo-vector or pseudo-tensor?
- Lepton flavor conservation in μ -decay
- VEV of the Higgs field: $\frac{G_F}{\sqrt{2}} = \frac{1}{2v^2}$
- Induced form-factors in nuclear μ -capture

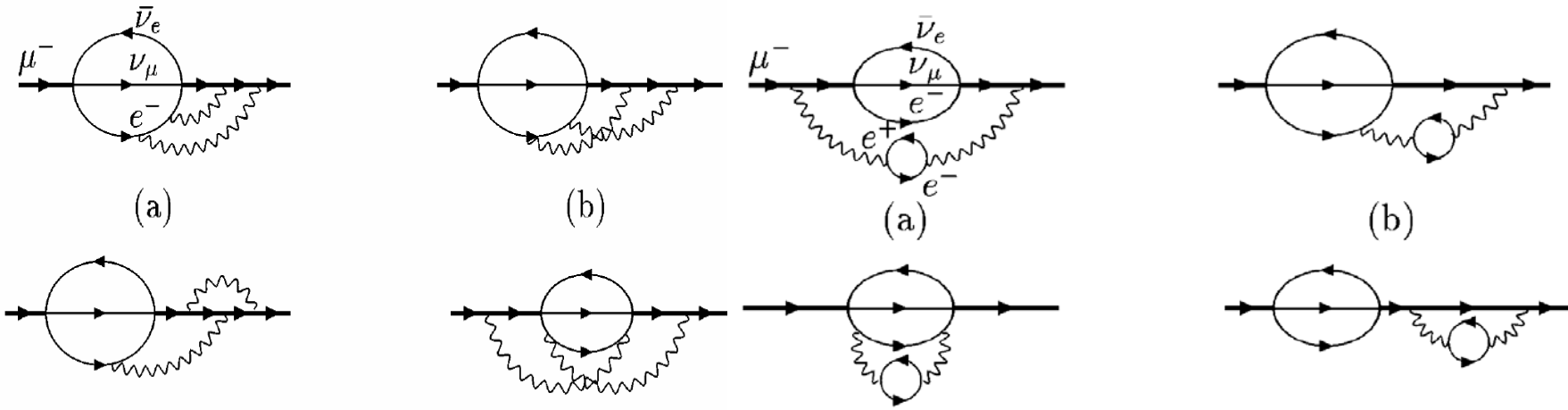
A precise measurement of τ_{μ^+} leads to a precise determination of the Fermi constant G_F



$$\frac{1}{\tau} = \frac{G_F^2 m_\mu^5}{192\pi^3} (1 + \delta)$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} (1 + \Delta r)$$

from radiative corrections

δ 

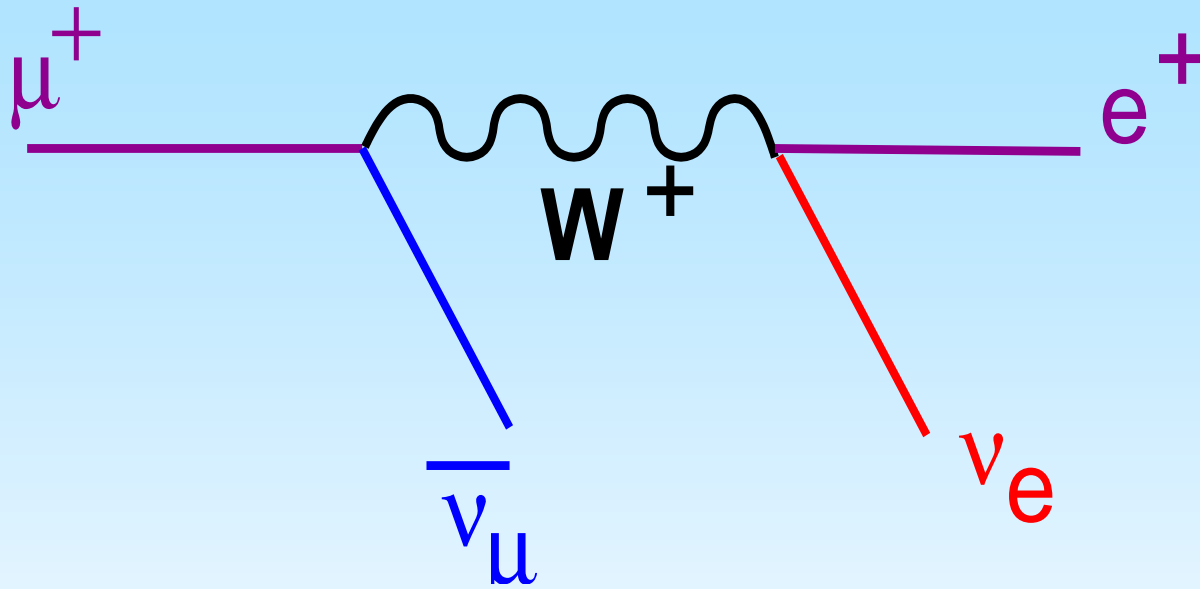
$$\tau_{\mu}^{-1} = \frac{G_F^2 m_{\mu}^5}{192\pi^3} F\left(\frac{m_e^2}{m_{\mu}^2}\right) \left(1 + \frac{3 m_{\mu}^2}{5 M_W^2}\right) \times \left[1 + \left(\frac{25}{8} - \frac{\pi^2}{2}\right) \frac{\alpha(m_{\mu})}{\pi} + C_2 \frac{\alpha^2(m_{\mu})}{\pi^2}\right],$$

where $F(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$, $\alpha(m_{\mu})^{-1} = \alpha^{-1} - \frac{2}{3\pi} \ln\left(\frac{m_{\mu}}{m_e}\right) + \frac{1}{6\pi} \approx 136$

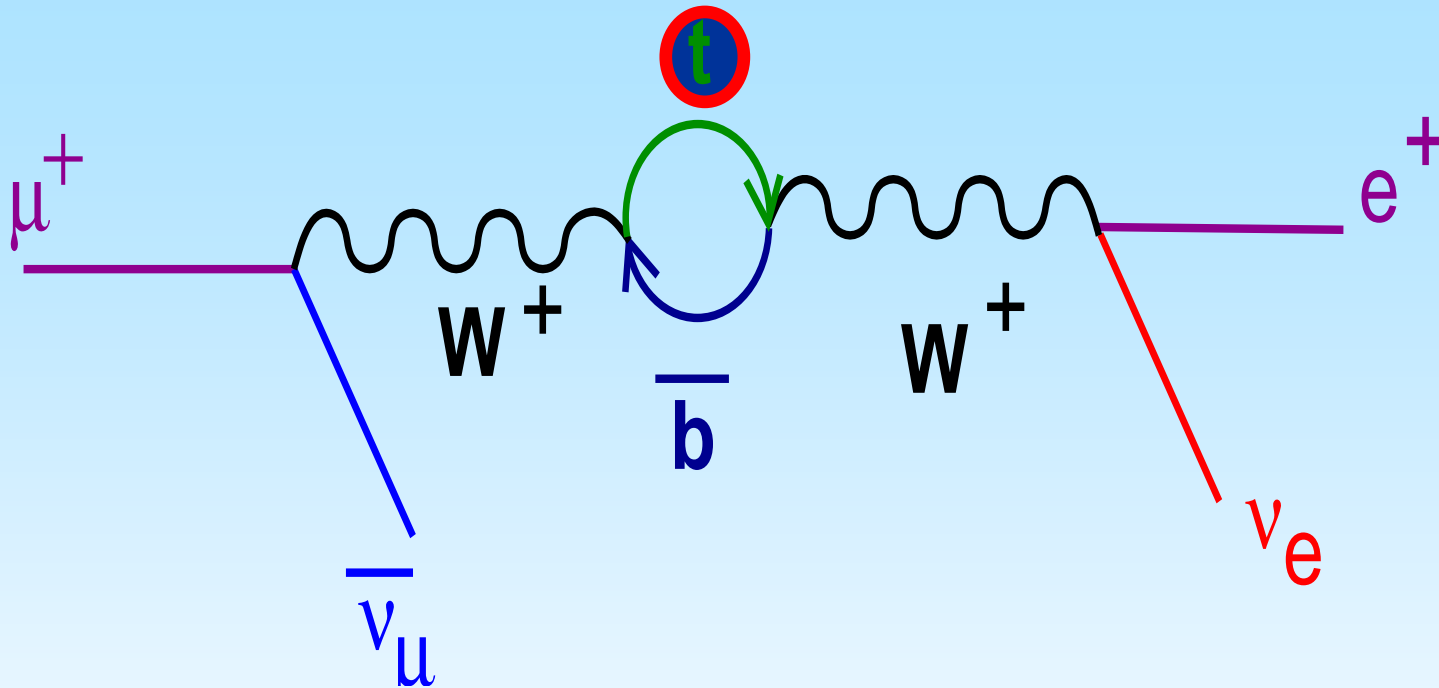
and $C_2 = \frac{156815}{5184} - \frac{518}{81}\pi^2 - \frac{895}{36}\zeta(3) + \frac{67}{720}\pi^4 + \frac{53}{6}\pi^2 \ln(2)$.

The $\mathcal{O}(\alpha^2)$ corrections to μ decay have been completed in 1999¹. The dominant theoretical uncertainty reduced from 16 to < 0.3 ppm.

τ_μ helped predict the mass of the top quark



τ_μ helped predict the mass of the top quark



Predictive power in weak sector of SM. Difference between the charged current $t\bar{b}$ and neutral current propagators $t\bar{t}$, $b\bar{b} \dots$. The radiative correction shown above depends on m_t^2 . Comparisons of charged, vrs. neutral currents gives information on m_t .

The Electro-Weak Working Group Fits:

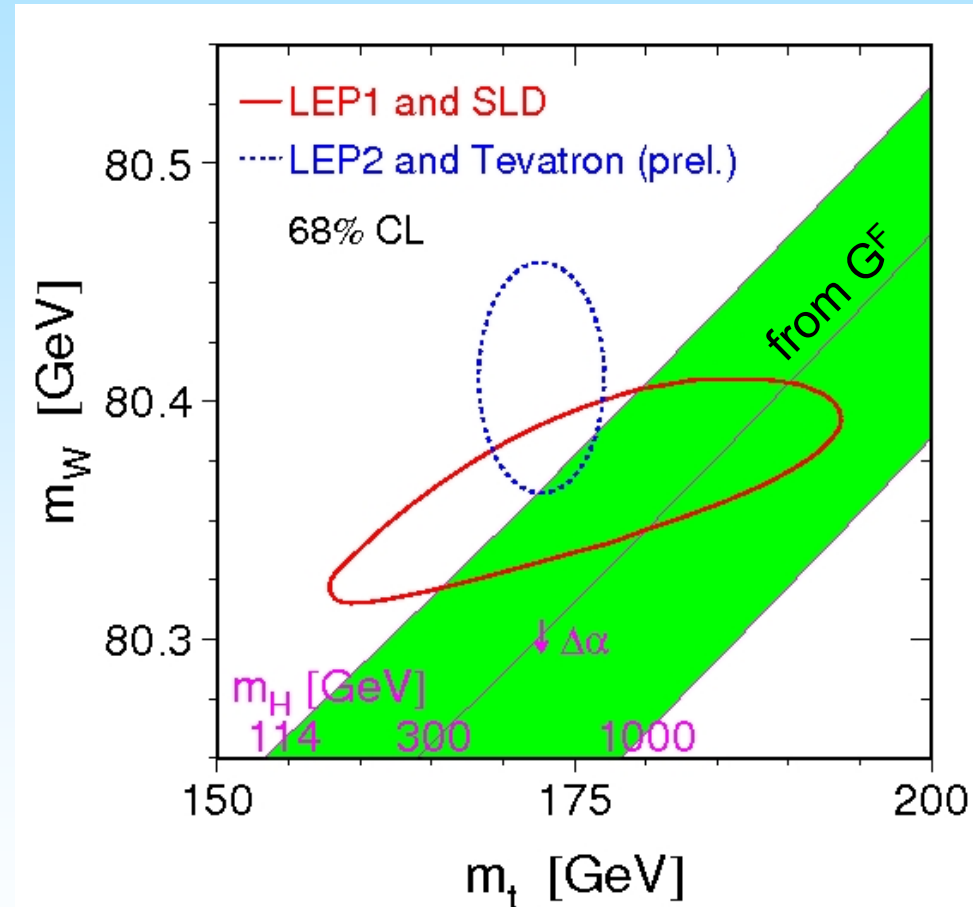
Predicted

$$m_t = 179_{-9}^{+12} \text{ GeV}$$

Input: G_F (17 ppm),
 α (4 ppb at $q^2=0$),
 αM_Z (23 ppm),

Measured:

$$m_t = 172.7 \pm 2.9 \text{ GeV}$$



The μ Lan experiment at PSI will accumulate 10^{12} μ -decays and measure G_μ to ~ 1 ppm. If LHC provides a Higgs Mass, then the precision of the confrontation with the SM will greatly improve

The Weak Lagrangian (Leptonic Currents)

- Lepton current is (vector - axial vector) "(V - A)"

$$J_{\sigma}^{\text{lept}} \propto \bar{u}(1 + \gamma_5)u$$

- It might have been: $V \pm A$ or $S \pm V \pm A$ or most general form:

$$\begin{aligned} & \text{Scalar} \pm \text{Vector} \pm \text{Weak-Magnetism} \\ & \pm \text{PseudoScalar} \pm \text{Axial-Vector} \pm \text{Tensor} \end{aligned}$$

There have been extensive studies at PSI by Gerber, Fetscher, et al. to look for other couplings in muon decay. Search continues with TWIST at TRIUMF.

At present, none have been found.

If the Strong Interaction is Present

- Then we have a more general current, which in principle can have 6 induced form factors in the current.

$$\mathcal{L} \propto J_{\lambda}^{\text{had}} J_{\text{lept}}^{\lambda} + HC$$

Leptonic and hadronic currents

- For nuclear μ^- capture (and also in β -decay) there are induced form-factors and the hadronic V-A current contains 6 terms.
 - in μ capture the induced pseudoscalar term becomes important

β - decay

$$\langle B | V_\lambda | A \rangle = \bar{u}(p') [g_V(q^2) \gamma_\lambda - g_{WM}(q^2) \sigma_{\lambda\nu} q_\nu - \cancel{ig_S(q^2) q_\lambda}] u(p)$$

vector
weak magnetism
scalar

2nd class

$$\langle B | A_\lambda | A \rangle = \bar{u}(p') [g_A(q^2) \gamma_\lambda \gamma_5 - \cancel{ig_P(q^2) q_\lambda \gamma_5} - g_T(q^2) \gamma_5 \sigma_{\lambda\nu} q_\nu] u(p)$$

axial vector
pseudoscalar
tensor

The Muon Trio:

- Muon Magnetic Dipole Moment a_μ chiral changing

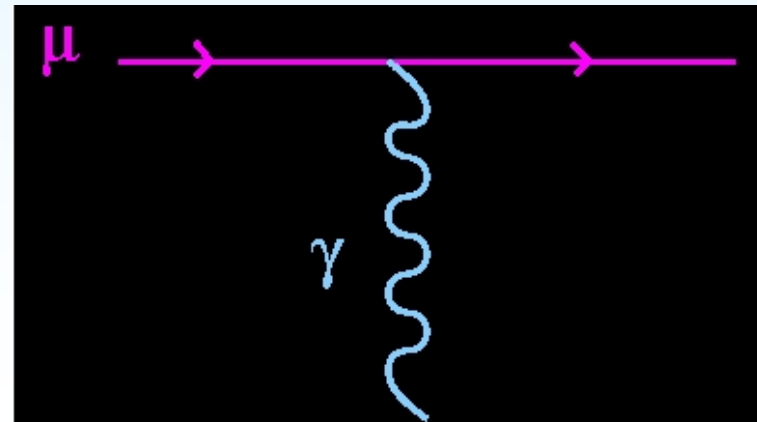
$$\bar{u}_\mu [ef_1(q^2)\gamma_\beta + \frac{ie}{2m_\mu} f_2(q^2)\sigma_{\beta\delta}q^\delta] u_\mu$$

$$\vec{\mu}_s = f_1(0) \left(\frac{e\hbar}{2m} \right) \vec{s} \quad f_2(0) = a_\mu \left(\frac{e\hbar}{2m} \right)$$

- Muon EDM

$$\bar{u}_\mu \left[\frac{ie}{2m_\mu} f_2(q^2) - f_3(q^2)\gamma_5 \right] \sigma_{\beta\delta}q^\nu u_\mu$$

$$f_2(0) = a_\mu \quad f_3(0) = d_\mu; \text{ EDM}$$



The Muon Trio:

- Muon Magnetic Dipole Moment a_μ **chiral changing**

$$\bar{u}_\mu [e f_1(q^2) \gamma_\beta + \frac{ie}{2m_\mu} f_2(q^2) \sigma_{\beta\delta} q^\delta] u_\mu$$

$$f_1(0) = 1 \quad f_2(0) = a_\mu$$

- Muon EDM

$$\bar{u}_\mu \left[\frac{ie}{2m_\mu} f_2(q^2) - f_3(q^2) \gamma_5 \right] \sigma_{\beta\delta} q^\nu u_\mu$$

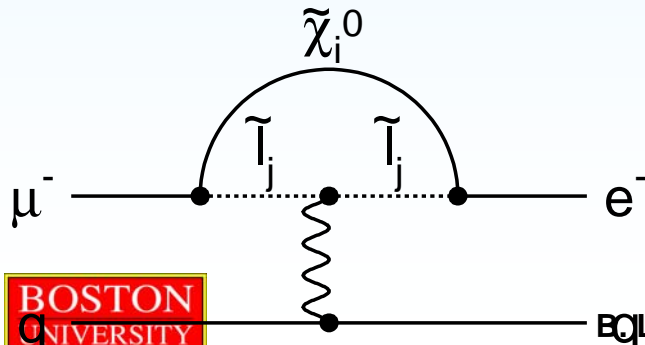
$$f_2(0) = a_\mu \quad f_3(0) = d_\mu; \text{ EDM}$$

- Lepton Flavor Violation

$$\mu^- A \rightarrow e^- A$$

$$\mu^+ \rightarrow e^+ \gamma$$

$$\mu^+ \rightarrow e^+ e^- e^+$$



Magnetic Moments (Field started by Stern)

1924.

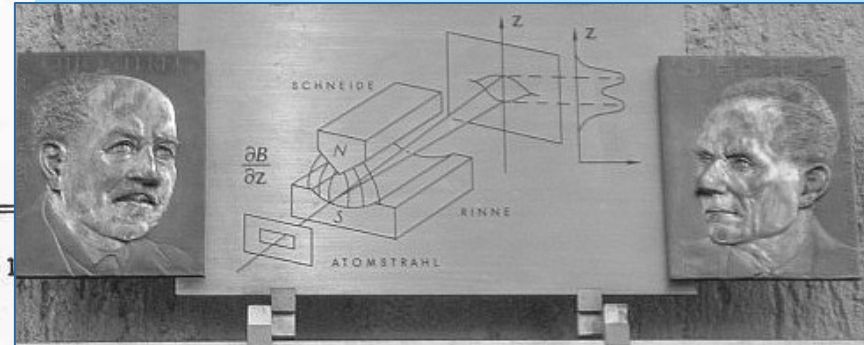
JUL 10.

ANNALEN DER PHYSIK. VIERTE FOLGE. BAND 74.

1. Über die Richtungsquantelung im Magnetfeld von Walther Gerlach und Otto Stern.

(Hierzu Tafel III.)

Nr. der Aufnahme	Entfernung des unabgelenkten Strahles von der Schneide	Mittlere Ablenkung des abgestoßenen Strahles	
		berechnet	beobachtet
15	0,32 mm	0,10 ₁ mm	0,10 ₀ mm
14	0,21 mm	0,14 ₈ mm	0,15 mm



IM FEBRUAR 1922 WURDE IN DIESEM GEBÄUDE DES PHYSIKALISCHEN VEREINS, FRANKFURT AM MAIN, VON OTTO STERN UND WALTHER GERLACH DIE FUNDAMENTALE ENTDECKUNG DER RAUMQUANTISIERUNG DER MAGNETISCHEN MOMENTE IN ATOMEN GEMACHT. AUF DEM STERN-GERLACH-EXPERIMENT BERUHEN WICHTIGE PHYSIKALISCH-TECHNISCHE ENTWICKLUNGEN DES 20. JHDTS., WIE KERNSPINRESONANZMETHODE, ATOMUHR ODER LASER. OTTO STERN WURDE 1943 FÜR DIESE ENTDECKUNG DER NOBELPREIS VERLIEHEN.

