暗黒物質の探索実験

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講義の内容

- 対象：
  - （素粒子）実験系の大学院生
- 暗黒物質の探索実験の現状
  - 常識的なintroduction
    - 宇宙物理・天文の話は、背伸びしない程度
    - なるべく、基礎に戻って説明
  - 直接、間接探索実験の現状
- XMASSの結果はまだない。

1. Brief History and Gravitational Observation of Dark Matter
2. Dark Matter and Cosmology
3. Galactic Dark Matter
4. DM candidates
5. Axion and Axion Like Particles
6. WIMPs Dark Matter
7. Indirect Search Experiments
8. Direct Search Experiments
9. XMASS

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1. Brief History and Gravitational Observation of Dark Matter

a. Cluster of Galaxies (Virial Mass) - Zwicky

b. Galaxy Rotation Velocity

c. Cluster of Galaxies

  1) Gravitational Lensing

  2) X-Ray Observation

d. Others

- When and how people have recognized the existence of ‘Dark Matter’
- What are the proofs of the existence of ‘Dark Matter’

かみのけ座銀河団
- 地球から～3億光年
- 我々の銀河系のディスクと法線方向
  ➔恒星があまりない。

1. Brief History and Gravitational Observation of Dark Matter

- Evidence of Dark Matter at various cosmic scale
  - In our laboratory ~1~10 m
    not yet (direct search)
  - Near-by earth ~10^{16}~10^{17} m
    not fully confirmed (indirect)
  - Our galaxy ~10^{21} m (Gravitation)
  - Near by galaxies ~10^{22} m (Gravitation)
  - Clusters of galaxies ~10^{24} m (Gravitation)
  - Cosmology and cosmological observation
    ~10^{26} m

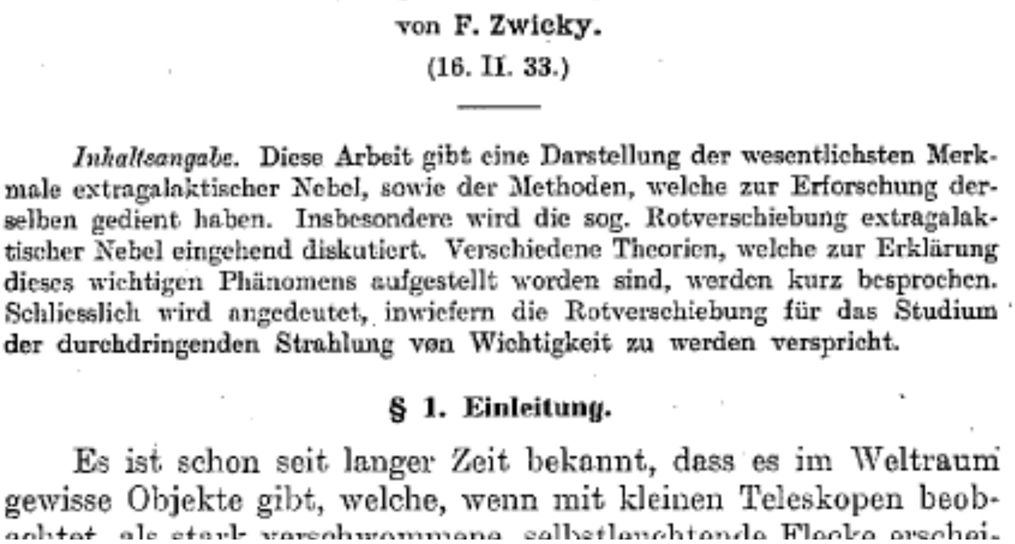
1光年	～10^{16} m
10万光年	～10^{21}m

1-a) Cluster of Galaxies (Virial Mass) - Zwicky

- Fritz Zwicky (フリッツ・ツビッキー)
  1898.02.14~1974.02.08
- スイス国籍の天文学者(ブルガリア生まれ):
  - Walter Baadeとともに超新星研究のパイオニア

- *1933年、銀河団の銀河の速度を計測し、Virial定理を応用し、その銀河団の全質量を推定した。*
  - 光学質量の’160倍’の‘見えない’質量の存在を推量。
  - 最初の暗黒物質の存在予想
- Ref) Zwicky, F. (1933), "Die Rotverschiebung von extragalaktischen Nebeln", *Helvetica Physica Acta* 6: 110–127

Paper in 1933



Redshift

Die Rotverschiebung von extragalaktischen Nebeln

von **F. Zwicky.**

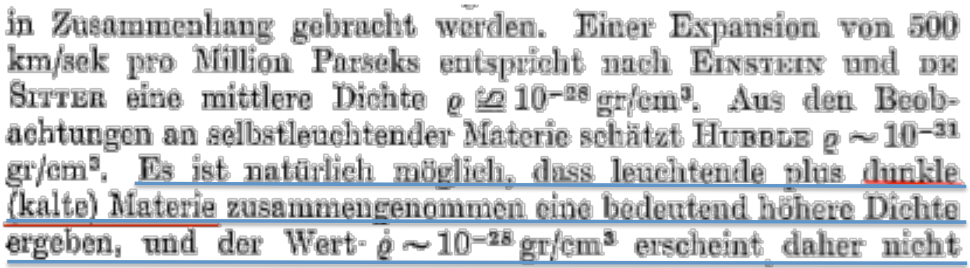
(16. II. 33.)

Inhaltsangabe. Diese Arbeit gibt eine Darstellung der wesentlichsten Merkmale extragalaktischer Nebel, sowie der Methoden, welche zur Erforschung derselben gedient haben. Insbesondere wird die sog. Rotverschiebung extragalaktischer Nebel eingehend diskutiert. Verschiedene Theorien, welche zur Erklärung dieses wichtigen Phänomens aufgestellt worden sind, werden kurz besprochen. Schliesslich wird angedeutet, inwiefern die Rotverschiebung für das Studium der durchdringenden Strahlung von Wichtigkeit zu werden verspricht.

§ 1. Einleitung.

Es ist schon seit langer Zeit bekannt, dass es im Weltraum gewisse Objekte gibt, welche, wenn mit kleinen Teleskopen beobachtet, als stark verschwommene, selbstleuchtende Flecke erschei-

Paper in 1933

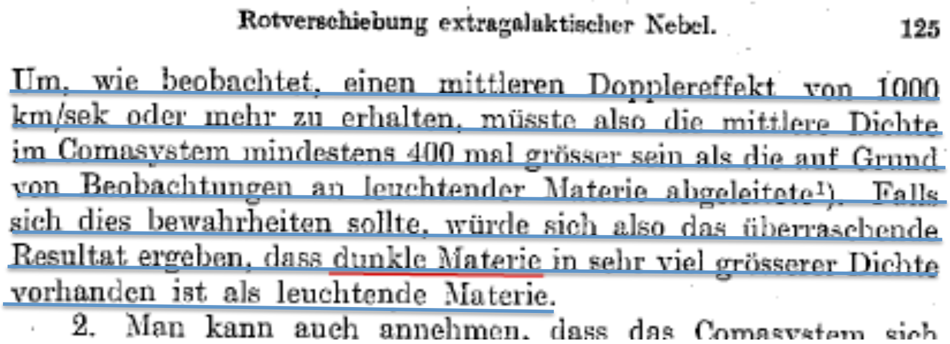


in Zusammenhang gebracht werden. Einer Expansion von 500 km/sek pro Million Parseks entspricht nach EINSTEIN und DE SITTER eine mittlere Dichte $\varrho \leqq 10^{-28}$ gr/cm³. Aus den Beobachtungen an selbstleuchtender Materie schätzt HUBBLE $\varrho \sim 10^{-31}$ gr/cm³. Es ist natürlich möglich, dass leuchtende plus dunkle (kalte) Materie zusammengenommen eine bedeutend höhere Dichte ergeben, und der Wert $\varrho \sim 10^{-28}$ gr/cm³ erscheint daher nicht

Of course it is possible that luminous plus dark (cold) matter together have a much higher density, and the value of $\rho \sim 10^{-28}$gr/cm^3 does therefore not seem unreasonable.

最初にDark Matter (dunkle Materie) を使ったときに、（冷たい）ということばが入っていたのは興味深い。この論文に、あと2回 Dark Matterが出てくるが、いずれも（Cold）抜きである。

Paper in 1933



Rotverschiebung extragalaktischer Nebel. 125

Um, wie beobachtet, einen mittleren Dopplereffekt von 1000 km/sek oder mehr zu erhalten, müsste also die mittlere Dichte im Comasystem mindestens 400 mal grösser sein als die auf Grund von Beobachtungen an leuchtender Materie abgeleitete[1]). Falls sich dies bewahrheiten sollte, würde sich also das überraschende Resultat ergeben, dass dunkle Materie in sehr viel grösserer Dichte vorhanden ist als leuchtende Materie.

2. Man kann auch annehmen, dass das Comasystem sich

To get the observed average doppler effect of 1000 km/s or more, the average density in the Coma cluster would have to be at least 400 times larger than that deduced from observations of luminous matter [1]). Should this prove to be true, it would yield the surprising result that dark matter exists in much larger density than luminous matter.

Coma Cluster

- Coma Cluster（かみのけ座銀河団）
  - >1,000 galaxies
  - Distance from the earth: 99 Mpc (321 million light years)

Image credit: NASA, ESA, Hubble Heritage (STScI/AURA)

(Note)
1 pc = 3.26 light years

(History)
Discovery of the Cluster of Galaxies
(contain 10～>1000 galaxies)
- Charles Messier   (1784)
- William Herschel (1785)
  - Virgo, Coma

Virial定理

- 重力場中の運動エネルギー（T）の時間平均は、系全体のポテンシャルエネルギー（U）の時間平均の-1/2に等しい。

$$2\overline{T} = -\overline{U}$$

Virial 定理の証明 1

- 同次関数とオイラーの定理
  - n次の同次関数:　$f(\lambda \mathbf{r}) = \lambda^n f(\mathbf{r})$
  - オイラーの定理:　f(**r**)がn次の同次関数の時

$$\mathbf{r} \cdot \frac{\partial f(\mathbf{r})}{\partial \mathbf{r}} = n f(\mathbf{r})$$

Virial 定理の証明 2

- オイラーの定理の証明

$$\begin{array}{rcl} \mathbf{r}^* & = & \lambda \mathbf{r} \qquad \text{とおく} \\ f^* & = & f(\mathbf{r}^*) = f(\lambda \mathbf{r}) = \lambda^n f(\mathbf{r}^*) \\ \frac{df^*}{d\lambda} & = & \frac{\partial f^*}{\partial \mathbf{r}^*}\frac{d\mathbf{r}^*}{d\lambda} = \frac{\partial f^*}{\partial \mathbf{r}^*}\mathbf{r} \\ \frac{df^*}{d\lambda} & = & n\lambda^{n-1} f(\mathbf{r}) \\ \therefore & & \frac{\partial f^*}{\partial \mathbf{r}^*}\mathbf{r} = n\lambda^{n-1} f(\mathbf{r}) \end{array}$$

全てのλで成立、λ＝1でも成り立つ(f＊→ f, r* → r)

$$\mathbf{r}\frac{\partial f}{\partial \mathbf{r}} = n f(\mathbf{r})$$

Virial 定理の証明 3

$$\sum_i \frac{\partial T}{\partial \mathbf{v}_i} \cdot \mathbf{v}_i = 2T$$

$$\frac{\partial T}{\partial \mathbf{v}_i} = \mathbf{p}_i$$

$$\begin{aligned}
\therefore 2T &= \sum_i \mathbf{p} \cdot \mathbf{v}_i \\
&= \frac{d}{dt}(\sum_i \mathbf{p}_i \cdot \mathbf{r}_i) - \sum_i \mathbf{r}_i \cdot \dot{\mathbf{p}}_i
\end{aligned}$$

Virial 定理の証明 4

- F(t)が有界とすると、その時間平均は、

$$\begin{aligned}
\bar{f} &= \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^\tau f(t) dt \\
&= \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^\tau \frac{dF}{dt} dt \\
&= \lim_{\tau \to \infty} \frac{F(\tau) - F(0)}{\tau} < \lim_{\tau \to \infty} \frac{G_{max} - G_{min}}{\tau} = 0
\end{aligned}$$

ゼロになる

Virial 定理の証明 5

$2T = \frac{d}{dt}(\sum_i \mathbf{p}_i \cdot \mathbf{r}_i) - \sum_i \mathbf{r}_i \cdot \dot{\mathbf{p}}_i$

- 限られた空間、有限の速さで、 $\Sigma \mathbf{p}_i \cdot \mathbf{r}_i$は有界

$$\begin{aligned} \overline{2T} &= -\overline{\sum_i \mathbf{r}_i \dot{\mathbf{p}}_i} \\ &= \overline{\sum_i \mathbf{r}_i \frac{\partial U}{\partial \mathbf{r}_i}} \qquad \left(\mathbf{F} = \dot{\mathbf{p}} = -\frac{\partial U}{\partial \mathbf{r}_i}\right) \end{aligned}$$

Uが同次関数として、

$$2\overline{T} = k\overline{U}$$

となる。K=2: 微小振動、k=-1: Newtonian

Virial Theorem for the cluster of galaxies (Coma Cluster) -01

$$2\overline{T} = -\overline{U}$$

Ref) Zwicky, F. (1937), "On the Masses of Nebulae and of Clusters of Nebulae", *Astrophysical Journal* 86: 217.

$$\begin{aligned} 2\overline{T} \leftarrow \overline{\sum_i m_i v_i^2} &= \sum_i m_i \overline{v_i^2} \\ &= M_{tot}\overline{\overline{v^2}} \end{aligned}$$

(*)

$\overline{\overline{v^2}}$: a double average taken over time and over masses

$$\overline{U} \leftarrow \sum_{i<j} \frac{G m_i m_j}{r_{ij}}$$

Virial Theorem for the cluster of galaxies (Coma Cluster) -02

- As a first approximation → uniformly distributed inside a sphere of radius R

$$M_{tot} = \frac{4}{3}\pi R^3 \rho$$

$$\overline{U} = -\int_0^R \frac{G_N \cdot \rho^2 \cdot \frac{4}{3}\pi r^3 \cdot 4\pi r^2 dr}{r}$$

$$= -\frac{3G_N M_{tot}^2}{5R} \qquad (**)$$

- From (*) and (**)