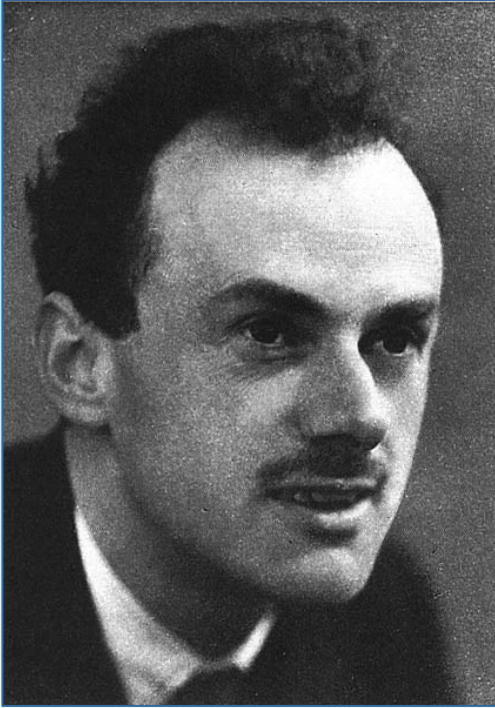
(in modern language)

(and in English)

$$\Rightarrow g = 2$$

$$\vec{\mu}_s = g_s \left(\frac{e\hbar}{2m} \right) \vec{s}$$

Dirac Equation Predicts $g=2$



Non-relativistic reduction of the Dirac Equation for an electron in a weak magnetic field.

$$i\hbar\frac{\partial\psi}{\partial t} = \left[\frac{p^2}{2m} - \frac{e}{2m}(\vec{L} + 2\vec{S}) \cdot \vec{B} \right] \psi$$

The Magnetic Moment of the Electron†

P. KUSCH AND H. M. FOLEY

Department of Physics, Columbia University, New York, New York

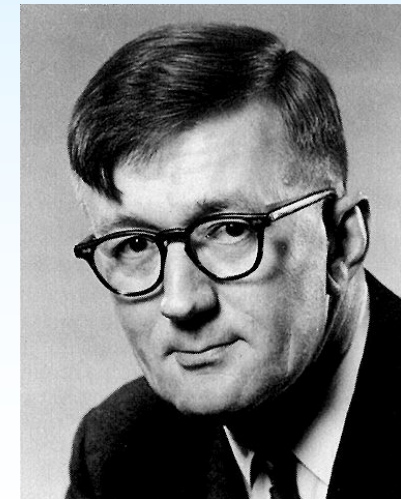
(Received April 19, 1948)

A comparison of the g_J values of Ga in the $^2P_{3/2}$ and $^2P_{1/2}$ states, In in the $^2P_{1/2}$ state, and Na in the $^2S_{1/2}$ state has been made by a measurement of the frequencies of lines in the hfs spectra in a constant magnetic field. The ratios of the g_J values depart from the values obtained on the basis of the assumption that the electron spin gyromagnetic ratio is 2 and that the orbital electron gyromagnetic ratio is 1. Except for small residual effects, the results can be described by the statement that The possibility that the observed effects may be explained by perturbations is precluded by the consistency of the result as obtained by various comparisons and also on the basis of theoretical considerations.

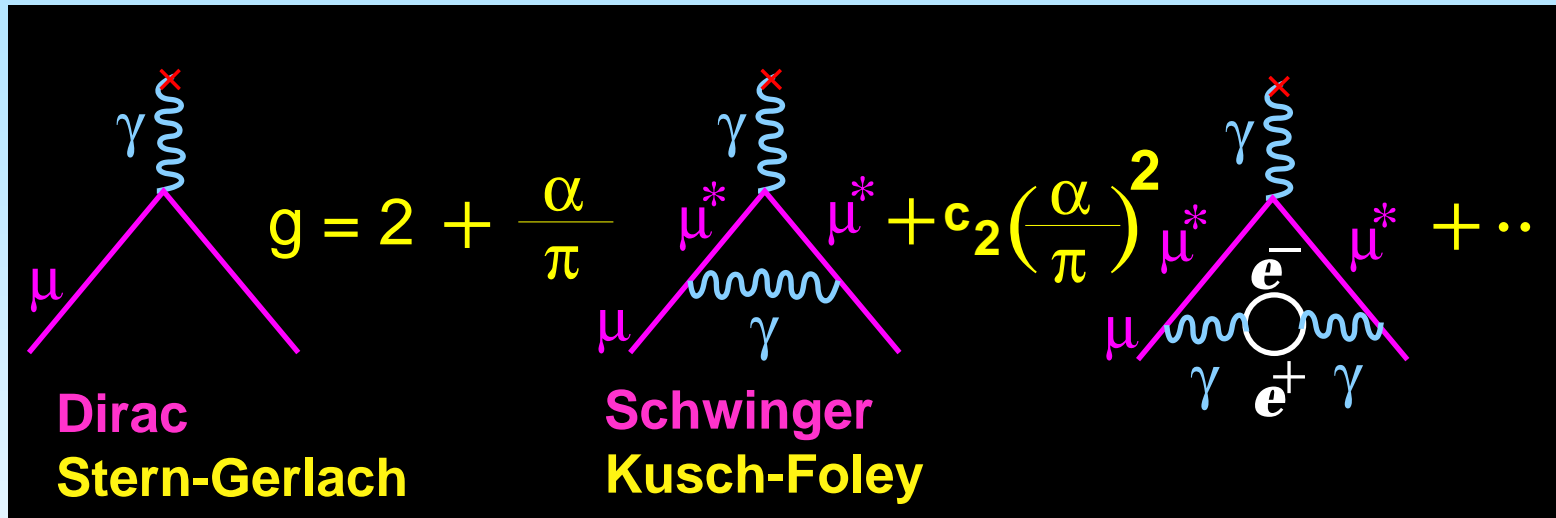
$$a = \frac{\alpha}{2\pi} = 0.001161 \quad \text{Schwinger}$$

$$\mu = (1 + a) \frac{e\hbar}{2m} \quad \text{Dirac + Pauli moment}$$

$$a = \frac{g - 2}{2}$$



Radiative corrections change g



$$a(\text{QED}) = \frac{1\alpha}{2\pi} + C_4 \left(\frac{\alpha}{\pi}\right)^2 + C_6 \left(\frac{\alpha}{\pi}\right)^3 + C_8 \left(\frac{\alpha}{\pi}\right)^4 + C_{10} \left(\frac{\alpha}{\pi}\right)^5 + \dots$$

a_μ is sensitive to a wide range of new physics

- substructure $\delta a_\mu(\Lambda_\mu) \simeq \frac{m_\mu^2}{\Lambda_\mu^2}$
- SUSY (with large $\tan\beta$)

$$a_\mu(\text{SUSY}) \simeq \frac{\alpha(M_Z)}{8\pi \sin^2 \theta_W} \frac{m_\mu^2}{\tilde{m}^2} \tan \beta \left(1 - \frac{4\alpha}{\pi} \ln \frac{\tilde{m}}{m_\mu} \right)$$
$$\simeq (\text{sgn}\mu) 13 \times 10^{-10} \tan \beta \left(\frac{100 \text{ GeV}}{\tilde{m}} \right)^2$$

- many other things (extra dimensions, etc.)

We measure the difference frequency between the spin and momentum precession

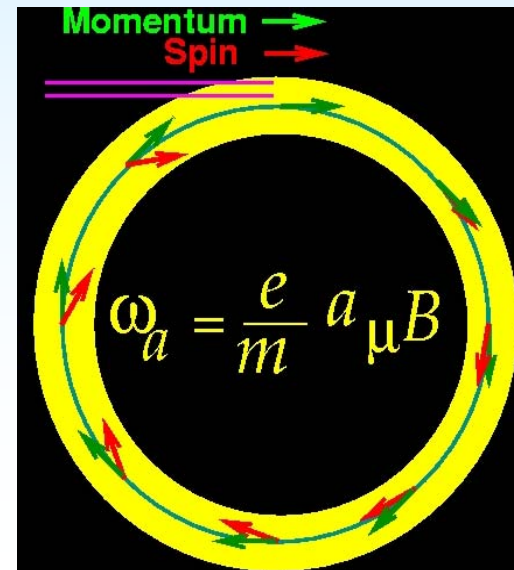
$$\omega_a = \omega_S - \omega_C = \left(\frac{g-2}{2} \right) \frac{eB}{mc} \quad B \Rightarrow \langle B \rangle_{\mu\text{-dist}}$$

With an electric quadrupole field for vertical focusing

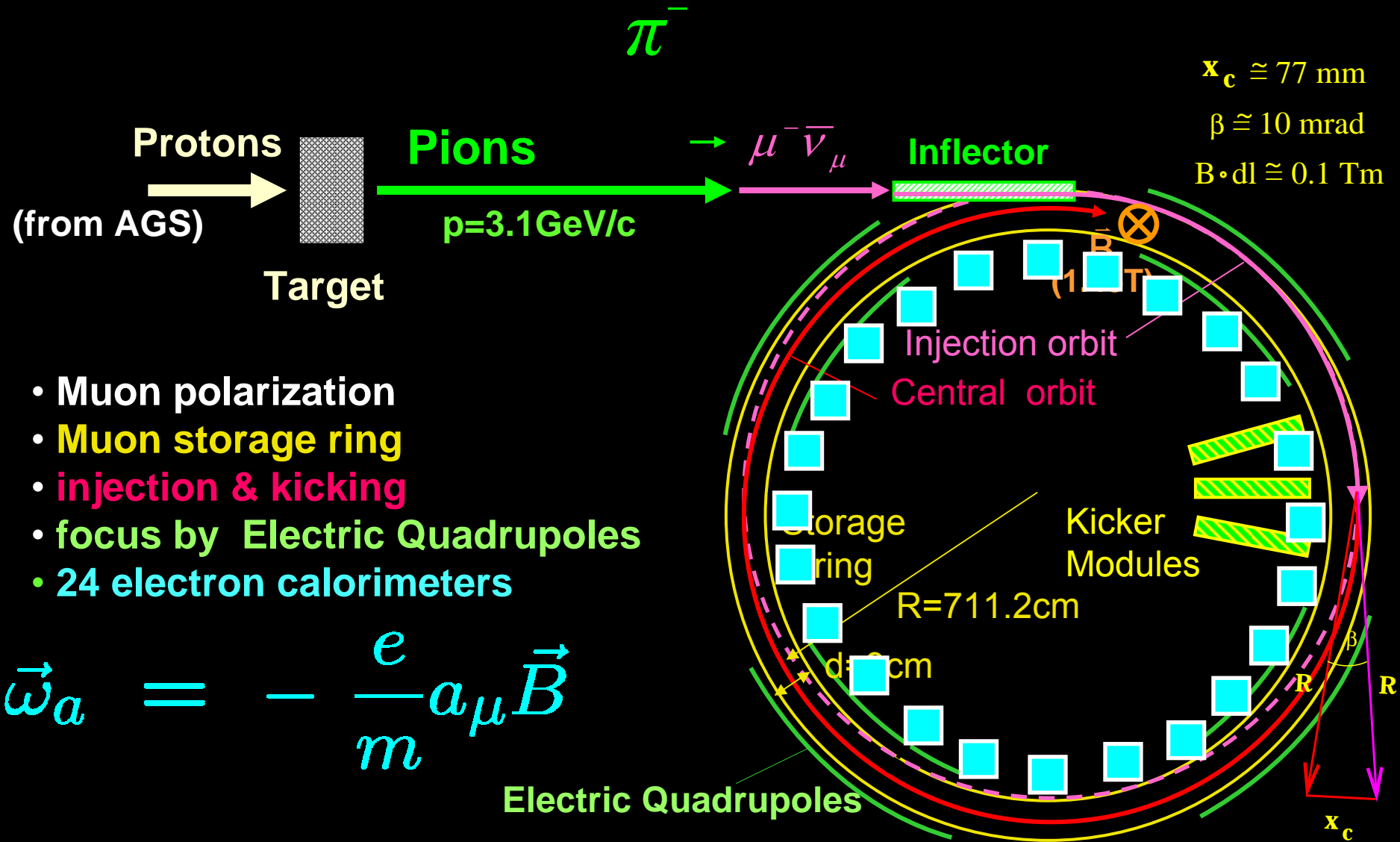
$$\vec{\omega}_a = - \frac{e}{m} \left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$

$$\gamma_{\text{magic}} = 29.3$$

$$p_{\text{magic}} = 3.09 \text{ GeV}/c$$

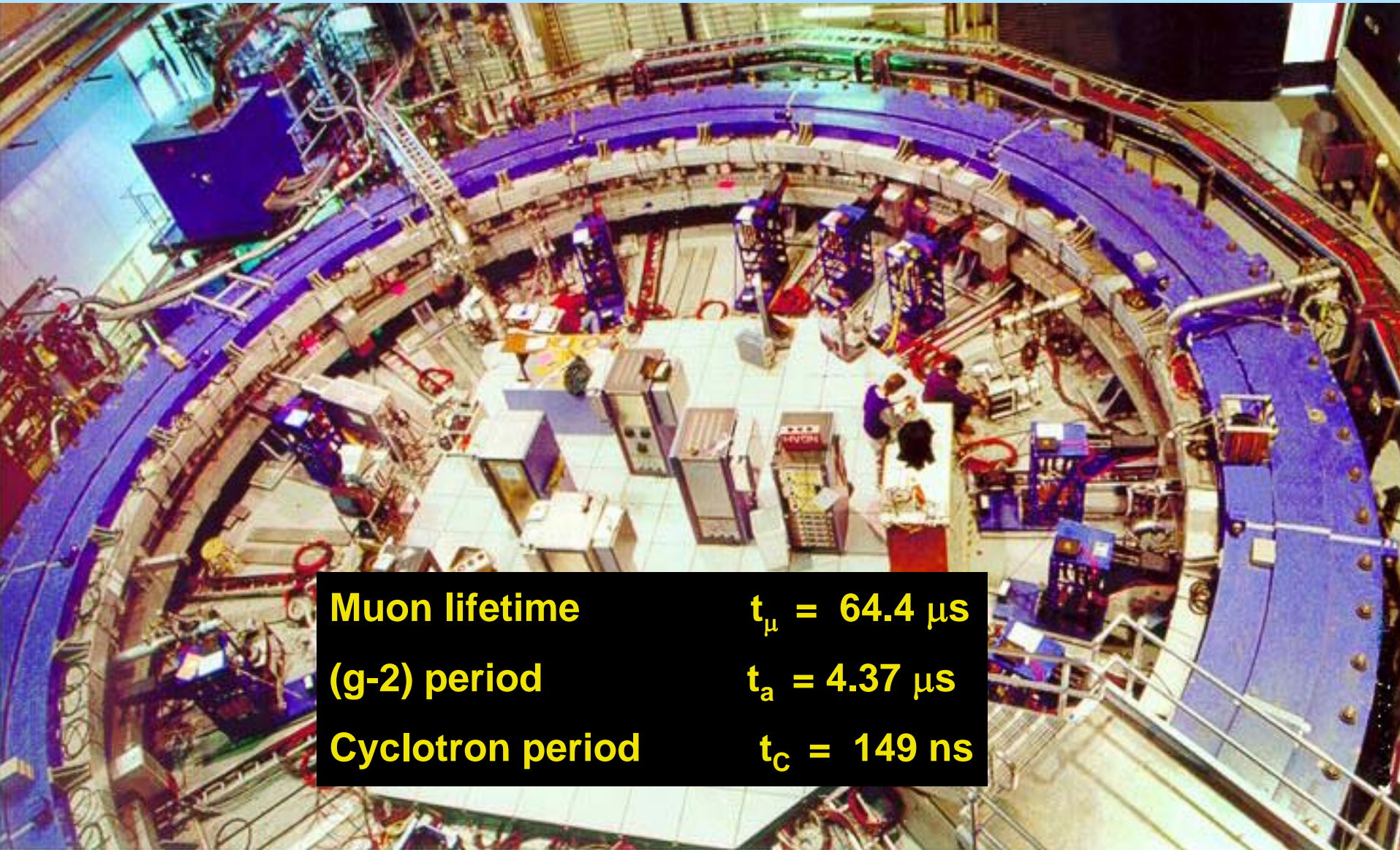


Experimental Technique



- Muon polarization
- Muon storage ring
- injection & kicking
- focus by Electric Quadrupoles
- 24 electron calorimeters

muon (g-2) storage ring



Muon lifetime

$$t_{\mu} = 64.4 \mu\text{s}$$

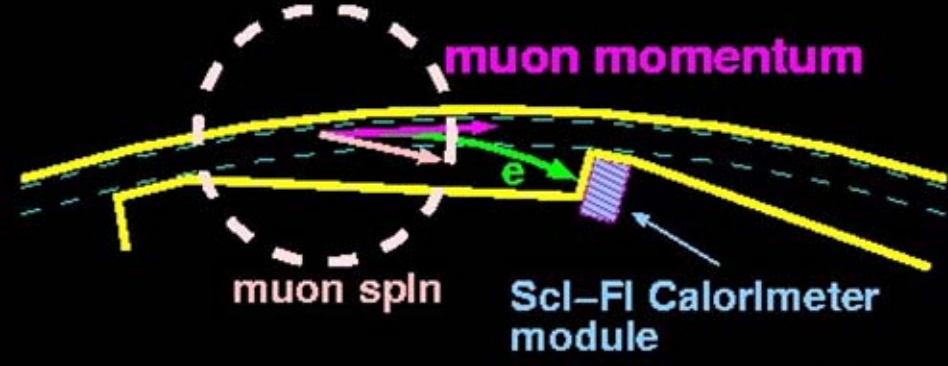
(g-2) period

$$t_a = 4.37 \mu\text{s}$$

Cyclotron period

$$t_c = 149 \text{ ns}$$

Detectors and vacuum chamber



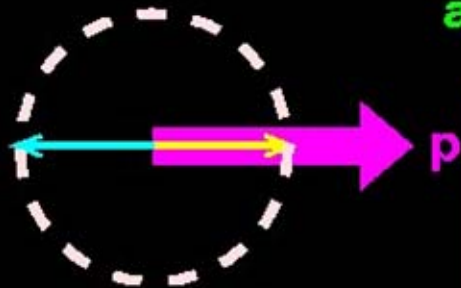
muon momentum

muon spin

Sci-Fi Calorimeter module

Measures Energy and time


e



p


spin forward, more high energy e

spin backward, less high energy e



76300 ns 76350 ns 76400 ns

400 MHz digitizer



R22

Sci-Fi Calorimeter module

400 MHz digitizer

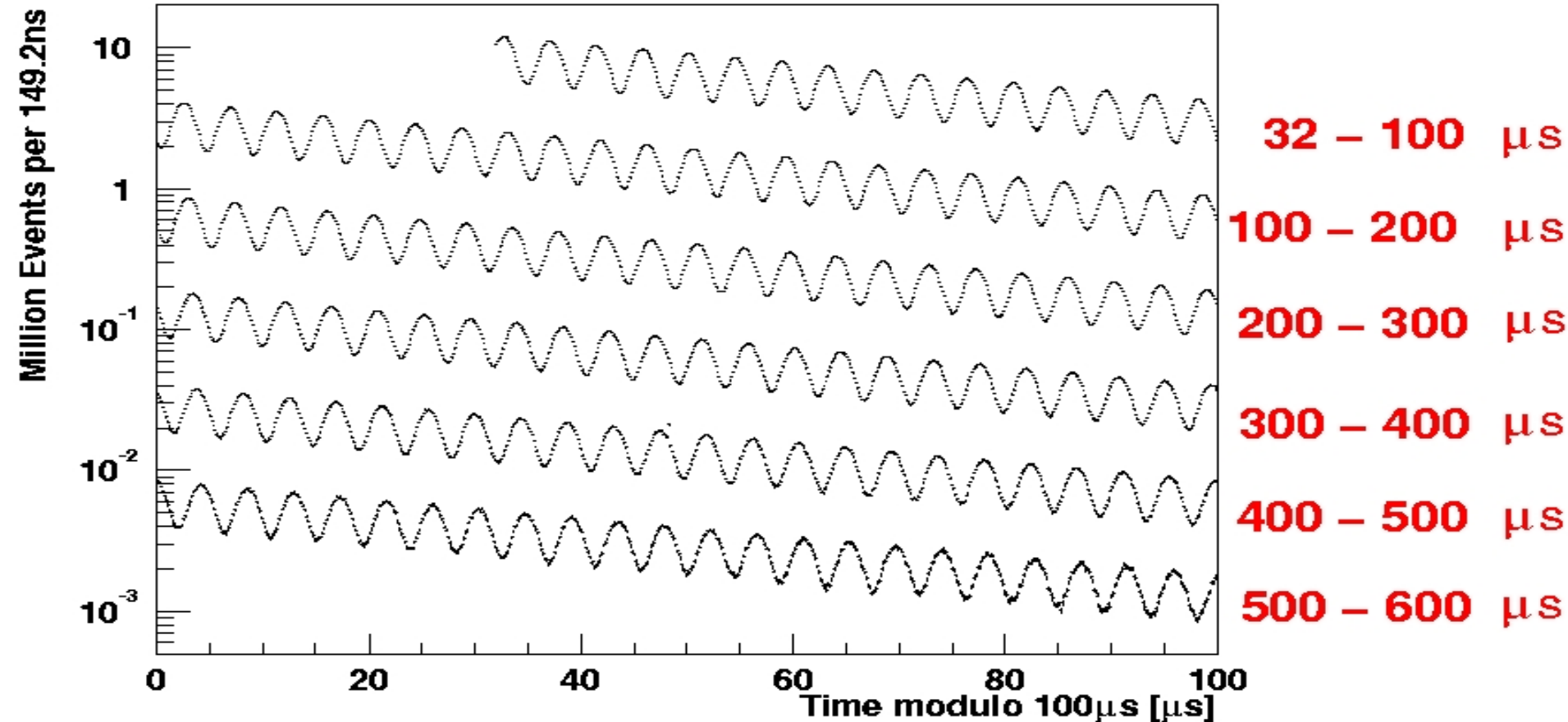
The photograph shows a blue metal rack containing several orange cylindrical modules, which are the Sci-Fi Calorimeter modules. The rack is labeled 'R22' and '400 MHz digitizer'. The modules are connected to various cables and components.