$2\overline{T} = M_{tot}\overline{\overline{v^2}}$

$$M_{tot} = \frac{5R\overline{\overline{v^2}}}{3G_N}$$

Virial Theorem for the cluster of galaxies (Coma Cluster) -03

$\overline{U}$ に対するsystematicsを見積もる

$\overline{U} = -\frac{3G_N M_{tot}^2}{5R}$

1. R → R/2 : $\overline{U}$ は2倍、M_{tot} は1/2
2. Massが2つか3つのgalaxyに集中、しかも距離は、R→R/10程度とする。それぞれ、

$$\overline{U} = \frac{-2.5GM_{tot}^2}{R} \qquad \overline{U} = \frac{-\frac{10}{3}GM_{tot}^2}{R}$$

M/2*M/2*10/R → 2.5

3*M/3*M/3*10/R → 10/3

- Conservative な見積もりとして、 $\overline{U} < \frac{-5GM_{tot}^2}{R}$

Virial Theorem for the cluster of galaxies (Coma Cluster) -04

Conservative limitとして、

$$M_{tot} > \frac{R\overline{\overline{v^2}}}{5G_N}$$

をとる。ここで、

$\overline{\overline{v^2}} = 3\overline{\overline{v_s^2}}$ v_s (velocity along the line of sight)

$$\therefore \quad M_{tot} > \frac{3R\overline{\overline{v_s^2}}}{5G_N}$$

Virial Theorem for the cluster of galaxies (Coma Cluster) -05

- $\overline{\overline{v_s^2}}$ = 5 x 10^{11} m^2s^{-1} (v ~ 1200 km/s)
- R ~ 2 M light years
- G_N = 6.67x10^{-11} $m^3kg^{-1}s^{-2}$

$$M_{tot} > \frac{3R\overline{\overline{v_s^2}}}{5G_N} = \frac{3 \times 2 \times 10^6 \times 3.15 \times 10^7 \times 3 \times 10^8 \times 5 \times 10^{11}}{5 \times 6.67 \times 10^{-11}}$$

= 9 x 10^{43} kg

- Av(M_{gal}) > 9 x 10^{40} kg = 4.5 x 10^{10} $M_\odot$ (1000 gal)
- Luminosity of an average galaxy ~ 8.5 x 10^7 $L_\odot$
- M/L ~ 500

$M_\odot$= 1.99x 10^{33}g = 1.99x 10^{30}kg

Virial Theorem for the cluster of galaxies (Coma Cluster) -06

- M/L ~ 3 for Kapteyn stellar system
  - カプタイン星 （13 light years from the earth)
    - 十数個の‘星’の集団
    - Nearest halo system to the sun
    - 速度が速い星団

M/L (Coma Cluster) / M/L (Kapteyn) ~ 160

➔ Existence of Dark Matter

Different History

- Oortが、Zwickyよりも先にrotational curveで、最初にDMの予言をしたという説もある。
- Oortは、Diskを横切る方向の速度で、質量をestimate. Halo DMではなくて、Diskの質量を計っていた、ということらしい。

1-b) Galaxy Rotation Velocity

- Rotation Curve of galaxies
  - Measure line of sight velocity of starts and gas
  - Extensive measurement of M31 by Rubin and Ford (1970)
    - via Doppler Shift
    - Hα in optical (HII) and 21 cm in radio (HI)

HI and HII

- I: 中性の原子、II: １階電離、III: ２階電離

  (H_2: 水素分子)
- HII 領域: 大きさ数百光年におよぶ。内部で星形成。生まれた若い大質量星からの紫外線によって励起
  - Hα: 電離水素の輝線、656.3 nm（赤）
- HI 領域: 星間雲中の水素の中性状態
  - 21 cm線による観測

M51: 腕に沿っての赤く光っている部分がHII領域 （HSTによる撮影）

水素のスペクトル

- Rydberg formula

$$\frac{1}{\lambda} = R\left(\frac{1}{n^2} - \frac{1}{n'^2}\right) \quad (n \leftarrow n')$$

$$R = 1.097373 \times 10^7 m^{-1}$$

n=1： Lyman series

n'	2	3	4	5	6	∞
nm	122	103	97.3	95.0	93.8	91.2

n=2： Balmer series

n'	3	4	5	6	7	∞
nm	656.3	486	434	410	397	365

H_α H_β

← 可視光 →

n=3： Paschen series, N=4 : ...

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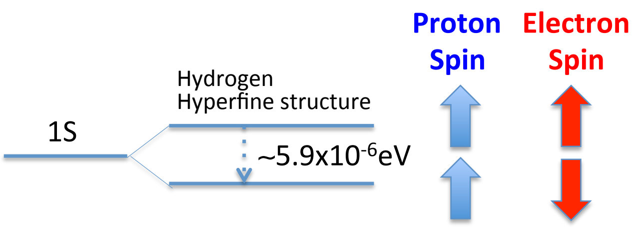
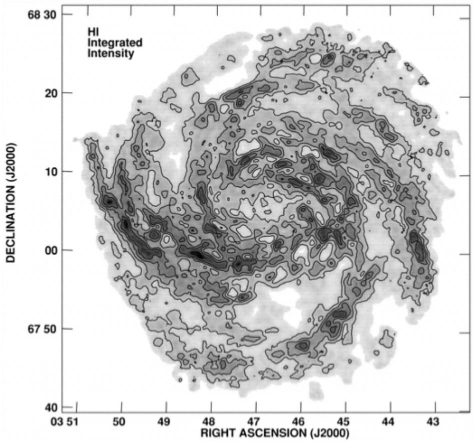
21 cm

- Spin-Spin相互作用による hyperfine structure



- 1420.40575 MHz,
- 21.106114 cm
- 遷移 ～1,000万年

HI map of IC342
Crosthwaite, Turner & Ho (2000)



HI により、銀河の大局的な構造をみることが出来る。

Doppler Shift

- 音のドップラーシフト：　空気という媒質がある
  - 音源が動く場合と、観測者が動く場合で違う
- 光のドップラーシフト：　光速一定
  - 音源が動こうと、観測者が動こうと同じ

（宇宙の観測での赤方偏移／青方偏移）
- 光源のlocalな運動（光のドップラーシフト）
- 宇宙の膨張（宇宙論的赤方偏移）
  - $Z = \Delta\lambda/\lambda_0 = (\lambda - \lambda_0)/\lambda_0$
  - Z=0.1 程度なら、$v = cZ$
  - Z = 7〜10 : 最遠方の銀河

  (Z=1089: 背景輻射）
- 強い重力場（重力赤方偏移）

音のドップラーシフト

- Observerが近づく
- Sourceが近づく

$$\begin{array}{ll} Source & V = f\lambda \\ Observer & V + v_0 = f'\lambda \\ Effect & f' = f\dfrac{V + v_0}{V} \end{array}$$

$$\begin{array}{ll} Source & V - v_s = f\lambda' \\ Observer & V = f'\lambda' \\ Effect & f' = f\dfrac{V}{V - v_s} \end{array}$$

$$f' = f\frac{V + v_0}{V - v_s}$$

光のドップラーシフト

- Doppler Shift → Home work

$$\nu = \nu_0 \frac{\sqrt{1-\left(\frac{v}{c}\right)^2}}{1-\left(\frac{v}{c}\right) cos\theta}$$

θ = 0°： 遠のく（赤方偏移）

$$\nu = \nu_0 \frac{\sqrt{\left(1-\frac{v}{c}\right)\left(1+\frac{v}{c}\right)}}{1+\frac{v}{c}} = \nu_0 \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

θ = 180°：近づく（青方偏移）

$$\nu = \nu_0 \frac{\sqrt{\left(1-\frac{v}{c}\right)\left(1+\frac{v}{c}\right)}}{1-\frac{v}{c}} = \nu_0 \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$$