We count high-energy electrons as a function of time.

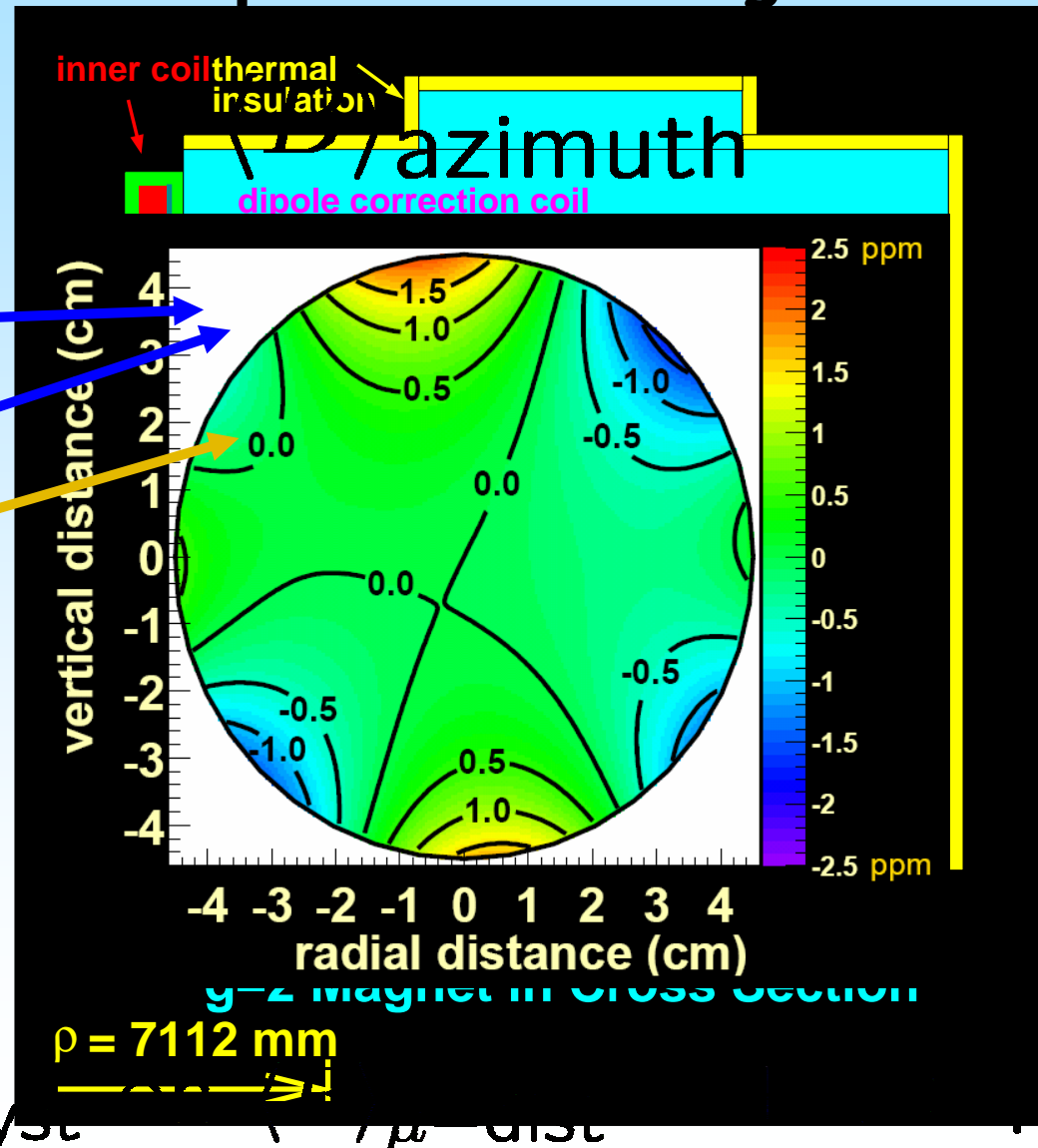
$$4 \times 10^9 \text{ e}, E_{e^-} \geq 1.8 \text{ GeV}$$

$$f(t) \simeq N_0 e^{-\lambda t} [1 + A \cos(\omega_a t + \phi)]$$

electron time spectrum (2001)



The ± 1 ppm uniformity in the average field is obtained with special shimming tools.



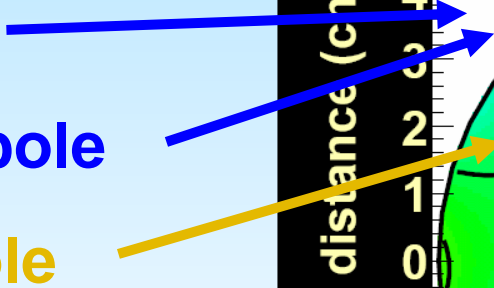
0.5 ppm contours

ppm

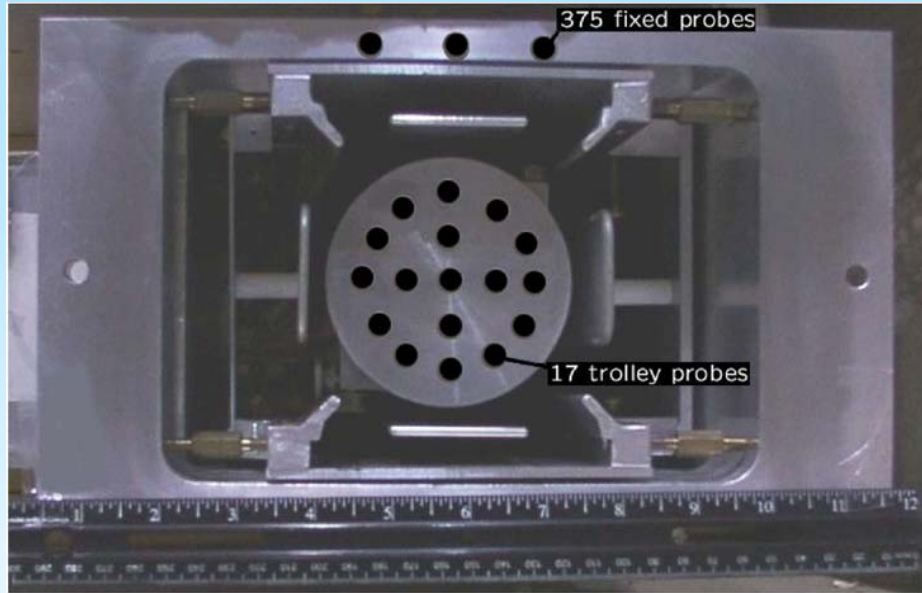
σ_{system}

$\frac{1}{\rho} \frac{dB}{dz}$

We can shim the
dipole,
quadrupole
sextupole
independently

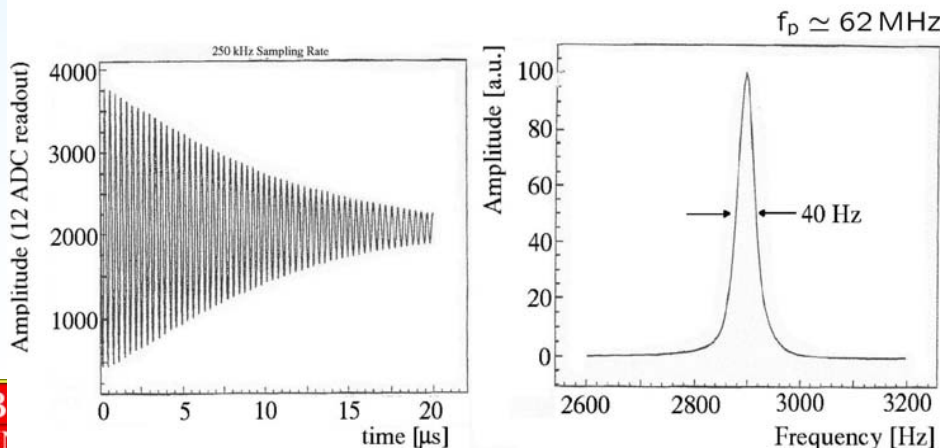


The magnetic field is measured and controlled using pulsed NMR and the free-induction decay.



- Calibration to a spherical water sample that ties the field to the Larmor frequency of the free proton ω_p .
- So we measure ω_a and ω_p

Free induction decay signals:

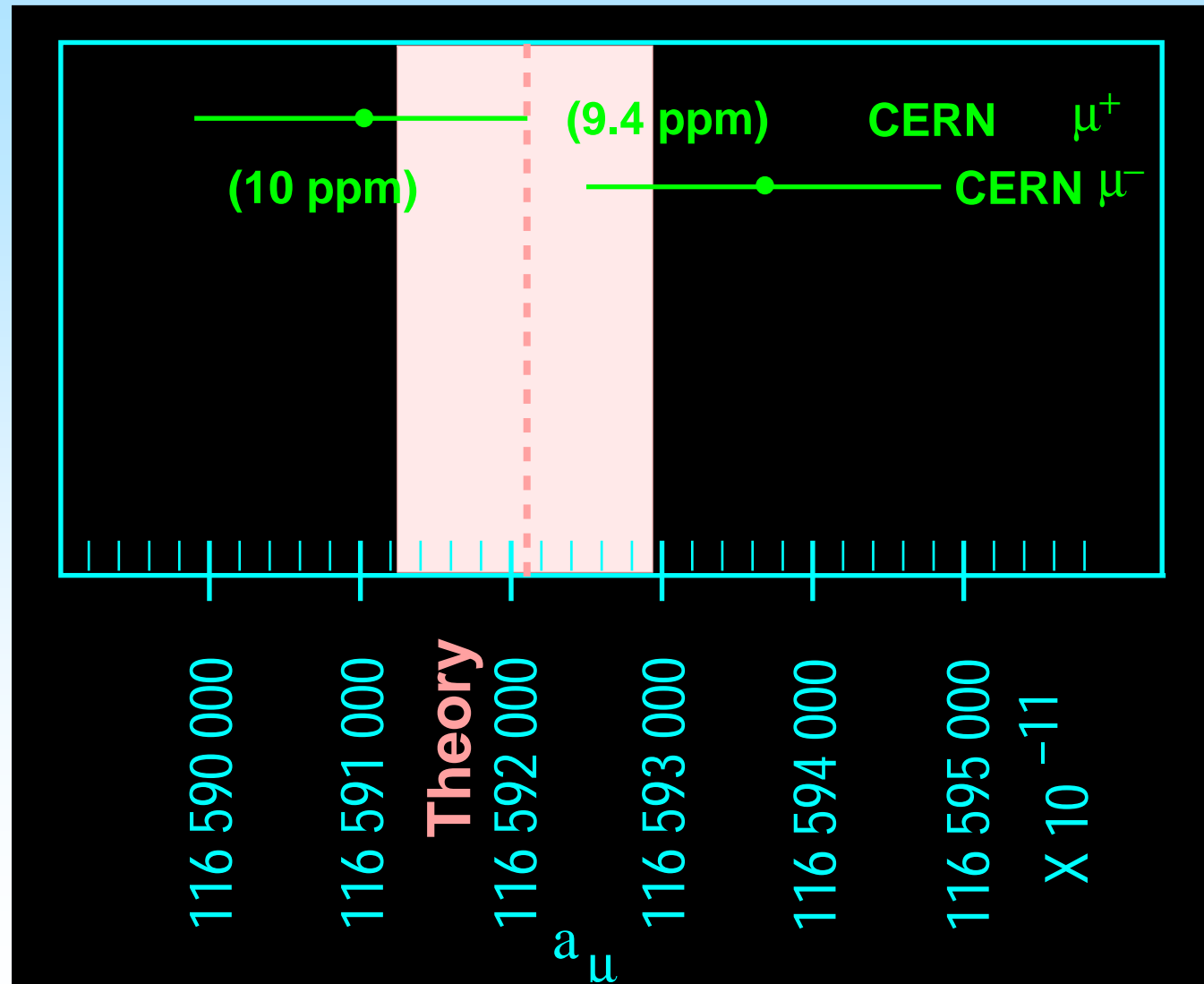


$$a_{\mu} = \frac{\frac{\omega_a}{\omega_p}}{\frac{\mu_{\mu}}{\mu_p} \frac{\omega_a}{\omega_p}}$$

When we started in 1983, theory and experiment were known to about 10 ppm.

Theory
uncertainty was
~ 9 ppm

Experimental
uncertainty was
7.3 ppm

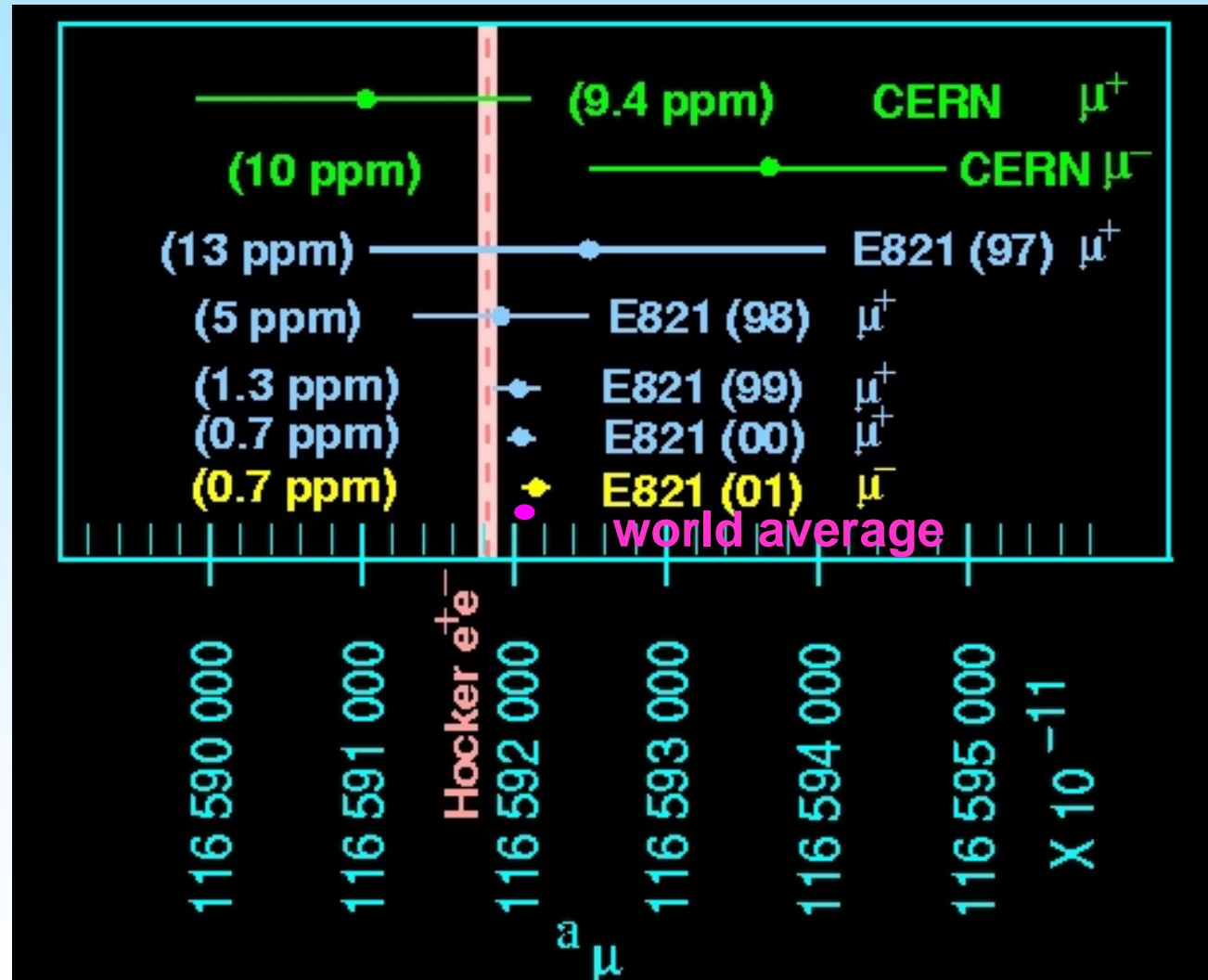


E821 achieved 0.5 ppm and the e^+e^- based theory is also at the 0.6 ppm level. Both can be improved.