光のドップラーシフト

- v/c <<1の場合 $\nu \sim \nu_0 \left(1 \pm \frac{v}{c}\right)$
  - 波長の測定精度 ➔ 速度の測定精度

余談）横ドップラーシフト(θ=90°)

$$\nu = \nu_0 \sqrt{1-\left(\frac{v}{c}\right)^2}$$

光源が固有時間内に出す振動(dn/dτ)を、観測者が座標時間で観測(dn/dt)する効果

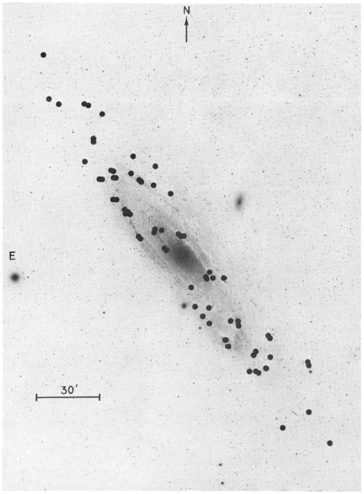
$$\nu = \frac{dn}{d\tau}\frac{d\tau}{dt} = \nu_0 \sqrt{1-\left(\frac{v}{c}\right)^2}$$

$$\frac{dn}{d\tau} = \nu_0, \quad d\tau = dt\sqrt{1-\left(\frac{v}{c}\right)^2}$$

Beginning of 1970's

V. C. Rubin and W. K. Ford, Jr, ApJ, 159 (1970)

- Observe M31
- Distance from the earth:
  - 778±17 kpc
  - 2.54±0.06 Mly
- 視角：190' x 60'
  - 太陽の視角0.53度(32')
- They measured 67 HII regions (3 to 24 kpc from the nucleus of M31)
  - Radial velocities (視線速度) from $H\alpha$
- Assumption of circular motions



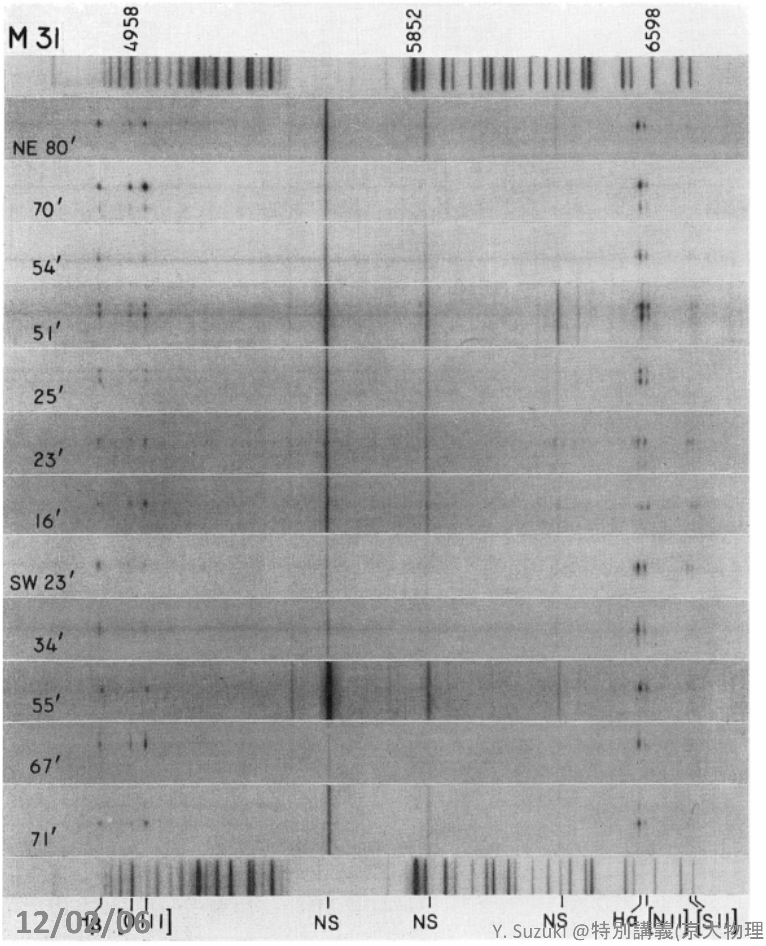
M31: Andromeda

視線速度の観測精度

Radial velocities (視線速度) from $H\alpha$ ➔ accuracy of ±10km/s !

- Image Tube Spectrograph
  - 逆分散 ($d\lambda/dx$): 135 Å mm^{-1} (分光器の性能)
  - $H\alpha$ = 6560 Å → 0.135 Å μm^{-1} → 2x 10^{-5} for μm
  - 3x 10^5 km/s (c) x 2x10^{-5} ➔ 6 km/s μm^{-1}
- 1 μmの精度で観測できるなら、速度の精度は 6km/sである。

分光



- H_{α}: 656 nm
- H_{β}: 486 nm
- NS: Night sky lines

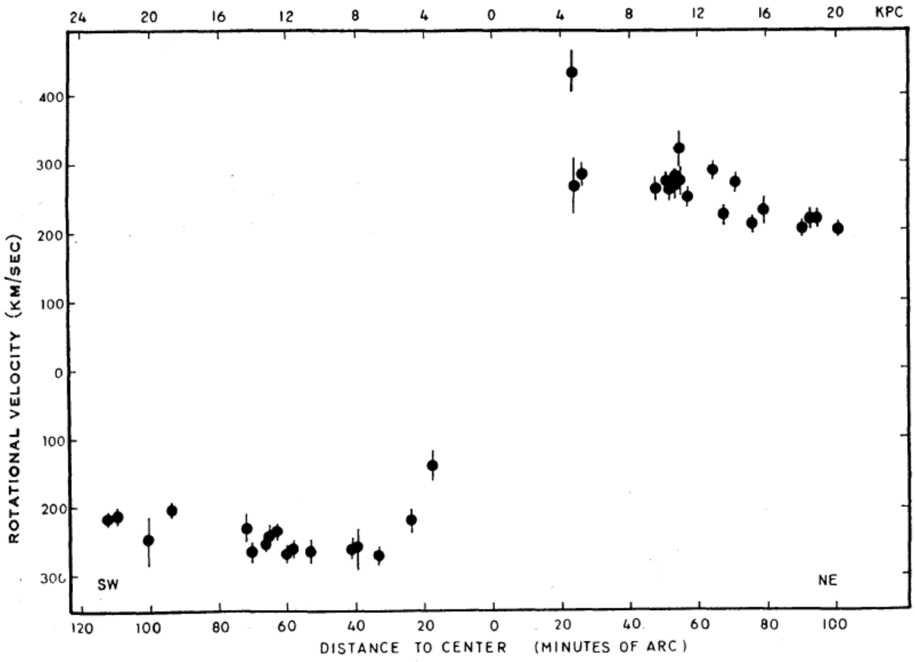
Circular Velocity

- Circular Velocity @R : V(R)