All E821 results were obtained with a “blind” analysis.

$$\sigma_{\text{stat}} = \pm 0.46 \text{ ppm}$$

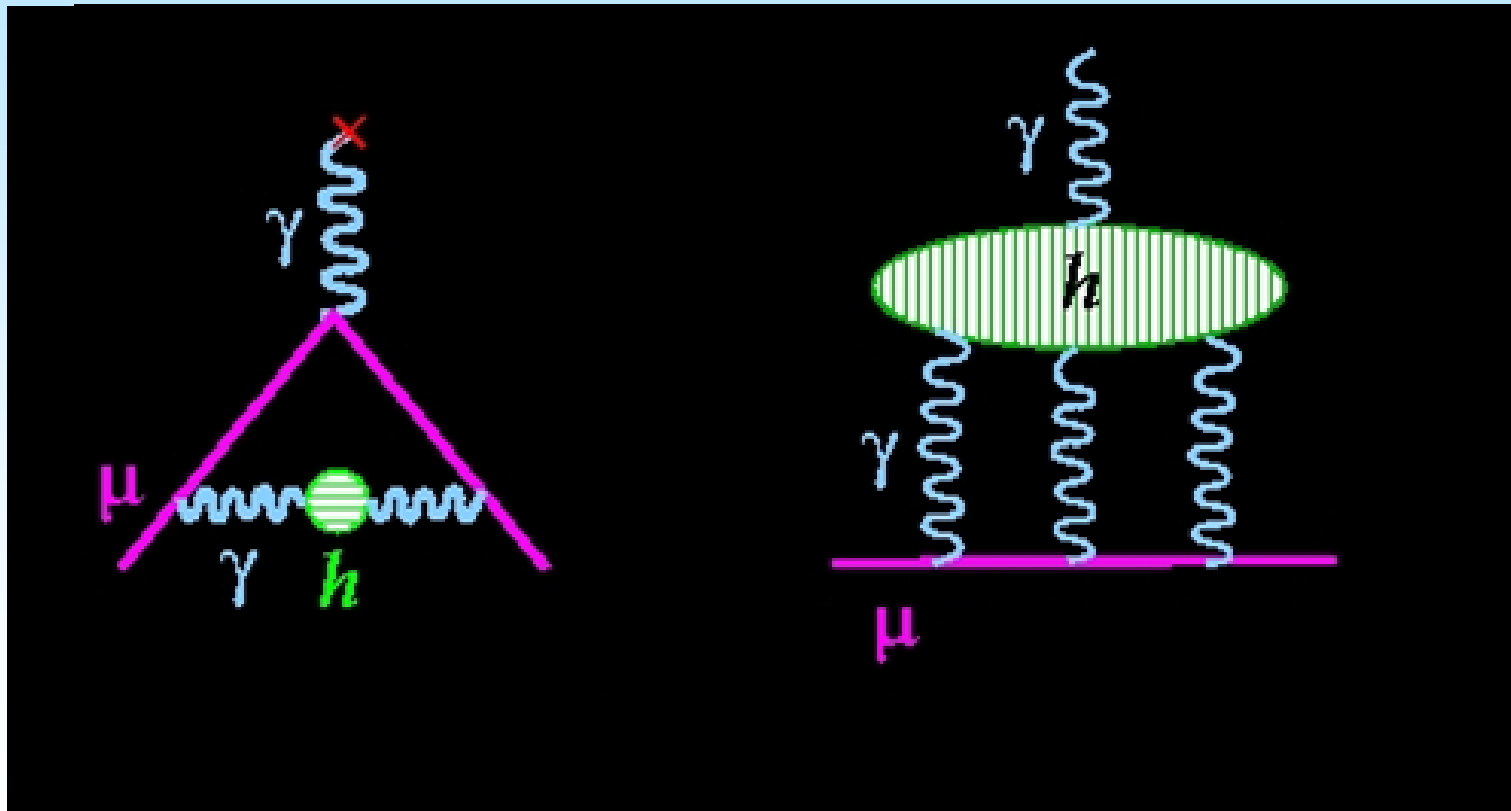
$$\sigma_{\text{syst}} = \pm 0.28 \text{ ppm}$$



$$a_\mu = 11\,659\,208(6) \times 10^{-10} \text{ (0.54 ppm)}$$

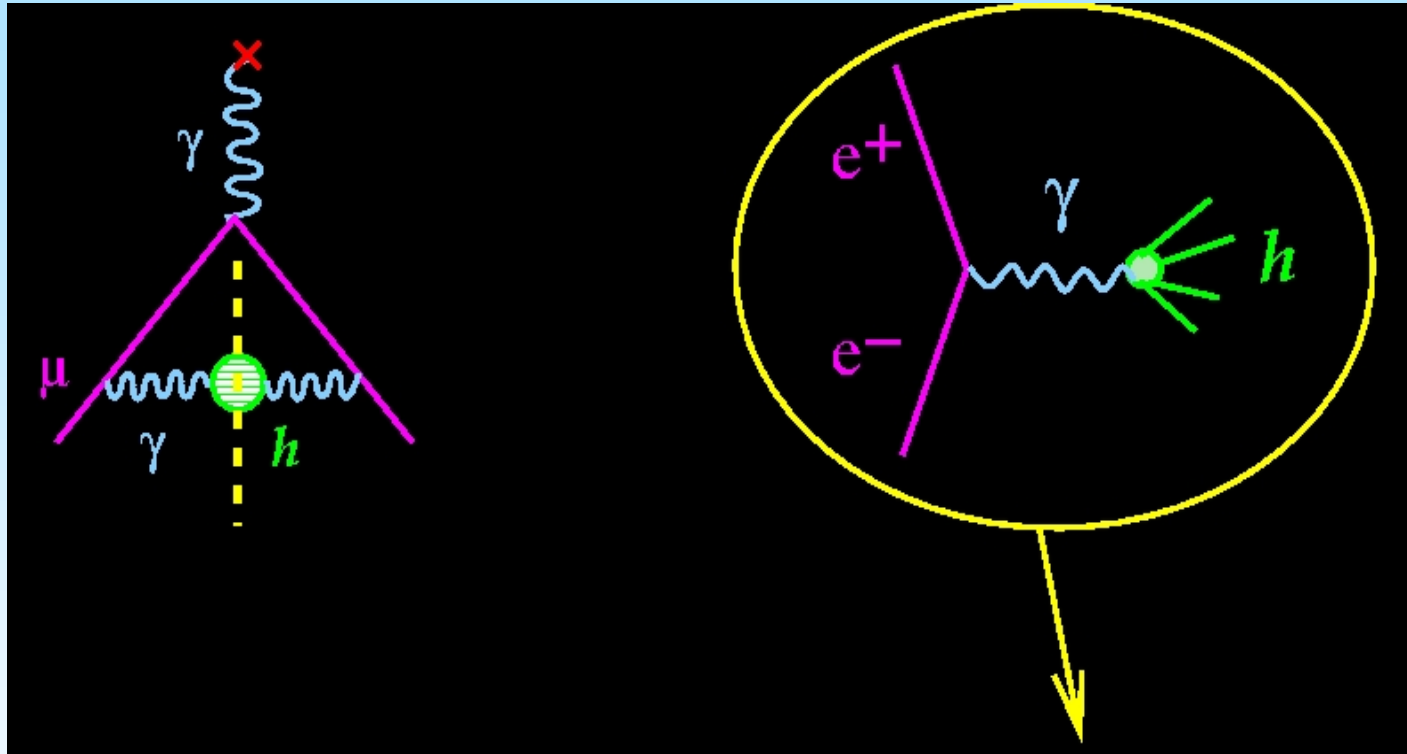
To compare with theory, there are two hadronic issues:

- Lowest order hadronic contribution
- Hadronic light-by-light



$$a_{\mu} = 11\,659\,208(6) \times 10^{-10} \text{ (0.54 ppm)}$$

Lowest Order Hadronic from e^+e^- annihilation



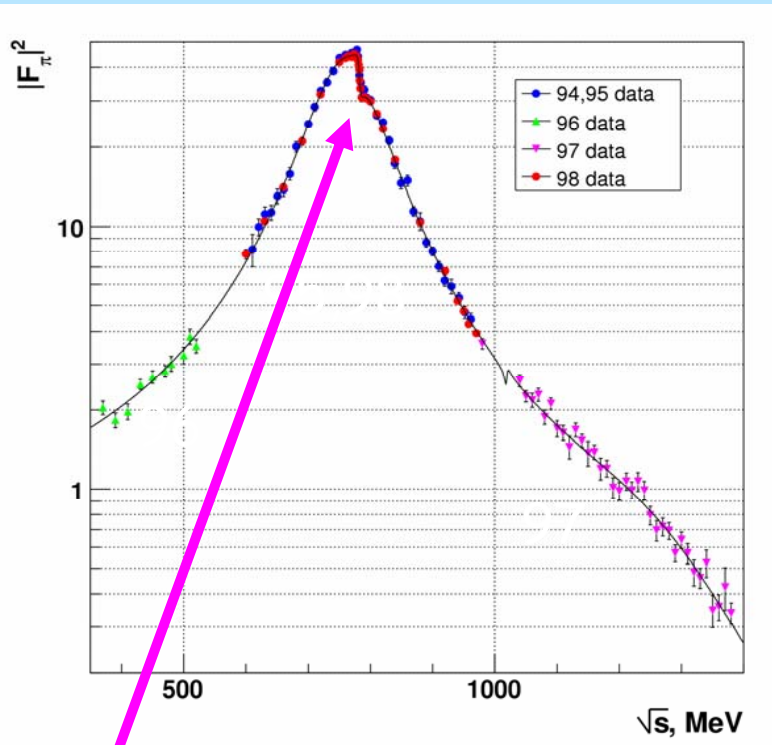
$$a_\mu(\text{had}) = \left(\frac{\alpha m_\mu}{3\pi} \right)^2 \int_{4m_\pi^2}^{\infty} \frac{ds}{s^2} K(s) \left(\frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \right)$$

Two experiments at the Budker Insitute at Novosibirsk have measured $R(s)$ to better than a percent.

CMD-2

1994-1995
114k $\pi^+\pi^-$

SND



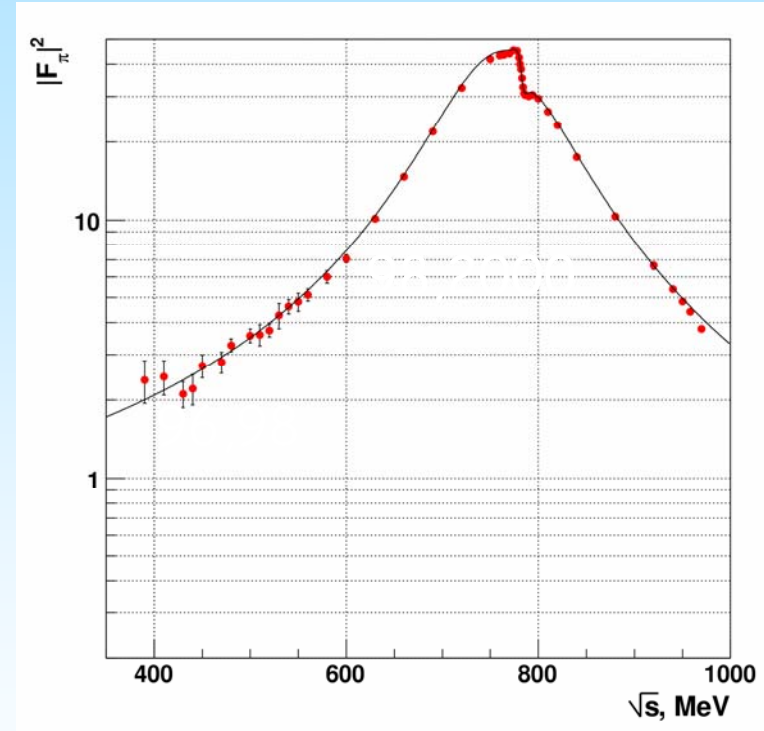
ρ - ω meson
interference

1996
4k $\pi^+\pi^-$

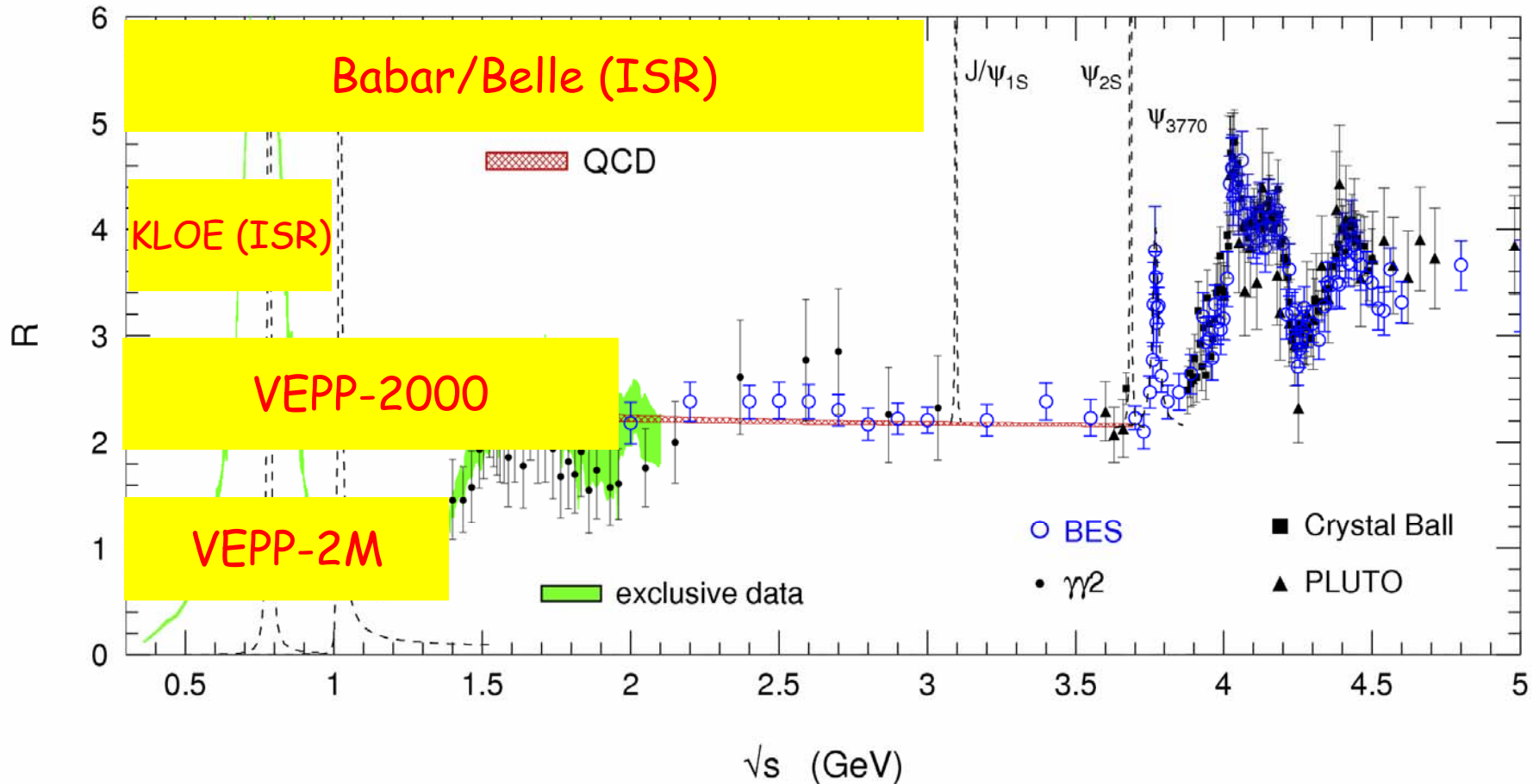
1997
33k $\pi^+\pi^-$

1998
 $\sim 1M$ $\pi^+\pi^-$

2000
 $\sim 2M$ $\pi^+\pi^-$



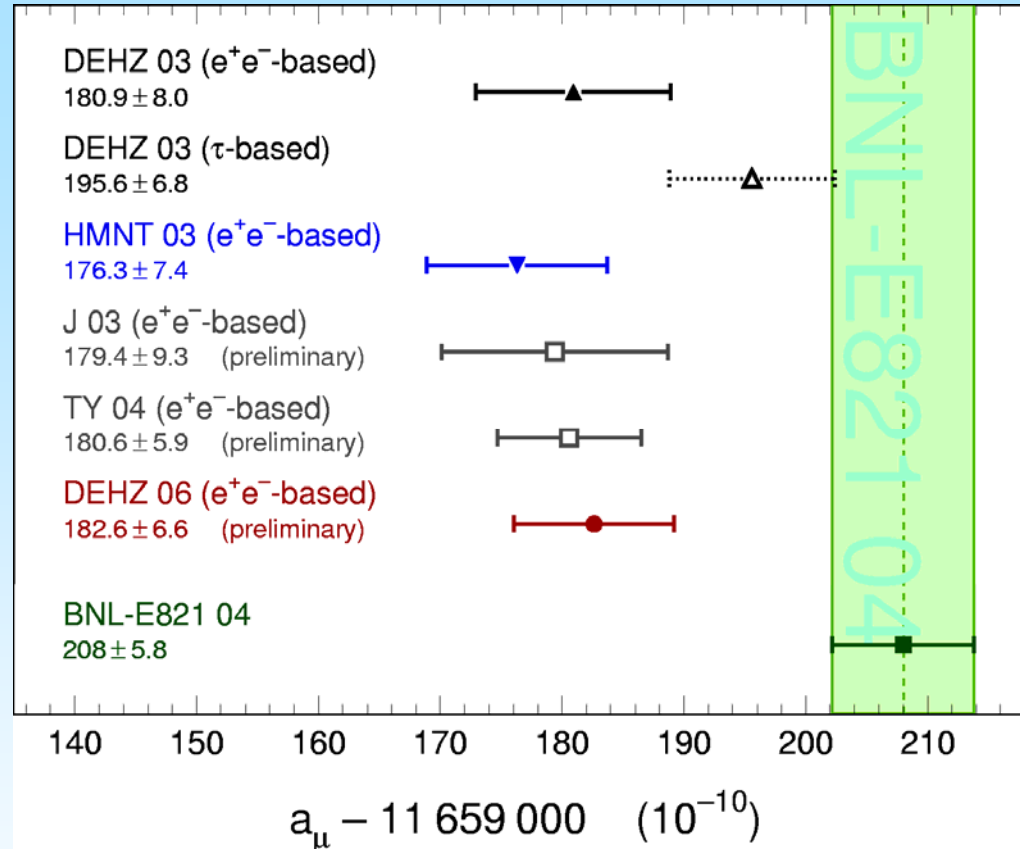
R(s) measurements at low s



At low s the cross-section is measured independently for each final state

from Davier/Höcker

Summary of muon ($g-2$) (from M. Davier at LM06)



$$a_\mu [\text{exp}] - a_\mu [\text{SM}] = (25.4 \pm 8.8) \times 10^{-10}$$

➔ 2.9 „standard deviations“

Electric and Magnetic Dipole Moments

Phys. Rev. 78 (1950)

LETTERS TO THE EDITOR

807

On the Possibility of Electric Dipole Moments for Elementary Particles and Nuclei

E. M. PURCELL AND N. F. RAMSEY

Department of Physics, Harvard University, Cambridge, Massachusetts

April 27, 1950

It is generally assumed on the basis of some suggestive theoretical symmetry arguments¹ that nuclei and elementary particles can have no electric dipole moments. It is the purpose of this note to point out that although these theoretical arguments are valid when applied to molecular and atomic moments whose electromagnetic origin is well understood, their extension to nuclei and elementary particles rests on assumptions not yet tested.

One form of the argument against the possibility of an electric dipole moment of a nucleon or similar particle is that the dipole's orientation must be completely specified by the orientation of the angular momentum which, however, is an axial vector specifying a direction of circulation, not a direction of displacement as would be required to obtain an electric dipole moment from electrical charges. On the other hand, if the nucleon should spend part of its time asymmetrically dissociated into opposite magnetic poles of the type that Dirac² has shown to be theoretically possible, a circulation of these magnetic poles could give rise to an electric dipole moment. To forestall a possible objection we may remark that this electric dipole would be a polar vector, being the product of the angular momentum (an axial vector) and the magnetic pole strength, which is a pseudoscalar in conformity with the usual

The argument against electric dipoles, in another form, raises directly the question of parity. A nucleon with an electric dipole moment would show an asymmetry between left- and right-handed coordinate systems; in one system the dipole moment

The authors wish to thank Mr. Smith for suggesting an important correction to our original calculation on the neutron-electron interaction experiment.

¹ A typical argument is given by H. A. Bethe, *Elementary Nuclear Theory* (John Wiley and Sons, Inc., New York).

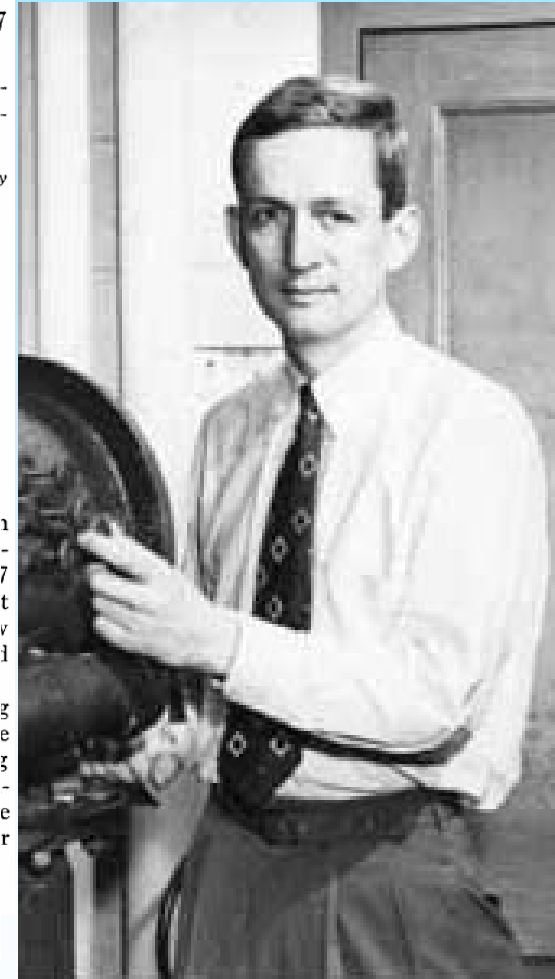
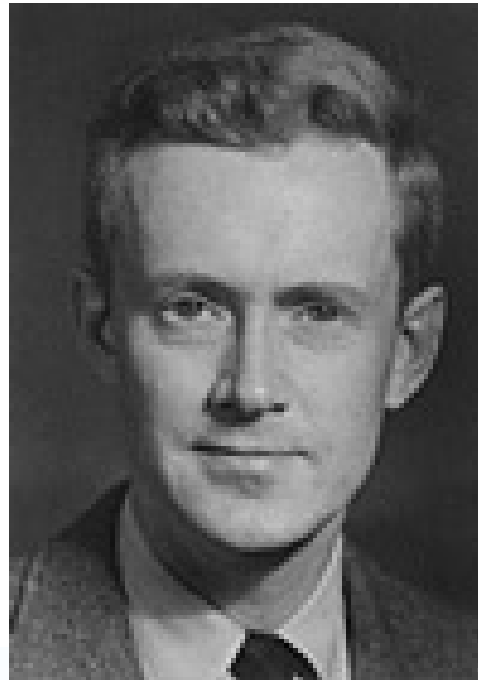
² P. A. M. Dirac, *Phys. Rev.* **74**, 817 (1948).