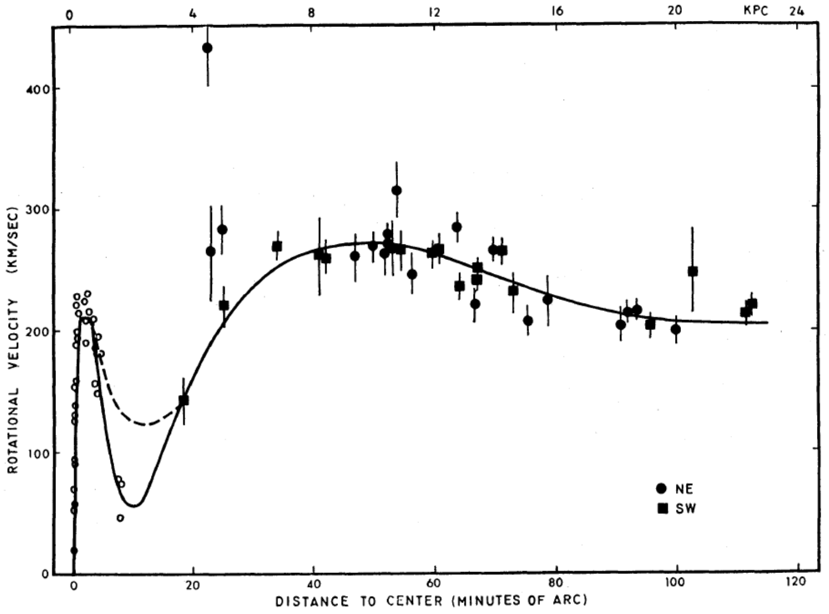
  $$V(R) = (V_{obs} - (\text{mean velocity}))/\alpha(x,y,z)$$

  - Mean velocity: 300 km/s
  - $\alpha(x,y,z)$: correction for the measured position and angle between ‘line of sight’ and perpendicular to the plane of the galaxy

Rotational Velocity of M31



Rotational Velocity of M31



Rotational Velocity

- Newtonian Dynamics:

$$F = \frac{mv^2}{r} = G_N \frac{mM}{r^2}$$

- If M: central, $v(r) \propto r^{-\frac{1}{2}}$
- For constant v

$$M(r) \propto r$$

$$\rho(r) \propto r^{-2}$$

Surface density: σ(r) ~ constant

お星様の場合

$$\rho(r) \propto e^{-\frac{r}{r_0}}$$

Mass determination

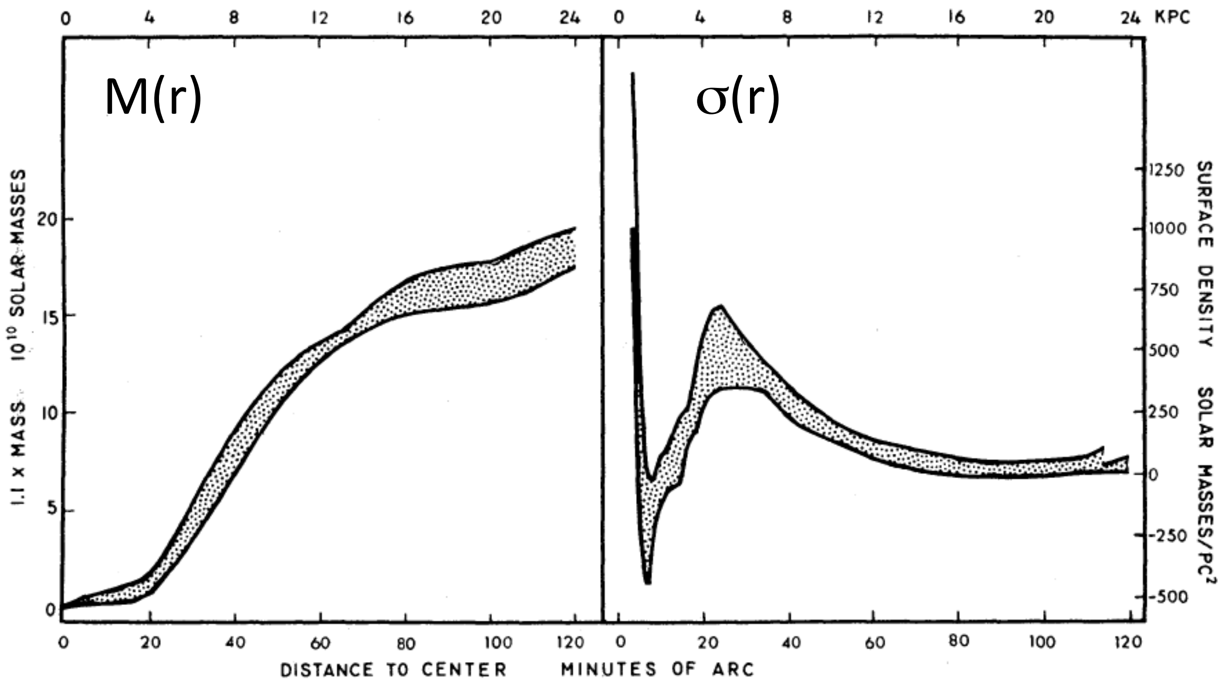
- Velocity measurement → mass determination,
- Need a model for density distribution
  - Ref) G.G.Kuzmin, Pub. Astr.Ob J.C. Brs. Tartu, 32, 211(1952); Brandt, ApJ, 131, 193 (1960)

$$M(R) = \frac{2}{G\pi} \int_0^R \frac{V^2(a)ada}{(R^2 - a^2)^{1/2}}$$

- By numerical integration by using IBM1130
  - ～18 x 10^{10} $M_\odot$
- from the optical measurements
  - 1.4 x 10^{10} $L_\odot$