³ Havens, Rabi, and Rainwater, *Phys. Rev.* **72**, 634 (1947).

⁴ E. Fermi and L. Marshall, *Phys. Rev.* **72**, 1139 (1947).

⁵ L. W. Alvarez and F. Bloch, *Phys. Rev.* **57**, 111 (1940).

⁶ N. F. Ramsey, *Phys. Rev.* **76**, 996 (1949).



Transformation Properties of Electric and Magnetic Dipole Moments

$$\mathcal{H} = -\vec{\mu} \cdot \vec{B} - \vec{d} \cdot \vec{E}$$

$$\vec{\mu} \quad \vec{d} \parallel \text{to } \vec{\sigma}$$

	\vec{E}	\vec{B}	$\vec{\mu}$ or \vec{d}
P	-	+	+
C	-	-	-
T	+	-	-

An EDM implies both P and T are violated. An EDM at a measurable level would imply non-standard model

~~CP~~.

However, it should be emphasized that while such arguments are appealing from the point of view of symmetry, they are not necessarily valid.

Ultimately the validity of all such symmetry arguments must rest on experiment.

indicates the existence of a 27-day recurring period which has thus far been followed through four cycles. Presumably, this variation is associated with a solar phenomenon, characterized by this period, which introduces a modulating effect upon the primary cosmic-ray flux. A detailed discussion of these results will be published later.

* Assisted by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission, and by the U. S. National Committee for the IGY through the National Science Foundation. Field operations sponsored by the National Geographic Society.

¹ See review articles by H. Elliott, in *Progress in Cosmic-Ray Physics*, edited by J. G. Wilson (North-Holland Publishing Company, Amsterdam, 1952), Vol. 1, and by V. Sarabhai and N. W. Nerurkar, in *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1956), Vol. 6, p. 1.

² S. E. Forbush, *J. Geophys. Research* **59**, 525 (1954).

³ H. V. Neher and E. A. Stern, *Phys. Rev.* **98**, 845 (1955).

⁴ P. Meyer and J. A. Simpson, *Phys. Rev.* **99**, 151 (1955).

⁵ J. R. Winckler and K. A. Anderson, *Phys. Rev.* **108**, 148 (1957).

⁶ J. Winckler (private communication).

⁷ M. A. Pomerantz, *Phys. Rev.* **77**, 830 (1950); and G. W. McClure, *Phys. Rev.* **86**, 536 (1952).

⁸ M. A. Pomerantz, *Phys. Rev.* **102**, 870 (1956), and earlier references contained therein.

Time Reversal, Charge Conjugation, Magnetic Pole Conjugation, and Parity

N. F. RAMSEY

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Cambridge, Massachusetts*

(Received November 14, 1957)

UNTIL recently various theorems on the properties of elementary particles and of nuclei were based on parity arguments whose validity was questioned¹ only rarely. It is now known from the theoretical work of Lee and Yang² and from the experiments of Wu, Ambler, *et al.*³ and of others^{4,5} that the parity arguments are often not valid. Recently some of the properties previously derived from parity arguments have been rederived from other symmetry properties such as time-reversal invariance,^{6,7} invariance under the combined operation (*TCP*) of time reversal, charge conjugation, and parity,⁸⁻¹¹ etc. Landau,⁶ for example, has shown from a time-reversal argument that particles cannot possess electric dipole moments.

However, it should be emphasized that while such arguments are appealing from the point of view of symmetry, they are not necessarily valid. Ultimately the validity of all such symmetry arguments must rest upon experiment. For example, if magnetic mono-

coupled to north and to south poles, the conclusions drawn from the normal symmetry arguments would be modified. Dirac¹² has shown that it is theoretically possible that such magnetic poles should exist and that

their possibility of existence might be related to the experimentally observed quantization of electric charge.

In a theory which includes the effects of magnetic poles, the *TCP* theorem would be replaced by a *TMCP* theorem where *T* represents simple time reversal, *M* magnetic pole conjugation, *C* electric charge conjugation, and *P* simple inversion of space coordinates. It is of course possible to express the theorem in various ways, such as

$$(TM) (MC) (PM) = T' C' P',$$

where *T'* indicates an extended time reversal whose definition includes magnetic pole conjugation as well, *C'* represents conjugation of both electric and magnetic charges, and *P'* represents a parity transformation which includes magnetic pole conjugation as well. This method of writing has the advantage that consistency with Maxwell's equations requires that a simple parity transformation be accompanied by either magnetic pole conjugation or electric charge conjugation (but not both) since otherwise a magnetic field would be a mixture of a vector and a pseudovector depending on its mixed origin from magnetic and electric poles. On the other hand, this requirement would equally well be satisfied if the theorem were written as (*TC*) (*MC*) (*PC*). A still different but equivalent procedure leading to a *TMCP* or equivalent *TC'P* theorem could be based on treating the magnetic charges in the fundamental equations as pseudoscalars with respect to both space and time reflections.

Since the experimental observations of parity non-conservation, it has been generally assumed,^{6,7} pending further experiments,⁷ that there should be invariance under the combination of *P* and *C* together in which case from the usual *TCP* theorem, invariance under *T* alone is inferred. On the other hand, with the possibility of magnetic poles, the above *TMCP* theorem would apply and invariance under *P* and *C* would imply invariance under *T* and *M* together and not each alone. If this were the case, the present proof⁶ for the non-existence of electric dipole moments for particles would no longer apply; an electric dipole moment could be proportional to the product of a magnetic pole and a spin angular momentum in which case each would change sign under *TM*, but their product and resulting electric field would not. A particle (such as all presently observed particles) whose magnetic monopole is zero could still possess an electric dipole moment by the above mechanism provided it were differently coupled to fields of north pole particles than to those of south poles. Such a coupling asymmetry, in addition to making possible the existence of an electric dipole moment, would also imply an added possible particle degeneracy since magnetic pole conjugation alone would provide a transformation to a particle of opposite magnetic pole coupling asymmetry and opposite electric dipole moment while the electric charge would be unaltered.

N.F. Ramsey,

Phys. Rev. 109, 225 (1958)

Experimental Limit to the Electric Dipole Moment of the Neutron

J. H. SMITH,* E. M. PURCELL, AND N. F. RAMSEY

Oak Ridge National Laboratory, Oak Ridge, Tennessee, and Harvard University, Cambridge, Massachusetts

(Received May 17, 1957)

An experimental measurement of the electric dipole moment of the neutron by a neutron-beam magnetic resonance method is described. The result of the experiment is that the electric dipole moment of the neutron equals the charge of the electron multiplied by a distance $D = (-0.1 \pm 2.4) \times 10^{-20}$ cm. Consequently, if an electric dipole moment of the neutron exists and is associated with its angular momentum, its magnitude almost certainly corresponds to a value of D less than 5×10^{-20} cm.

1. INTRODUCTION

SEVERAL years ago Purcell and Ramsey¹ pointed out that the usual parity arguments for the non-existence of electric dipole moments for nuclei and elementary particles, although appealing from the point of view of symmetry, were not necessarily valid. In particular they pointed out that the validity of the parity assumption must rest on experimental evidence and that the experimental evidence was not as conclusive as then generally supposed in the case of nuclei and elementary particles, even though there was abundant evidence for the assumption in the case of electromagnetic forces. Analysis of the experimental evidence against the existence of electric dipole moments

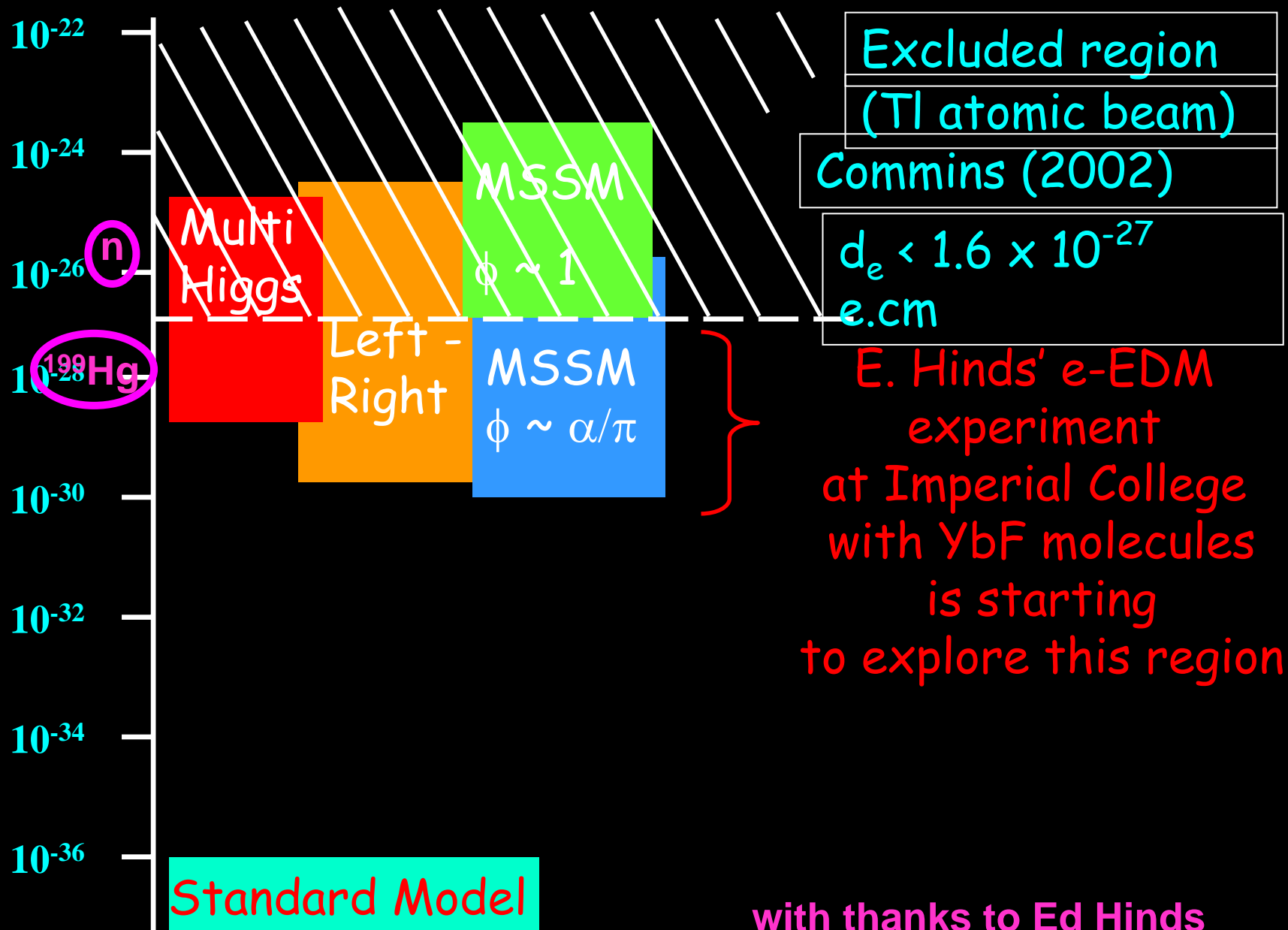
decay, the angular distributions of π - μ - e decays, and existence of electric dipole moments of particles. The effects of the first two of these have been observed by Wu, Ambler, Hayward, Hoppes, and Hudson,⁷ and by Garwin, Lederman, and Weinrich.⁸ Since electric moments are primarily determined by the strong forces, Lee and Yang⁶ showed that the effect of mixed parity should produce an electric dipole moment even smaller than the upper limit set by the experiment described in the present paper. In their most recent theories, Lee and Yang⁹ no longer anticipate the existence of an electric dipole moment for the neutron, and arguments involving time-reversal invariance^{9,10} can be advanced against its existence. These arguments, however, like the original ones of parity, can be questioned

Present EDM Limits

<i>Particle</i>	<i>Present EDM limit (e-cm)</i>	<i>SM value (e-cm)</i>
n	3×10^{-26} (90%CL)	10^{-32} to 10^{-31}
e^-	1.6×10^{-27} (90%CL)	$< 10^{-41}$
μ	$< 10^{-18}$ (CERN) $\sim 10^{-19}$ * (E821)	$< 10^{-38}$
^{199}Hg	2.1×10^{-28} (95%CL)	

*not yet final

e EDM (e.cm)



with thanks to Ed Hinds

Muon EDM: Naïve scaling would imply that

$$\left| \frac{d_\mu}{d_e} \right| \sim \frac{m_\mu}{m_e} \Rightarrow d_\mu < \mathcal{O}(10^{-25}) e \text{ cm}$$

but in some models the dependence is greater.

Enhanced Electric Dipole Moment of the Muon in the Presence of Large Neutrino Mixing

K. S. Babu,¹ B. Dutta,² and R. N. Mohapatra³

¹*Department of Physics, Oklahoma State University, Stillwater, Oklahoma 74078*

²*Center for Theoretical Physics, Department of Physics, Texas A & M University, College Station, Texas 77843*

³*Department of Physics, University of Maryland, College Park, Maryland 20742*

(Received 12 July 2000)

The electric dipole moment (edm) of the muon (d_μ^e) is evaluated in supersymmetric models with nonzero neutrino masses and large neutrino mixing arising from the seesaw mechanism. It is found that if the seesaw mechanism is embedded in the framework of a left-right symmetric gauge structure, the interactions responsible for the right-handed neutrino Majorana masses lead to an enhancement in d_μ^e to values as large as $5 \times 10^{-23} e \text{ cm}$, with a correlated value of $(g - 2)_\mu \approx 13 \times 10^{-10}$. This should provide a strong motivation for improving the edm of the muon to the level of $10^{-26} e \text{ cm}$ as has recently been proposed.

$$\Delta_{\text{now}} = (25.4 \pm 8.8) \times 10^{-10}$$

Spin Frequencies: μ in B field with MDM & EDM

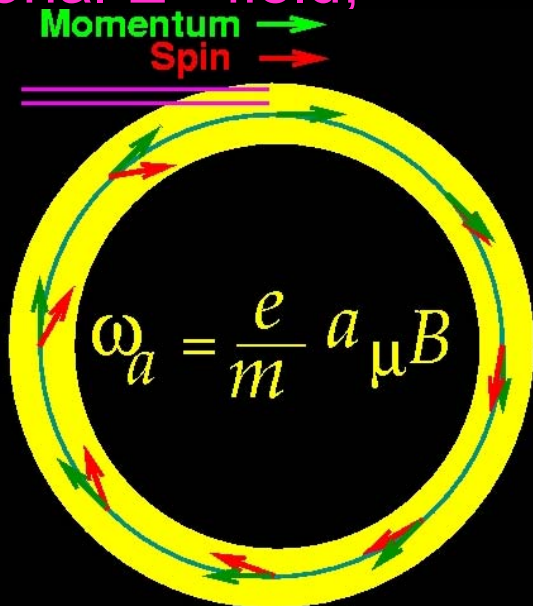
$$\omega_C = \frac{eB}{mc\gamma}$$

$$\omega_S = \frac{geB}{2mc} + (1 - \gamma) \frac{eB}{\gamma mc}$$

$$\omega_a = \omega_S - \omega_C = \left(\frac{g - 2}{2} \right) \frac{eB}{mc}$$

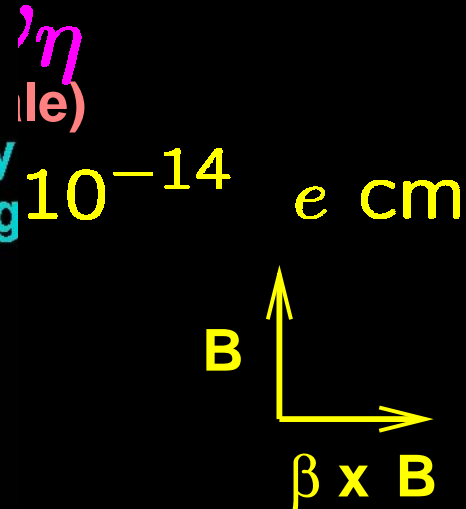
$$\gamma_{\text{magic}} = 29.3 + \frac{e a \eta}{m} \left[\frac{\vec{E} mc}{-c\omega_C} + \vec{\beta} \times \vec{B} \right]$$

The motional E - field, $\beta \times B$, is than lab fields ($\sim C$)



The EDM spin to p of plane.

The highest energy decay e^{\pm} are along the muon spin direction



Dedicated EDM Experiment

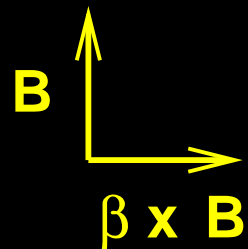
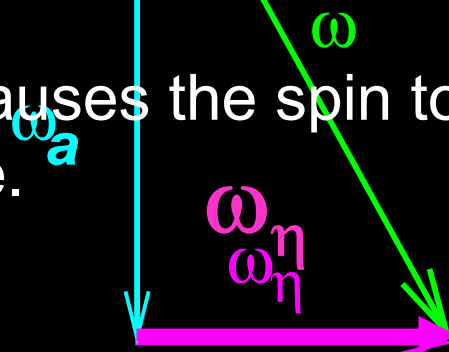
$$\vec{\omega} = -\frac{e}{m} \left[a_{\mu} \vec{B} - \left(a_{\mu} - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right] \quad 0$$

Use a radial E-field to turn off the ω_a precession

$$+ \frac{e}{m} \left[\frac{\eta}{2} \left(\frac{\vec{E}}{c} + \vec{\beta} \times \vec{B} \right) \right]$$

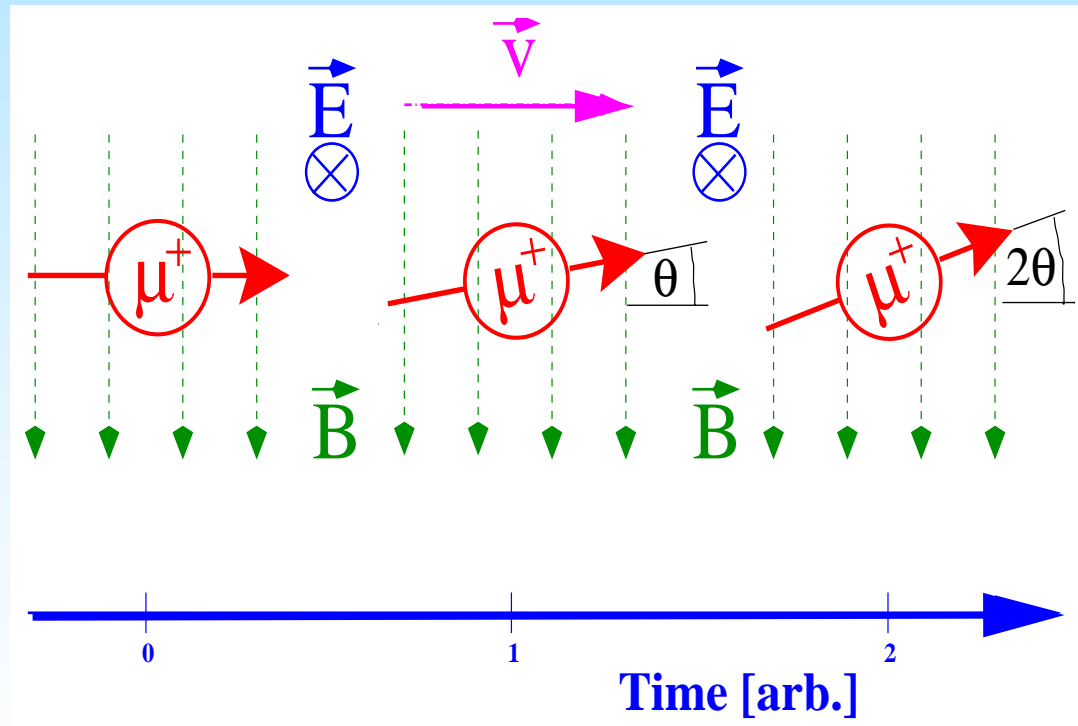
(not to scale)

With $\omega_a = 0$, the EDM causes the spin to steadily precess out of the plane.



"Frozen spin" technique

- Turn off the $(g-2)$ precession with radial E
- Look for an up-down asymmetry building up with time



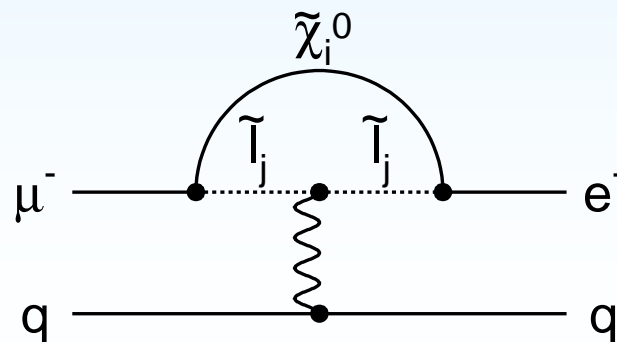
Lepton Flavor Violation (the transition moment)

$$\mu^- A \rightarrow e^- A$$

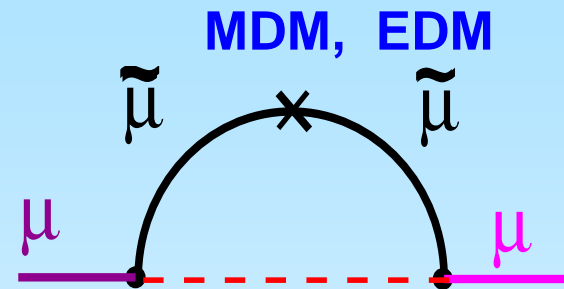
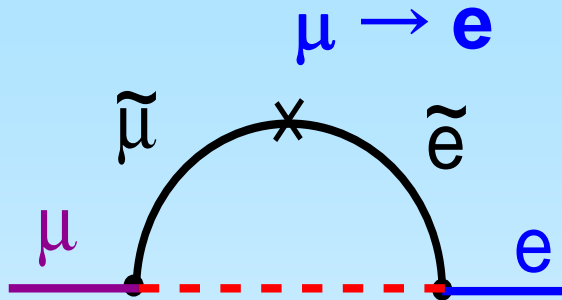
$$\mu^+ \rightarrow e^+ \gamma$$

$$\mu^+ \rightarrow e^+ e^- e^+$$

- The standard-model gauge bosons do not permit leptons to mix, but new physics at the TeV scale such as SUSY does.



SUSY connection between a_μ , D_μ , $\mu \rightarrow e$



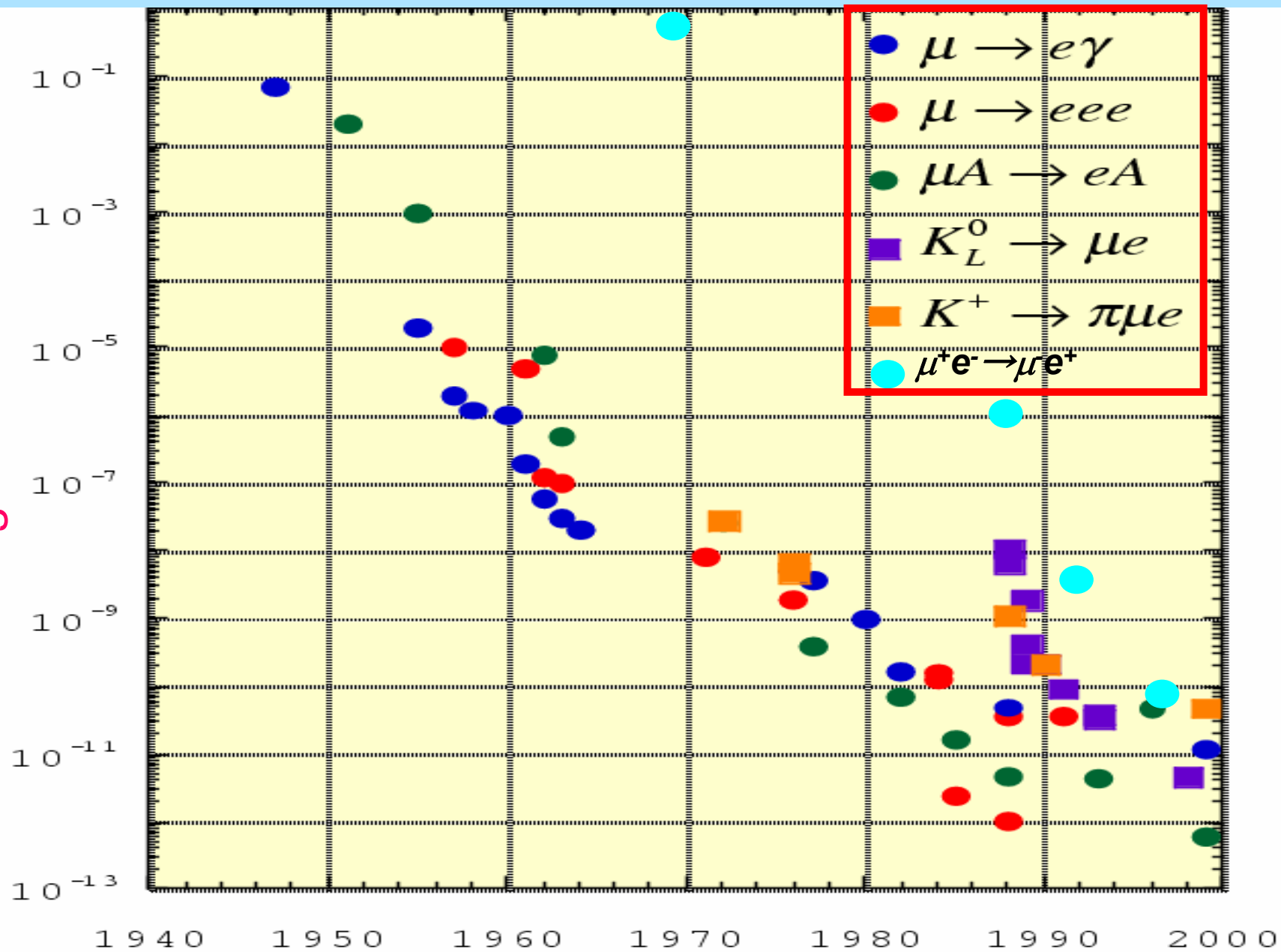
\tilde{B}

\tilde{B}

$$\left(\begin{array}{ccc} m_{\tilde{e}\tilde{e}}^2 & \Delta m_{\tilde{e}\tilde{\mu}}^2 & \Delta m_{\tilde{e}\tilde{\tau}}^2 \\ \Delta m_{\tilde{\mu}\tilde{e}}^2 & m_{\tilde{\mu}\tilde{\mu}}^2 & \Delta m_{\tilde{\mu}\tilde{\tau}}^2 \\ \Delta m_{\tilde{\tau}\tilde{e}}^2 & \Delta m_{\tilde{\tau}\tilde{\mu}}^2 & m_{\tilde{\tau}\tilde{\tau}}^2 \end{array} \right)$$

Past and Future of LFV Limits

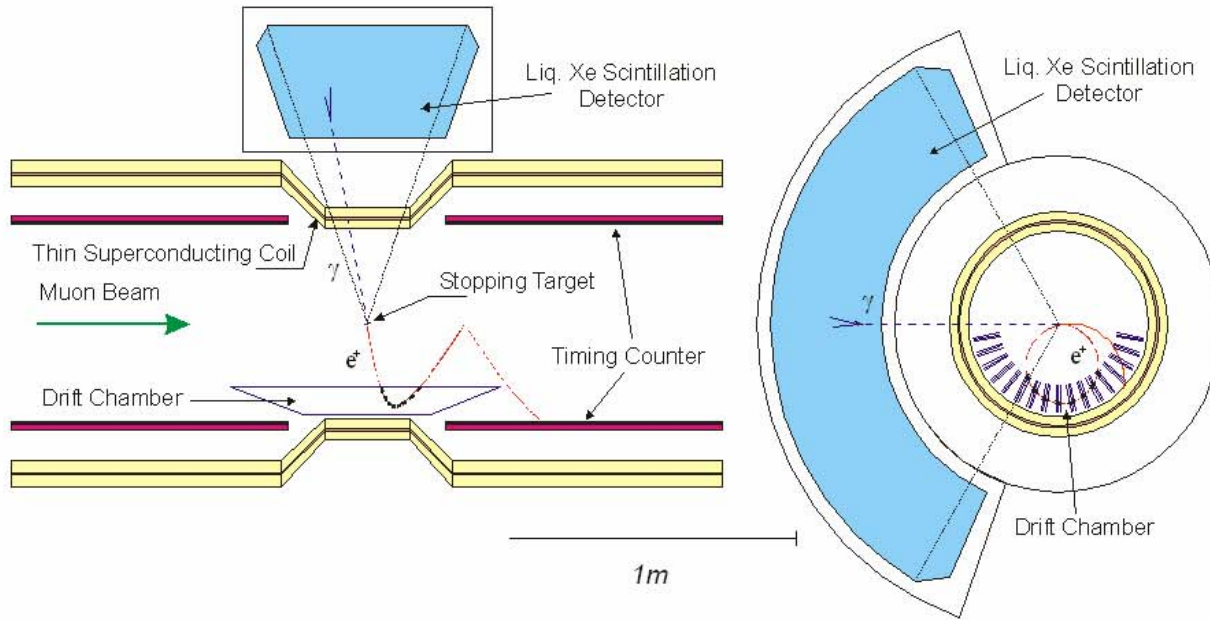
Branching Ratio Limit



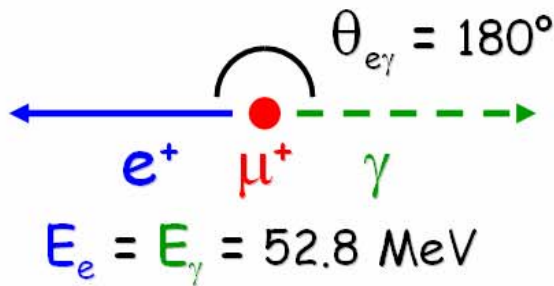
MEG Experiment (@PSI)

$$\mu^+ \rightarrow e^+ \gamma$$

The Detector



- Muon beam stopped in a 150 μm target
- Liquid Xenon calorimeter for γ detection (using scintillation light)
- (Thin wall) solenoid spectrometer & drift chambers for e^+ momentum
- Scintillation counters for e^+ timing



Easy signal selection for μ^+ decaying at rest

Discovery Potential: Single event sensitivity $\sim 1 \times 10^{-13}$

Summary and Outlook

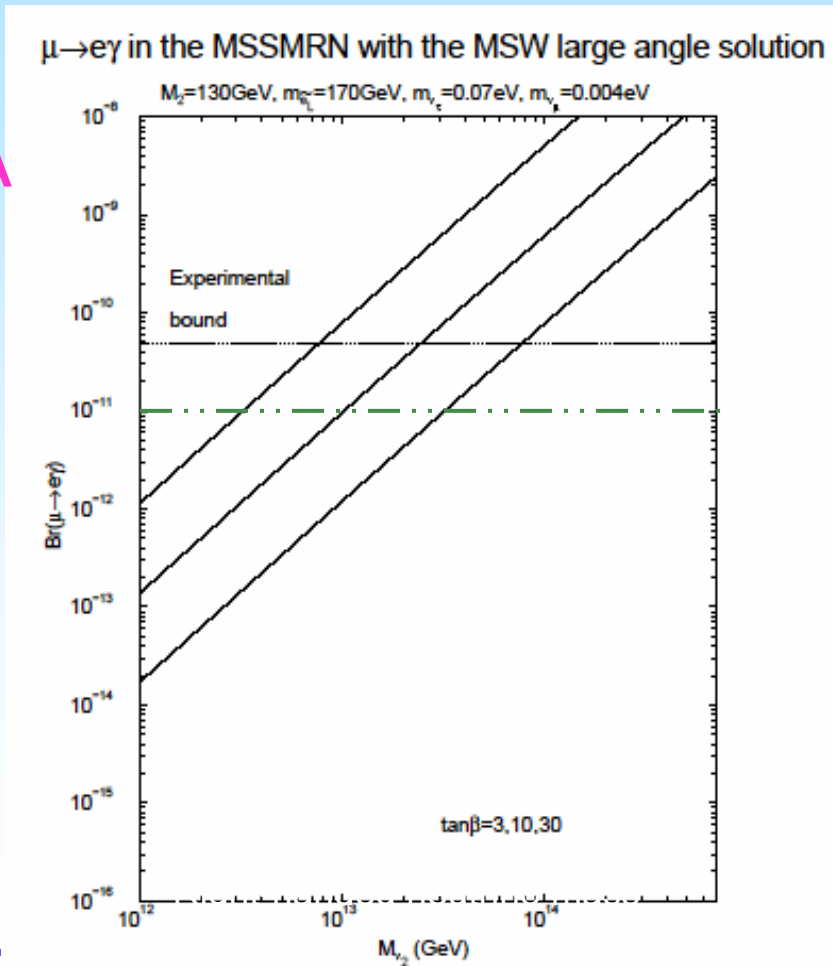
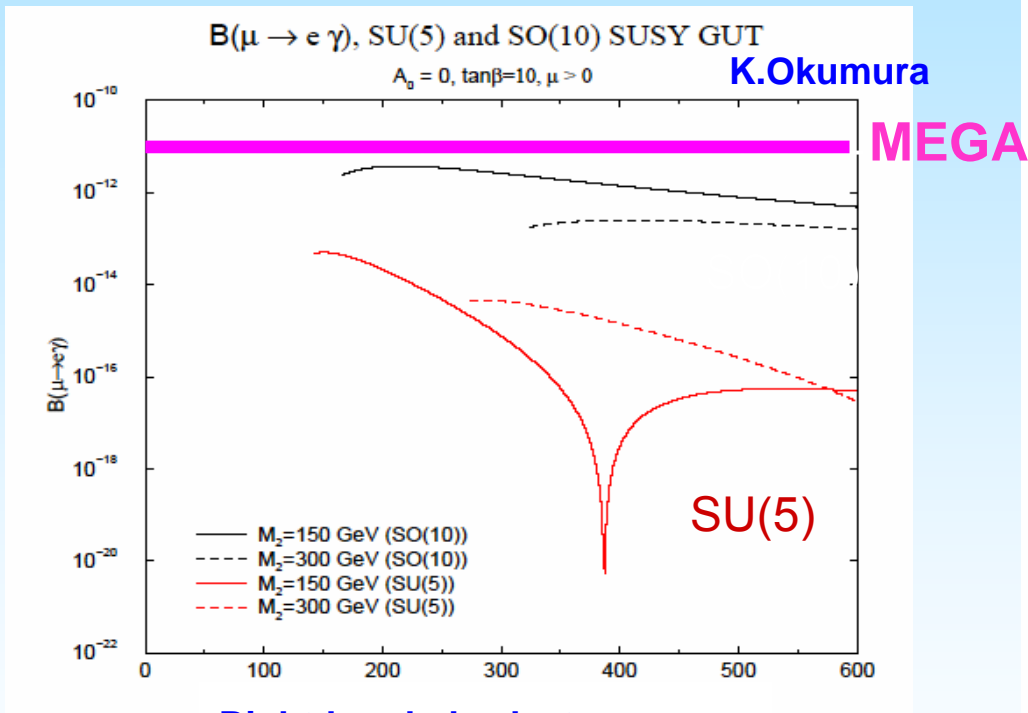
- The muon has provided us with much knowledge on how nature works.
- New experiments on the horizon continue this tradition.
- Muon ($g-2$), with a precision of 0.5 ppm, has a 2.9σ discrepancy with the standard model.
 - Upgrade, E969 waits for funding decision in 10/06
- This new physics, if confirmed, would show up in an EDM and perhaps LFV as well.
- MEG is set to turn on and collect data in the next year.
- Muon-electron conversion holds the best experimental prospect for going much further on LFV.
- **There is plenty of room for new surprises!**

$\mu \rightarrow e \gamma$ branching ratio (typical example)

SUSY seesaw model

SU(5) and SO(10) SUSY GUT

J.Hisano and D.Nomura,2000



The branching ratio can be large in particular for SO(10) SUSY GUT model.

thanks to Y. Okada

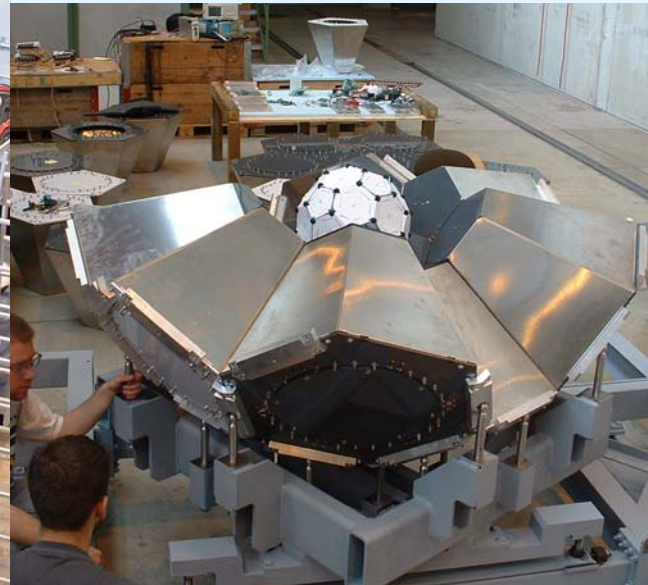
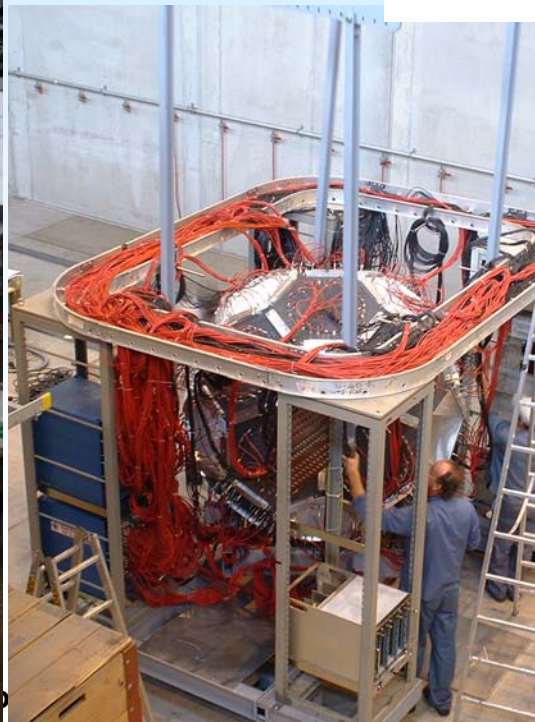
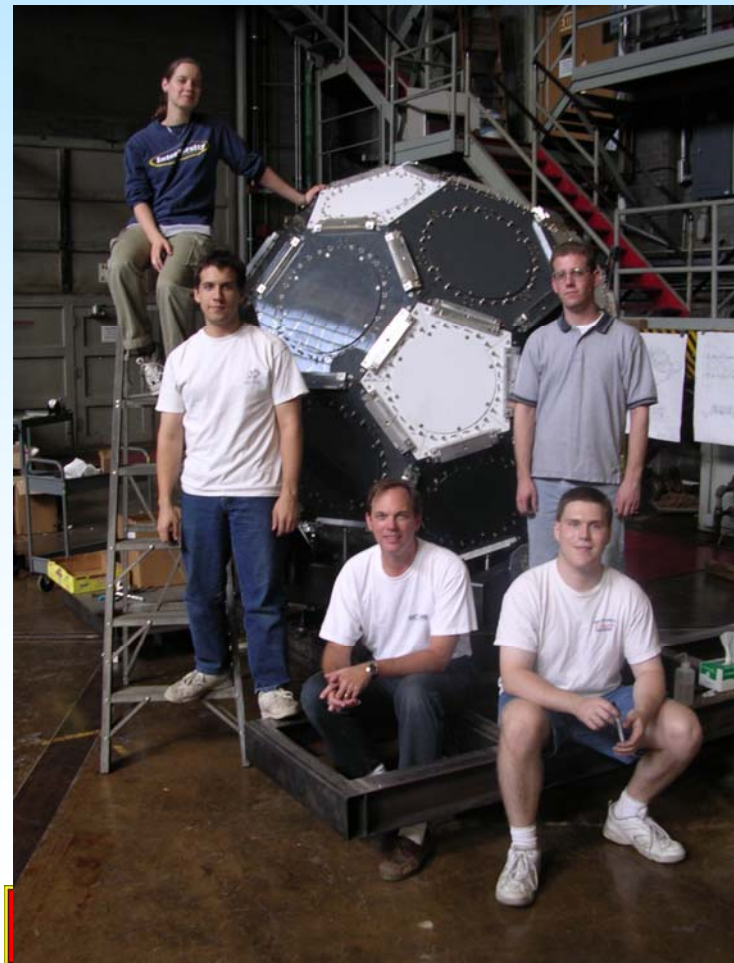
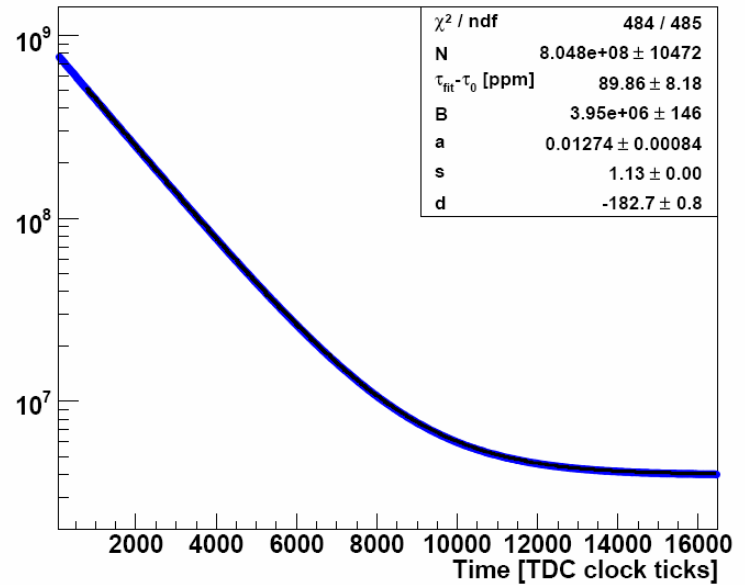
Comparison of three muon processes in various new physics models