➔ M/L ～ 13±0.7

Results of Mass Determination



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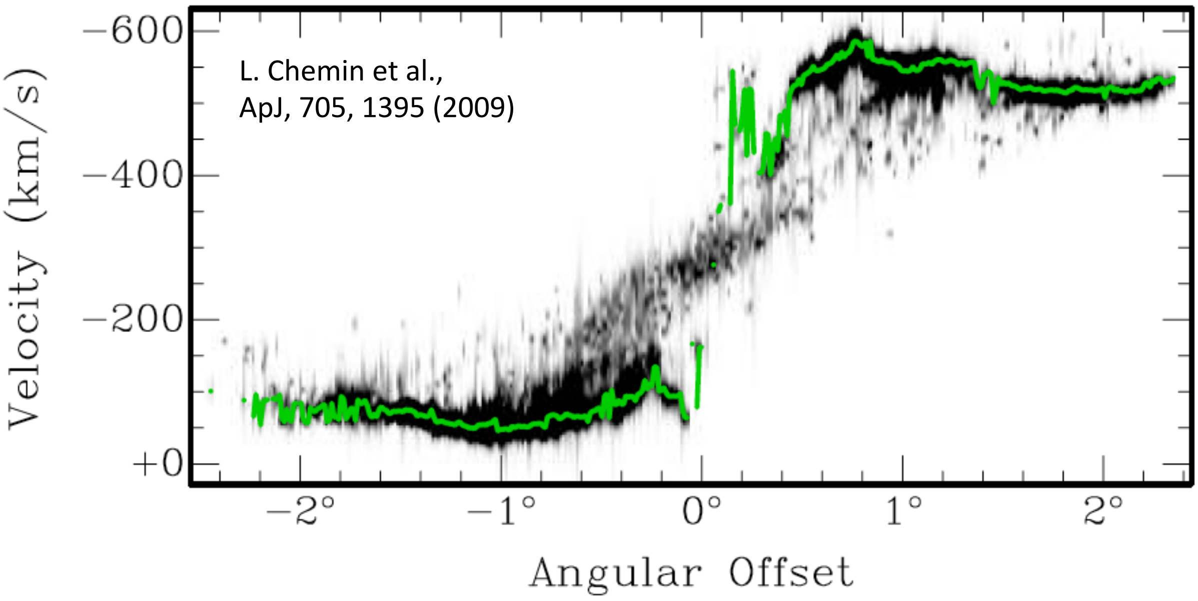
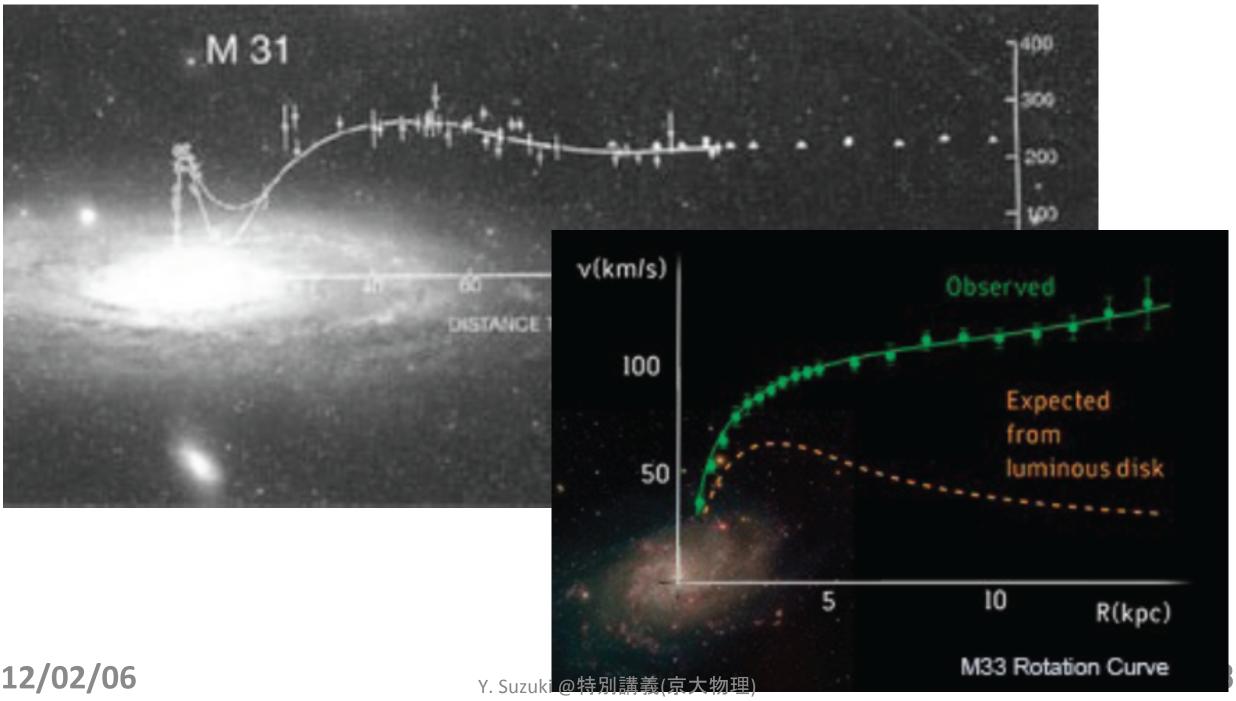
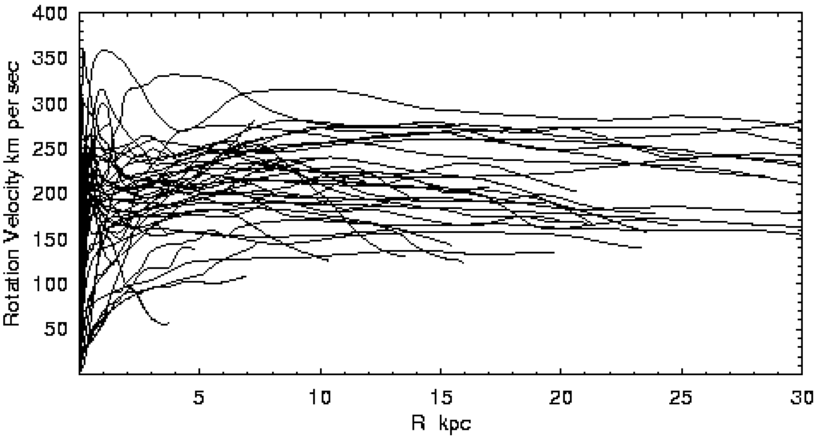
21 cm measurement of M31



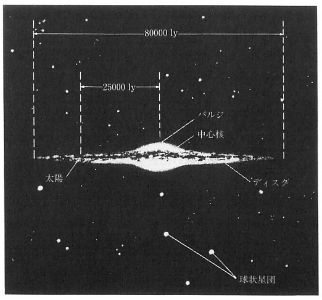
Rotation Curves

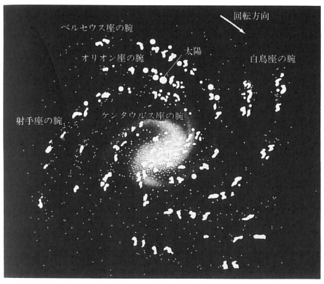


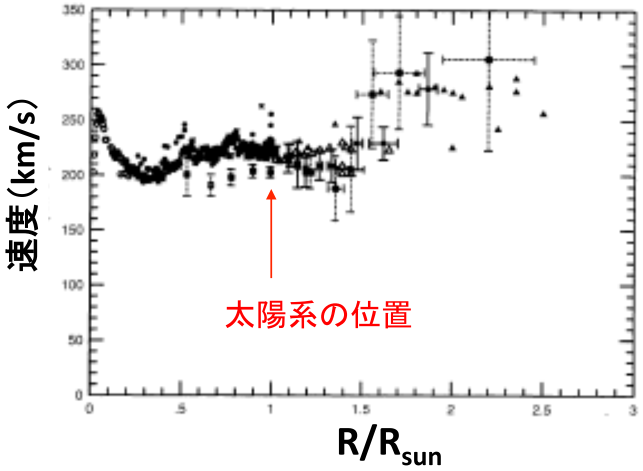


- Rotation curves of spiral galaxies: combining CO data for the central regions, optical for disks, and HI for outer disk and halo
- (ref) (Fig.4) Y. Sofue, V. Rubin, Rotation Curves of Spiral Galaxies, Ann. Rev. Astron. Astrophyscis, 39 (2001)

我が銀河







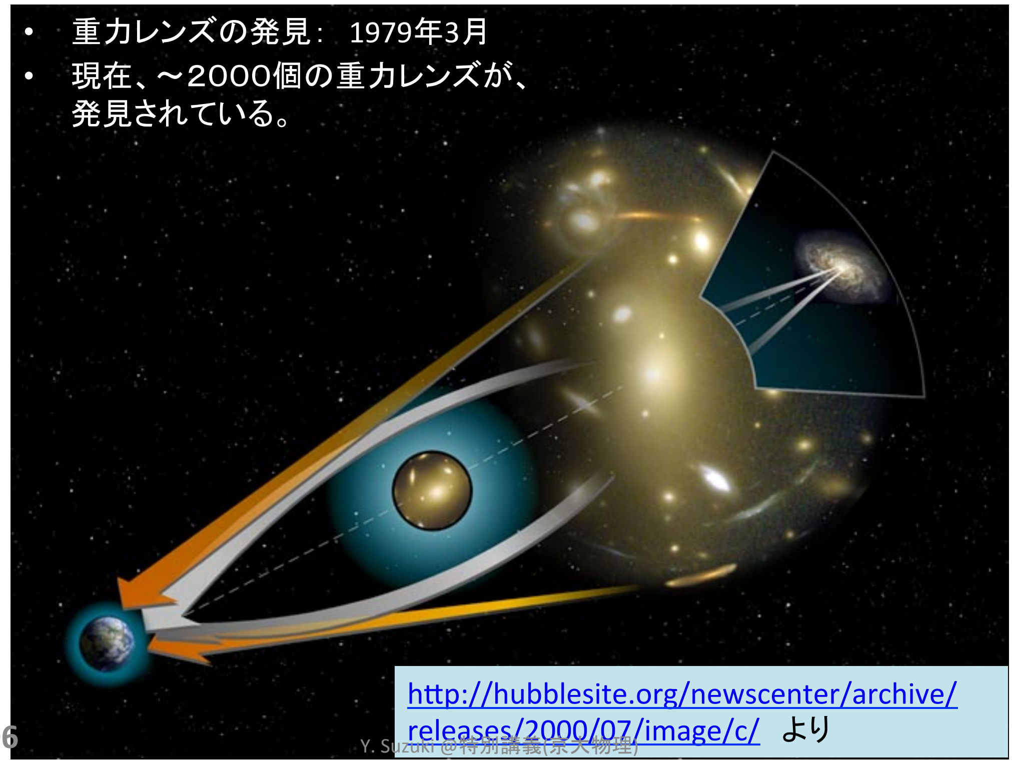
我々の銀河にもダークマターがある。

21cm線： 1420.40575 MHz (21.106114cm)

中性の水素： $E_{spin平行}>E_{spin反平行}$：遷移~1000万年

1-c)-1. Gravitational Lensing

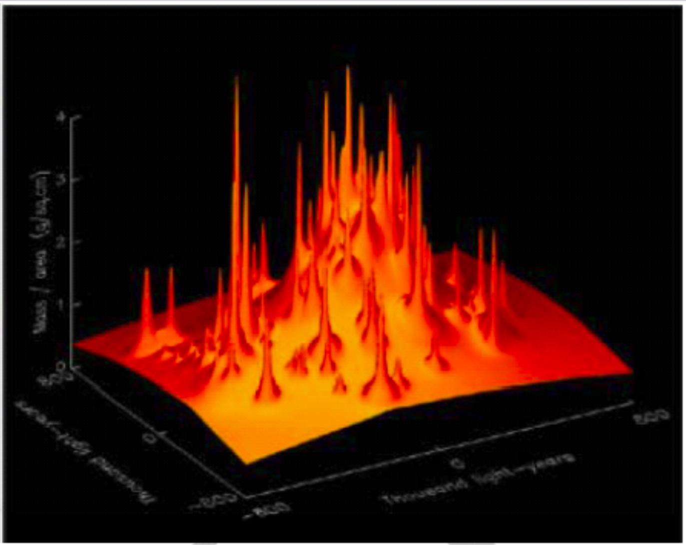
重力レンズ



- 重力レンズの発見： 1979年3月
- 現在、～2000個の重力レンズが、発見されている。

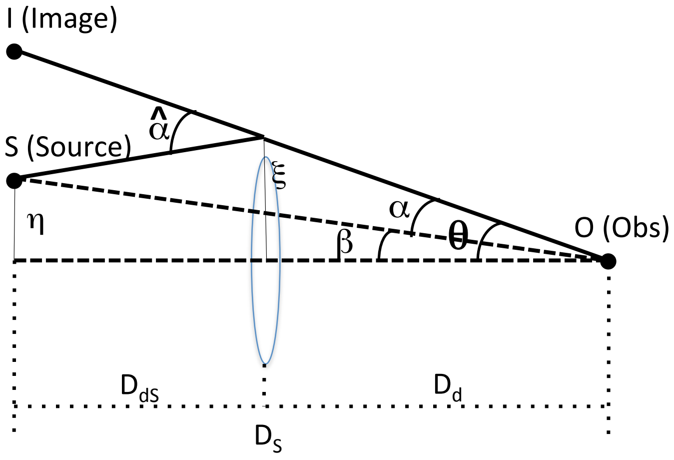
http://hubblesite.org/newscenter/archive/releases/2000/07/image/c/ より

銀河群にまとわりついている暗黒物質 ← 重力レンズ



- 左図：手前の銀河群による重力レンズ効果。遠くの銀河が重力レンズ効果により複数個見える（青色で表示）。
- 右図：レンズ効果により得られた手前の銀河群の‘質量’分布。連続した分布が暗黒物質によるもの。

Deflection Angle



Deflection Angleの導出は、

(Ref) Lecture notes on
Gravitational Lensing,
R. Narayan and M. Bartelmann,
astro-ph/9606001v2

などを、参照にすること。

$$\hat{\alpha} = \frac{4GM}{c^2\xi}$$

: Deflection Angle

M: lens Mass, ξ: Impact Parameter

Deflection Angle/Schwarzschild半径

- Schwarzschild半径 R_s=2GM/c^2
  - Deflection angle: $\hat{\alpha} = \dfrac{R_s}{\xi/2}$

余談 (Shwarzschild半径）
古典的にもescape velocityから求められる

$$\frac{1}{2}mv_{esc}^2 = G\frac{mM}{r}$$

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

$$\rightarrow \quad R_s = \frac{2GM}{c^2}$$

Schwarzschild半径

- （例）太陽
- $R_{\odot s} = 2GM_{\odot}/c^2$

$$= (2\mathrm{x}6.67\mathrm{x}10^{-11}\mathrm{x}1.99\mathrm{x}10^{30})/(3\mathrm{x}10^{8})^2$$

$$= 2.95\ \mathrm{km}$$

$$\begin{aligned}\hat{\alpha}_{\odot}^{rim} &= 2.95/(6.96\times10^{5}/2)\\ &= 0.848\times10^{-5}rad\\ &= 1.75''\end{aligned}$$

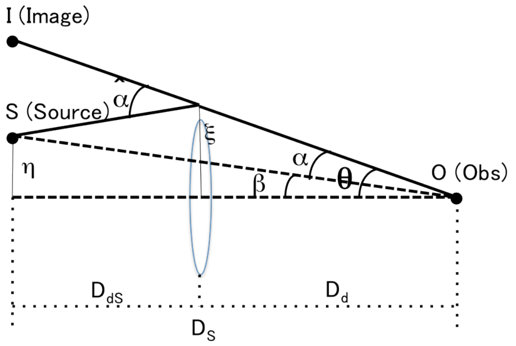
Q: 地球のSchwarzschild半径は？
Q: 10^8 $M_{\odot}$ のblack holeの場合は？

- $M_{\odot}$= 1.99x 10^{33} g
- G_N= 6.67 x 10^{-11} $m^3kg^{-1}s^{-2}$
- $R_{\odot}$= 6.96 x 10^5 km

Lens　Equation

- Reduced deflection angle

$$\vec{\alpha} = \frac{D_{ds}}{D_s}\vec{\hat{\alpha}}$$



$$\theta D_s = \beta D_s + \hat{\alpha} D_{ds}$$

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

Source ⇔ image relation
Lens equation (ray-tracing equation)

- Non-linear ➔ multiple image of $\vec{\theta}$ from a single source $\vec{\beta}$
- Euclidian relation: separation=angle x distance
  ➔ Curved space?
  ➔ Distance → <u>defined</u> as to satisfy 'the relation'
  ➔ $D_{ds} \neq D_s - D_d$