SUSY GUT/Seesaw	$B(\mu \rightarrow e \gamma) \gg B(\mu \rightarrow 3e) \sim B(\mu A \rightarrow eA)$ Various asymmetries in polarized μ decays.
SUSY with large $\tan \beta$	$\mu \rightarrow e$ conv. can be enhanced. Z-dependence in $\mu \rightarrow e$ conv. branching ratio.
Triplet Higgs for neutrino	$B(\mu \rightarrow 3e) > \text{or} \sim B(\mu \rightarrow eg) \sim B(\mu A \rightarrow eA)$
RL model	$B(\mu \rightarrow 3e) \gg B(\mu \rightarrow eg) \sim B(\mu A \rightarrow eA)$ Asymmetry in $\mu \rightarrow 3e$
RPV SUSY	Various patterns of branching ratios and asymmetries

want to measure all three LFV processes to disentangle the models

μLan @ PSI aims for a factor of 20 improvement

Fit to 2004 data set,
 $\sigma_{\text{stat}} \approx 8.2$ ppm



Recent Developments:

The induced pseudoscalar coupling in nuclear μ -capture

$$\langle B | A_\lambda | A \rangle = (\bar{u}(p') [g_A(q^2) \gamma_\lambda \gamma_5 + \underbrace{ig_P(q^2) q_\lambda \gamma_5}_{\text{induced pseudoscalar}} + g_T(q^2) \gamma_5 \sigma_{\lambda\nu} q_\nu] u)$$

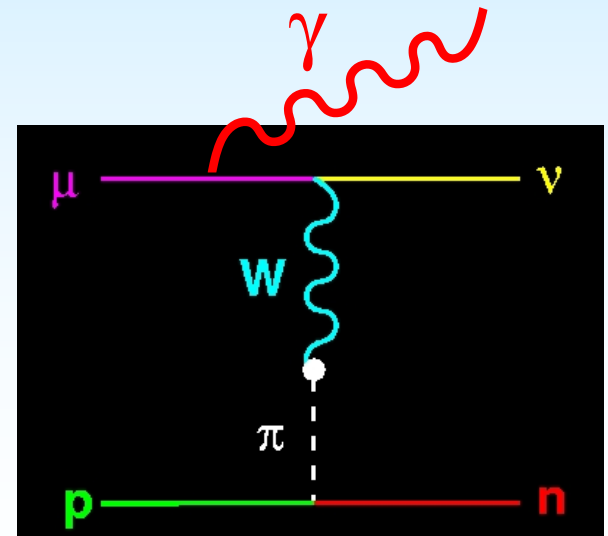
further enhanced in radiative muon capture (RMC).

TRUIMF experiment saw a 3σ discrepancy with PCAC prediction.

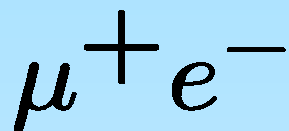
A new ordinary muon capture (OMC) experiment at PSI, MuCap, hopes to resolve this 3σ discrepancy.

However, a new TRIUMF measurement of the atomic “ortho to para transition rate” seems to remove much of this problem.

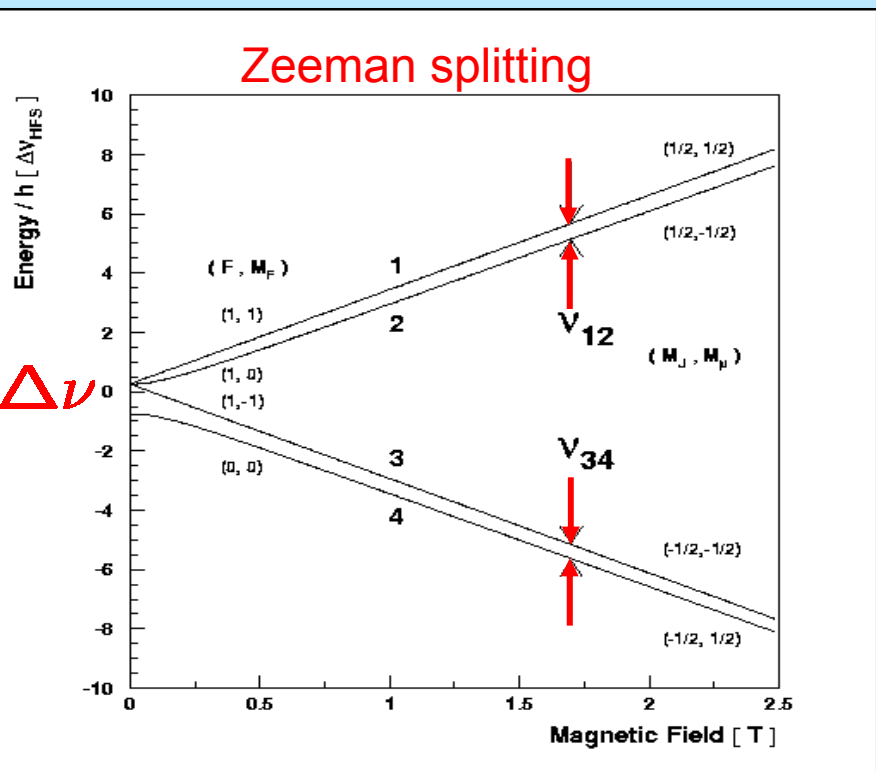
Clark, et al., PRL 96, 073401 (2006)



Muonium

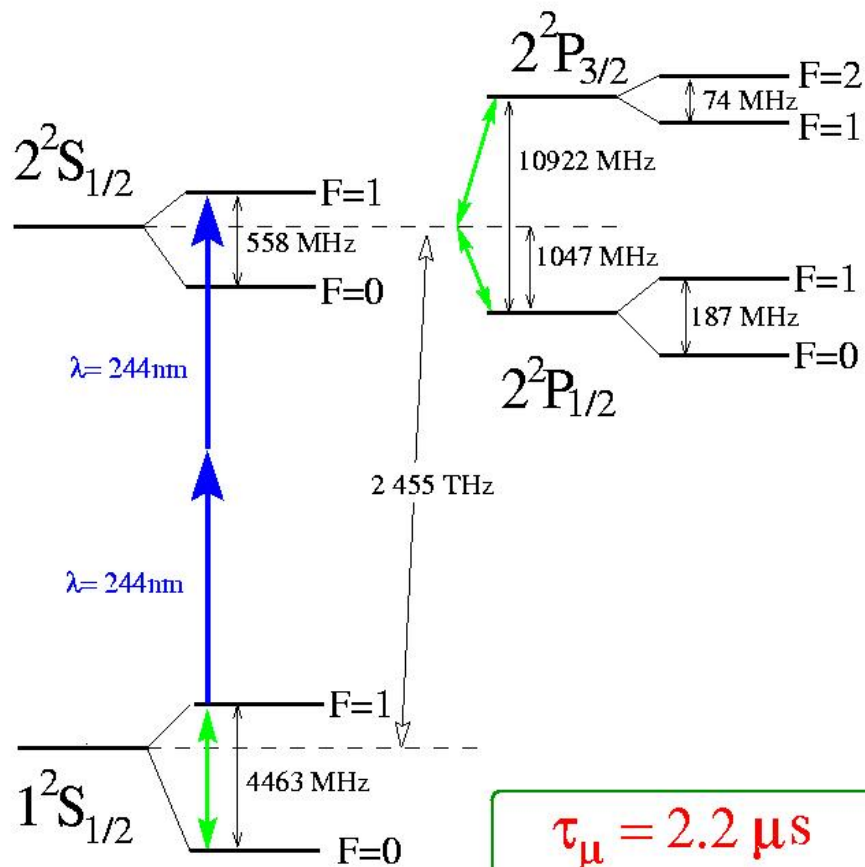


Hydrogen (without the proton)



Muonium ($M=\mu^+e^-$) Energy Levels

n=1 and n=2



$\tau_\mu = 2.2 \mu\text{s}$

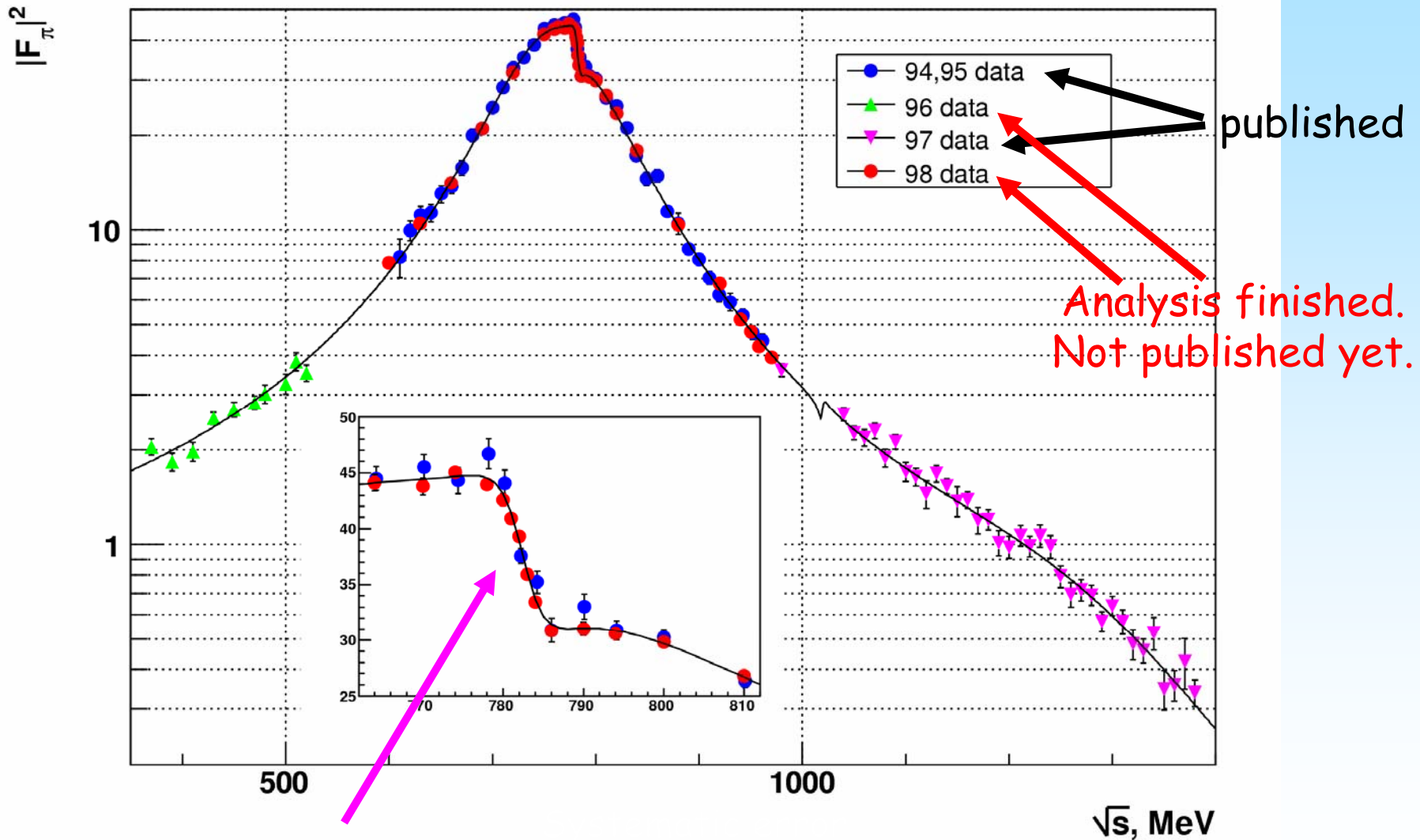
$\min \Delta\nu_{\text{nat}} = 145 \text{ kHz}$

(not to scale)

$\mu_\mu/\mu_p = 3.183\ 345\ 24(37)$ (120 ppb)

where μ_p comes from proton NMR in the same B field

Pion formfactor (CMD-2)



from: I Logashenko

Implications for a_μ

Davier, Marciano-2004:

$$\Delta a_\mu = (23.9 \pm 7.2_{\text{had,LO}} \pm 3.5_{\text{other}} \pm 5.8_{\text{exp}}) \cdot 10^{-10}$$

• $0.610 < \sqrt{s} < 0.960 \text{ GeV}$

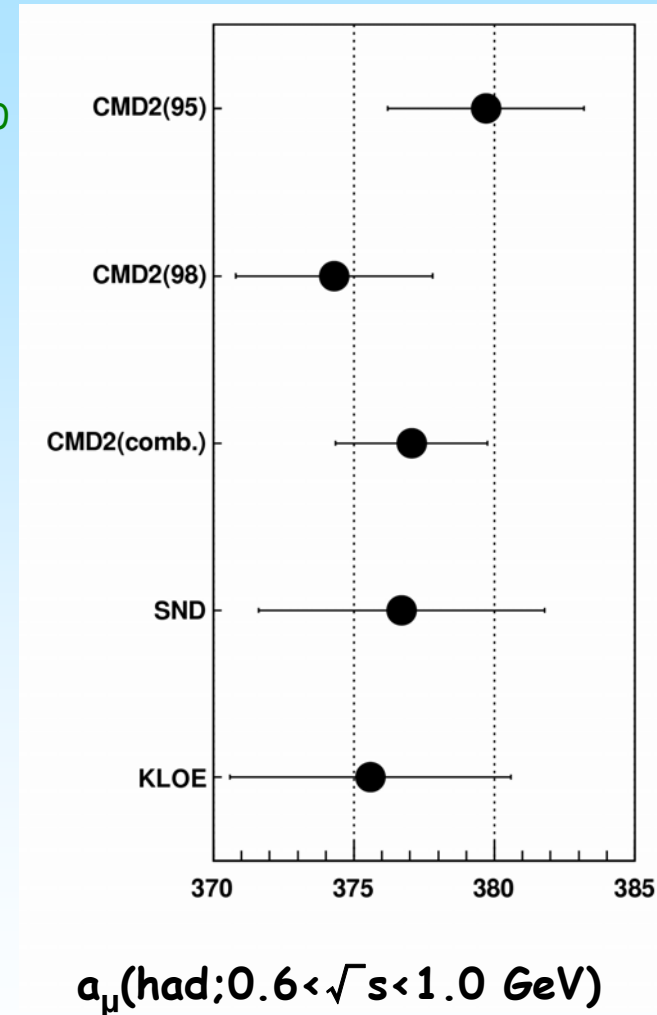
CMD-2 (95): $379.7 \pm 2.6 \pm 2.3$ (3.5)

CMD-2 (98): $374.3 \pm 1.8 \pm 3.0$ (3.5)

CMD-2 (comb): $377.05 \pm 2.2 \pm 1.5$ (2.7)

SND: $376.7 \pm 1.3 \pm 4.9$ (5.1)

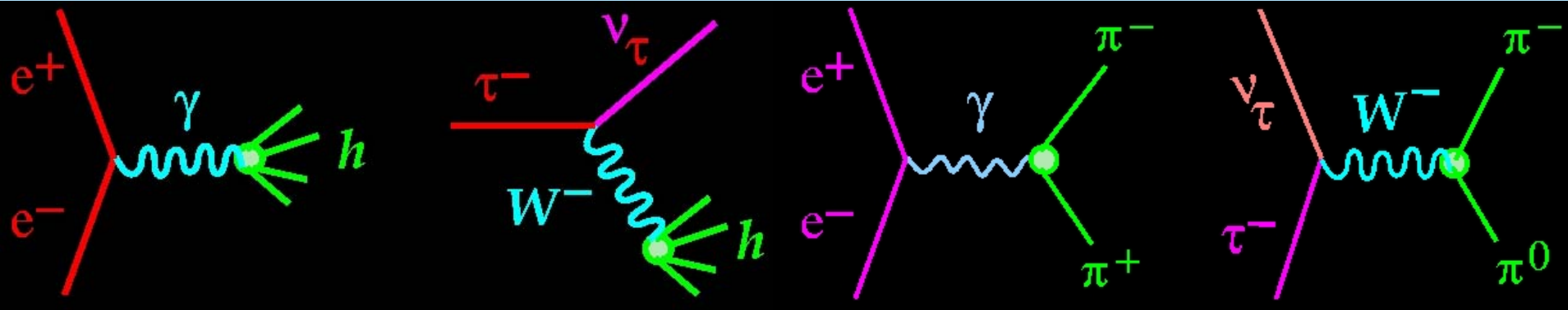
KLOE: $375.6 \pm 0.8 \pm 4.9$ (5.0)



S. Eidelman, private communication

from I. Logaseheko

$a(\text{had})$ from hadronic τ decay?

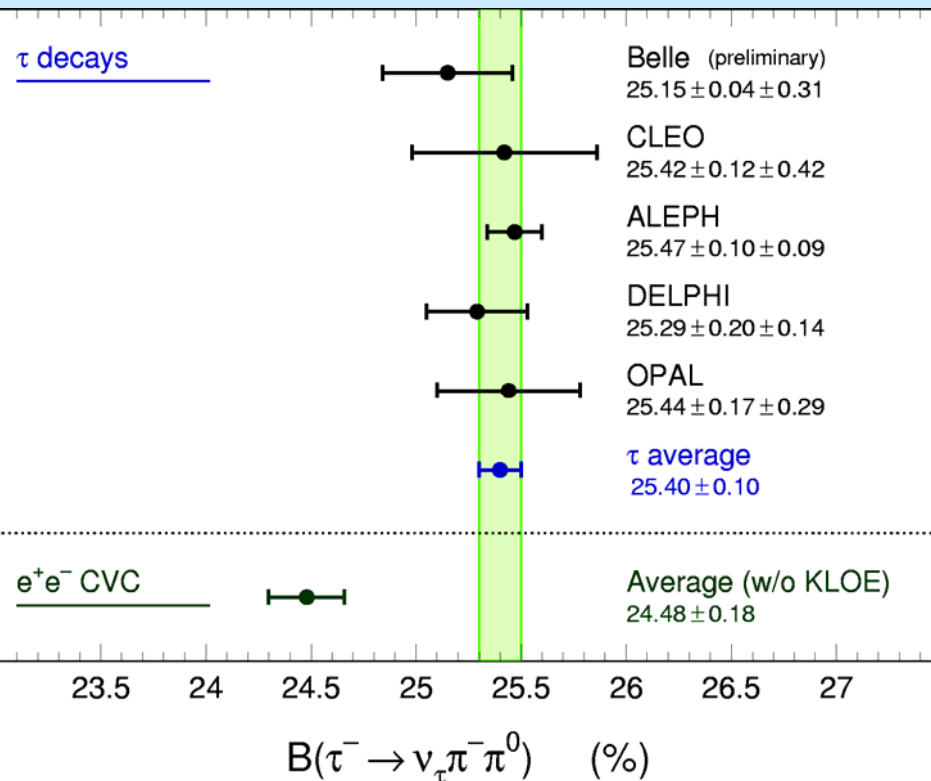


- Assume: CVC, no 2nd-class currents, isospin breaking corrections.
 - e^+e^- goes through neutral ρ
 - while τ -decay goes through charged ρ
- n.b. τ decay has no isoscalar piece, e^+e^- does
- Many inconsistencies in comparison of e^+e^- and τ decay:

Testing CVC with one number

Infer τ branching fractions (more robust than spectral functions) from e^+e^- data:

$$\text{BR}_{\text{CVC}}(\tau^- \rightarrow \pi^- \pi^0 \nu_\tau) = \frac{6\pi |V_{ud}|^2 S_{EW}}{m_\pi^2} \int_0^{m_\tau} ds \text{kin}(s) \nu^{SU(2)\text{-corrected}}(s)$$



Difference: $\text{BR}[\tau] - \text{BR}[e^+e^- \text{ (cvc)}]$:

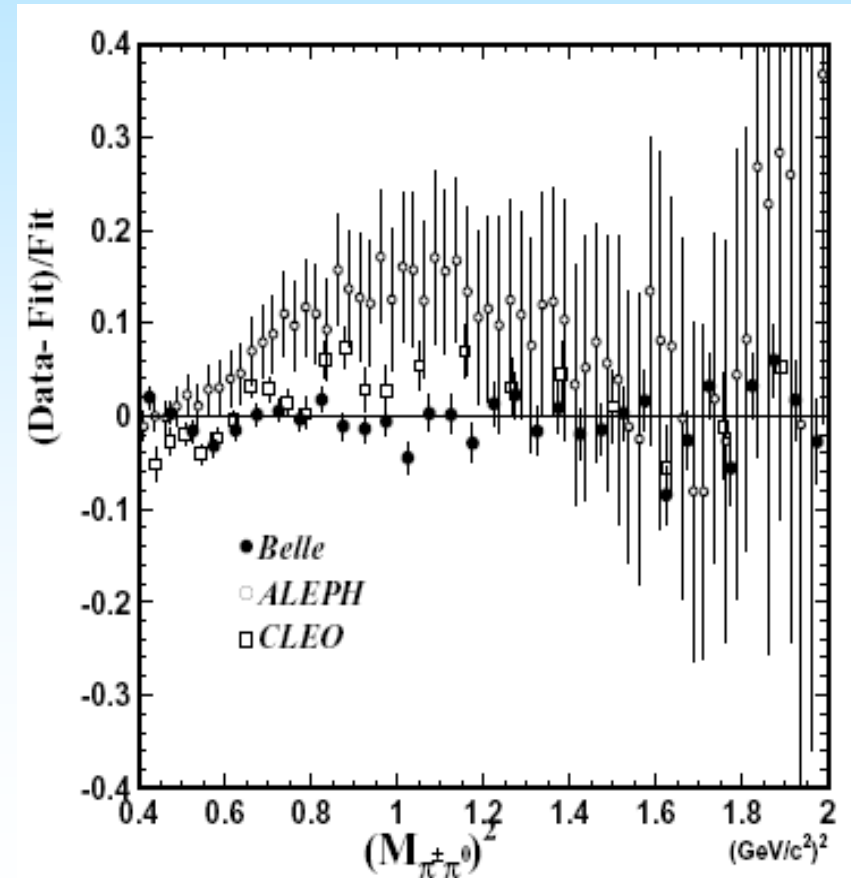
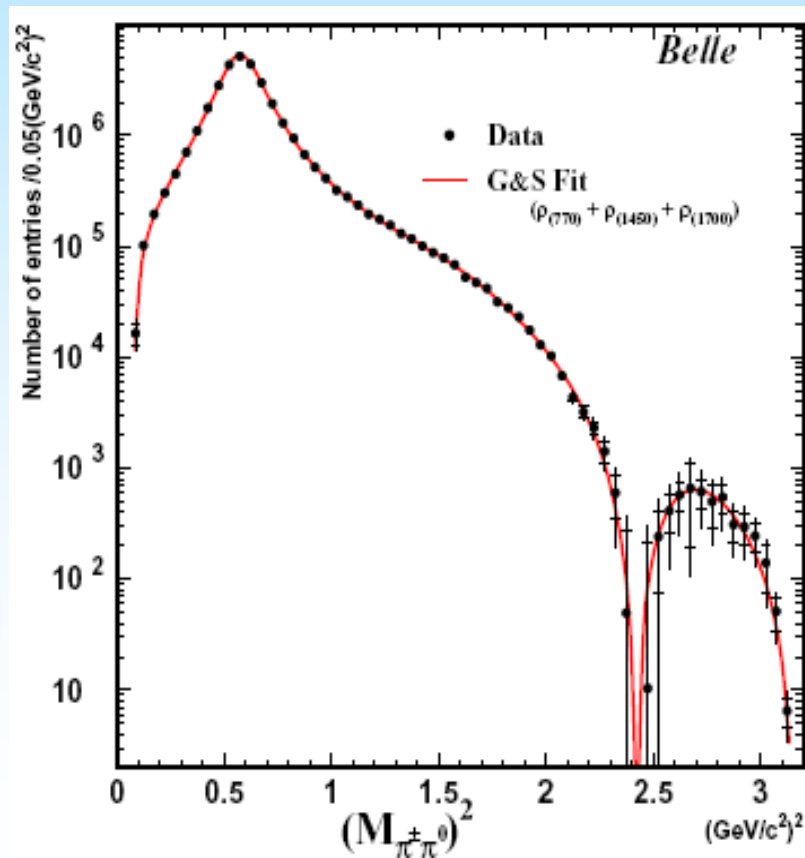
Mode	$\Delta(\tau - e^+e^-)$	'Sigma'
$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$	$+ 0.92 \pm 0.21$	4.5
$\tau^- \rightarrow \pi^- 3\pi^0 \nu_\tau$	$- 0.08 \pm 0.11$	0.7
$\tau^- \rightarrow 2\pi^- \pi^+ \pi^0 \nu_\tau$	$+ 0.91 \pm 0.25$	3.6

ee data on $\pi^- \pi^+ \pi^0 \pi^0$ not satisfactory

from Michel Davier

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$: preliminary results from BELLE

- preliminary results from BELLE on $\tau \pi\pi$ spectral function presented at EPS 2005
- high statistics: see dip at 2.4 GeV² for first time in τ data
- discrepancies with ALEPH/CLEO at large mass and ee data at low mass



The use of τ -decay is in question

- Until the discrepancies between the individual τ data sets can be resolved, and CVC can be shown to hold independently it's clear that only the e^+e^- data can be used to determine $a_\mu(\text{had})$

➤ **The agreement between SND and CMD-2 invalidates the use of τ data until a better understanding of the discrepancies is achieved (an interesting question as such)**

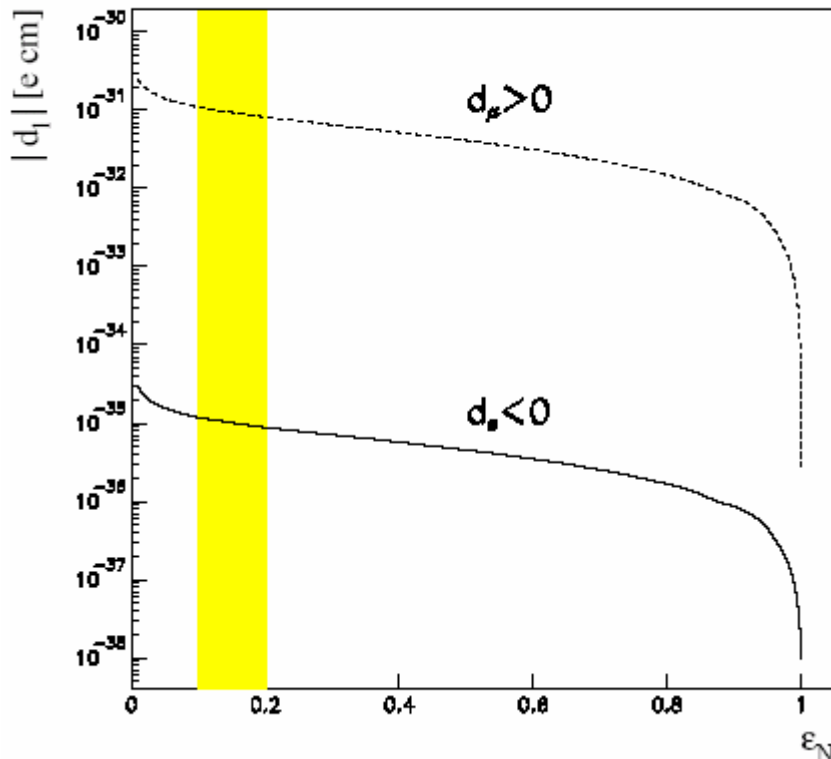
Michel Davier at Lepton Moments, June 2006

Model calculations of μ EDM

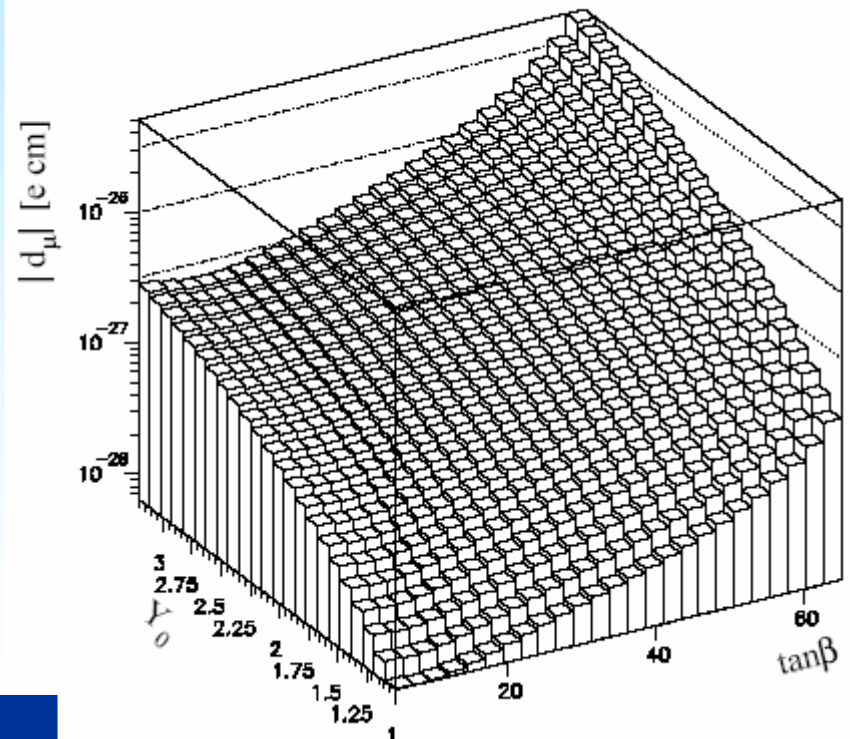
μ EDM may be enhanced
above $m_\mu/m_e \times e$ EDM

Magnitude increases with
magnitude of ν Yukawa couplings
and $\tan \beta$

Electric Dipole Moments



Muon Electric Dipole Moment

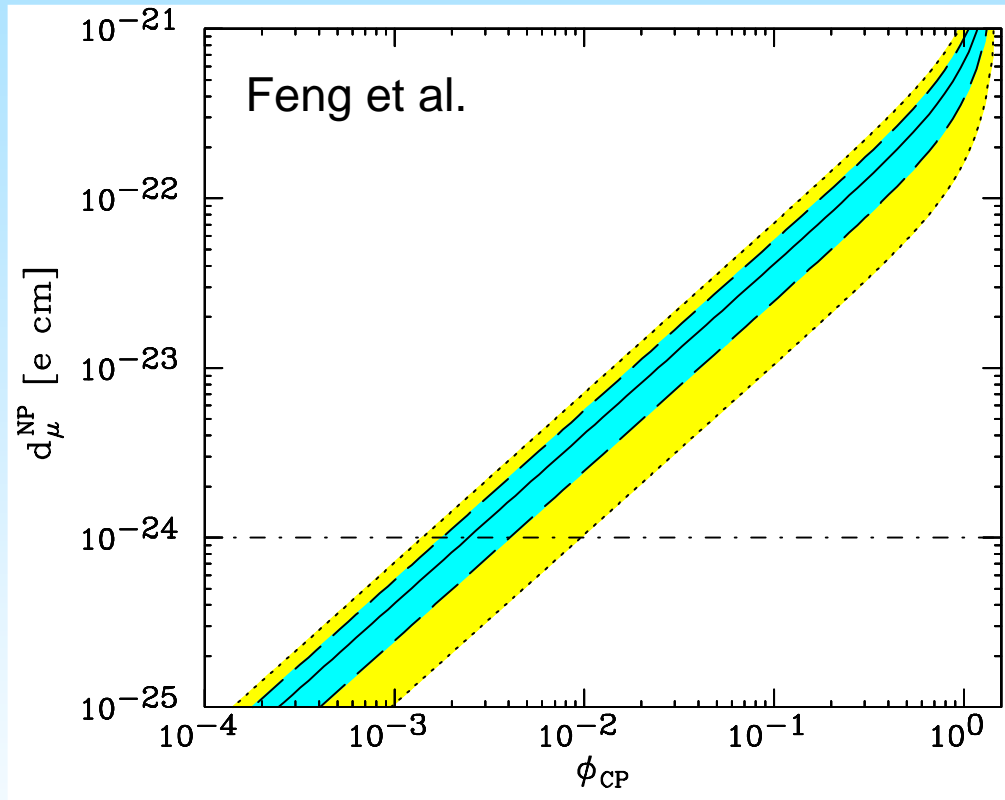


μ EDM greatly enhanced
when heavy neutrinos non-degenerate

arXiv:hep-ph/0111324 v2 30 Nov 2001

from John Ellis

a_μ implications for the muon EDM



assuming that
 $a_\mu^{\text{NP}} = 3(1) \times 10^{-9}$

$\pm 2\sigma$

$\pm 1\sigma$

$$d_\mu^{\text{NP}} \simeq 3 \times 10^{-22} \left(\frac{a_\mu^{\text{NP}}}{3 \times 10^{-9}} \right) \tan \phi_{CP} \text{ e} \cdot \text{cm}$$

where ϕ_{CP} is a CP violating phase.

mu2e @ FNAL???

Cancelled

Future Experiments:

- ~~MESQ Experiment @ BNL~~



~~- 10^{-17} single event sensitivity~~

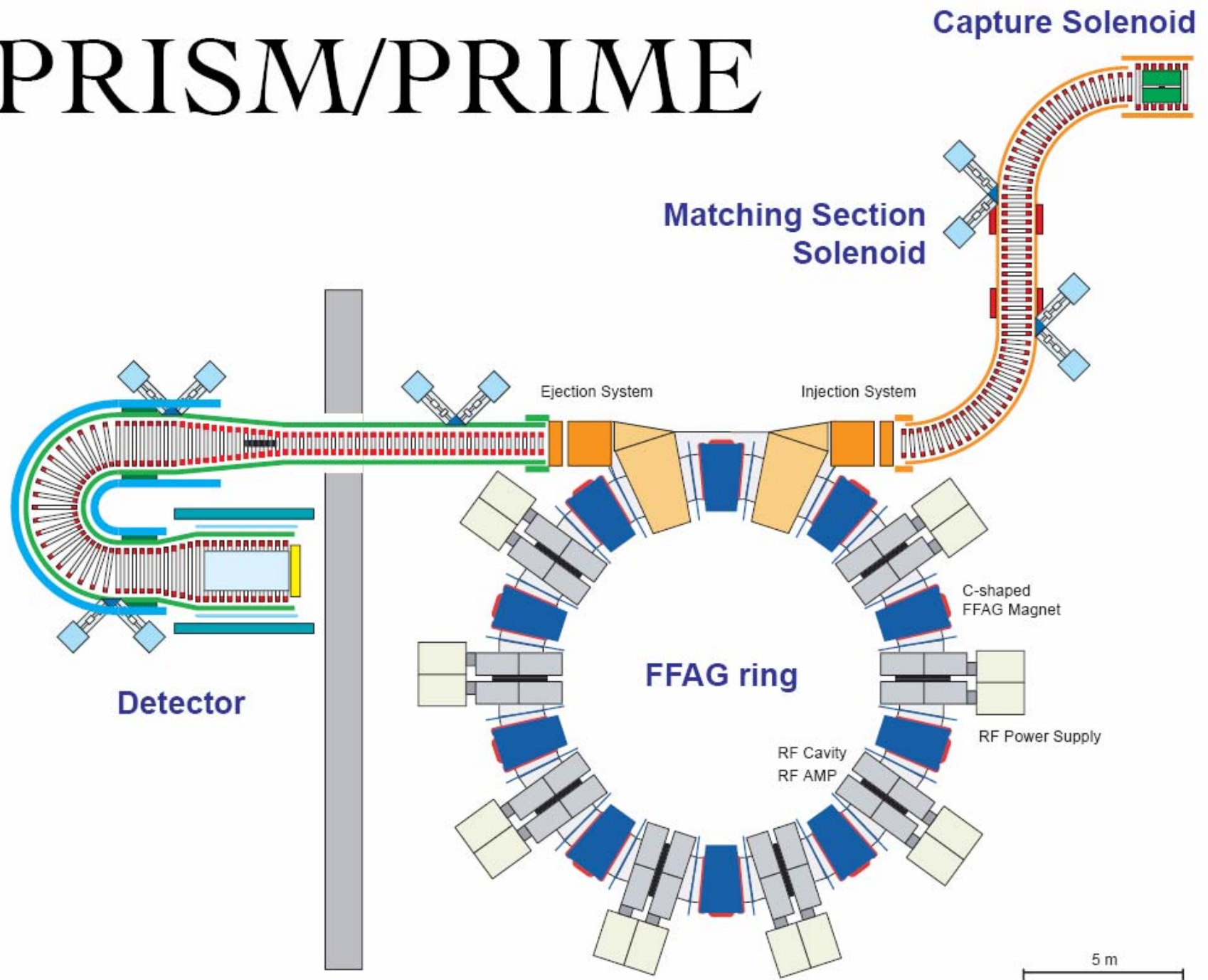
Cancelled

- MEG Experiment @PSI



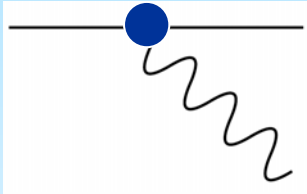
- Under Construction Data Begins in 2006

PRISM/PRIME

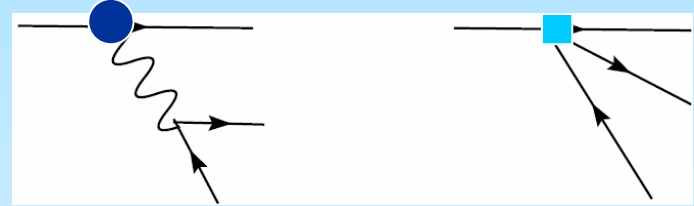


Comparison of three processes

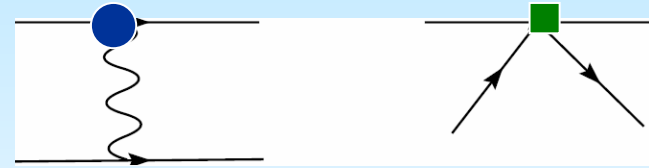
$$\mu^+ \rightarrow e^+ \gamma$$



$$\mu^+ \rightarrow e^+ e^+ e^-$$



$$\mu^- A \rightarrow e^- A$$



If the photon penguin process dominates, there are simple relations among these branching ratios.

$$B(\mu^+ \rightarrow e^+ e^+ e^-) \sim 6 \times 10^{-3} B(\mu \rightarrow e \gamma)$$

$$\frac{\sigma(\mu^- Ti \rightarrow e^- Ti)}{\sigma(\mu^- Ti \rightarrow \text{capture})} \sim 4 \times 10^{-3} B(\mu \rightarrow e \gamma)$$

This is true in many, but not all SUSY modes.

LFV Experiments:

- ~~MESQ Experiment @ BNL~~



- ~~- 10^{-17} single event sensitivity~~

Cancelled

- MEG Experiment

- Under Construction Data Begins in 2006

- PRISM-PRIME



- LOI to J-PARC FFAG under construction
- 10^{-18} - 10^{-19} single event sensitivity