Einstein Radius

- Circularly symmetric and arbitrary mass profile

$$\hat{\alpha}(\xi) = \frac{4GM(\xi)}{c^2\xi}$$

$$\alpha(\theta) = \frac{D_{ds}}{D_s}\alpha(\hat{\theta}) \qquad \xi = D_d\theta$$

$$\beta = \theta - \alpha(\theta) = \theta - \frac{D_{ds}}{D_s}\frac{4GM(\theta)}{D_d c^2\theta}$$

- A source on β = 0 ➔ ring
- Einstein ring (radius): $\theta_E = \sqrt{\frac{D_{ds}}{D_s D_d}\frac{4GM(\theta_E)}{c^2}}$

Example

- Lensing by a star in our Galaxy

$$\theta_E = (0.9\ \mathrm{mas})\left(\frac{M}{M_\odot}\right)^{1/2}\left(\frac{D}{10\ \mathrm{kpc}}\right)^{-1/2}$$

mas: milli arc second

- Lensing by a cluster of galaxies

$$\theta_E = (90'')\left(\frac{M}{10^{15}M_\odot}\right)^{1/2}\left(\frac{D}{\mathrm{Gpc}}\right)^{-1/2}$$

Example

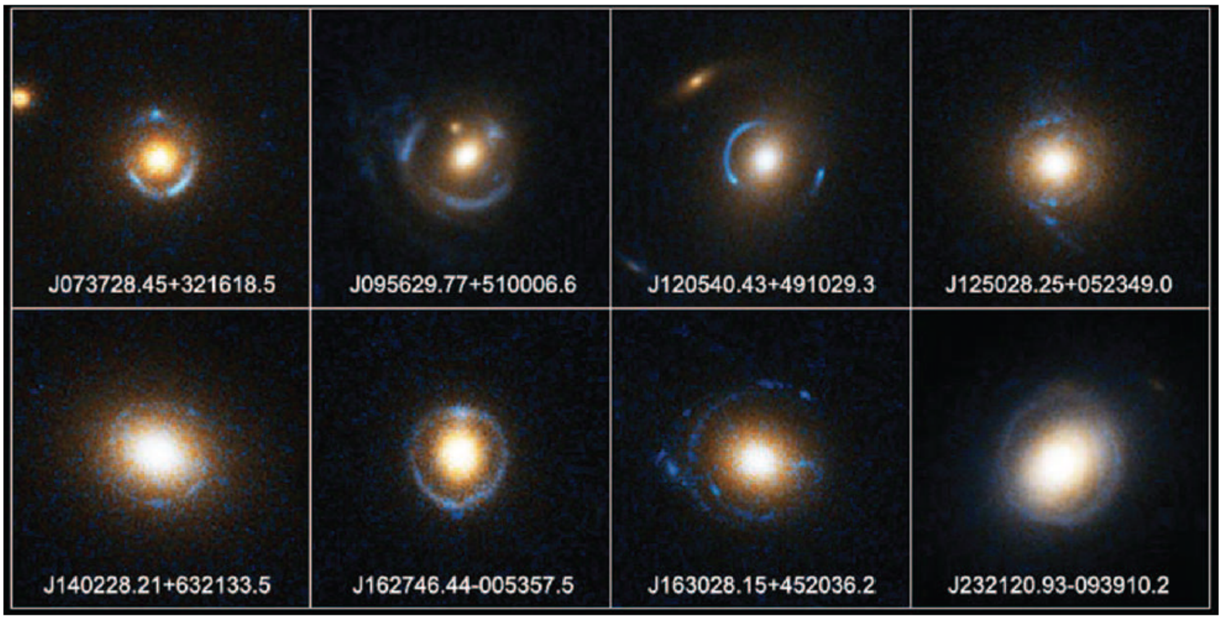


Figure 3. Einstein rings produced by a galaxy behind the lensing galaxy. The sources are actually extended and that is why one sometimes sees arcs rather than complete rings. (Photo credit: NASA, ESA, and the SLACS Survey team: A. Bolton (Harvard/Smithsonian), S. Burles (MIT), L. Koopmans (Kapteyn), T. Treu (UCSB), and L. Moustakas (JPL/Caltech).)

Galaxy Cluster Abell2218, HST

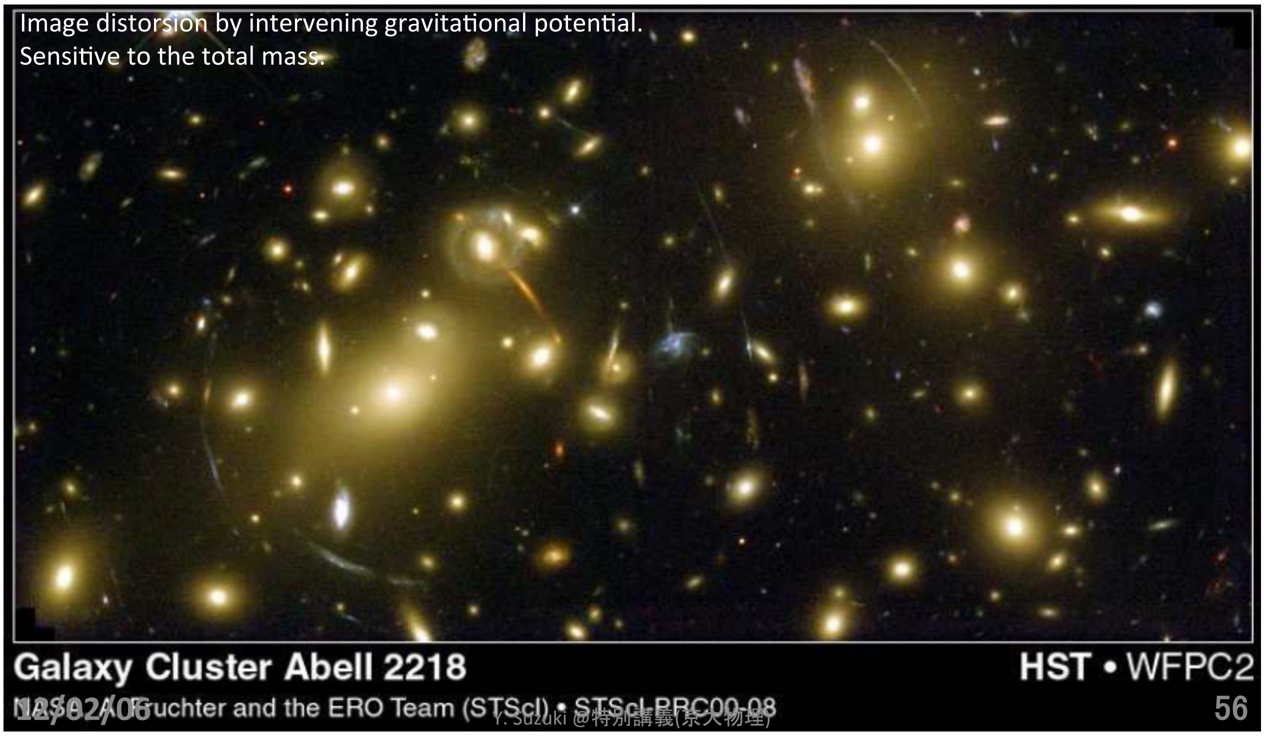
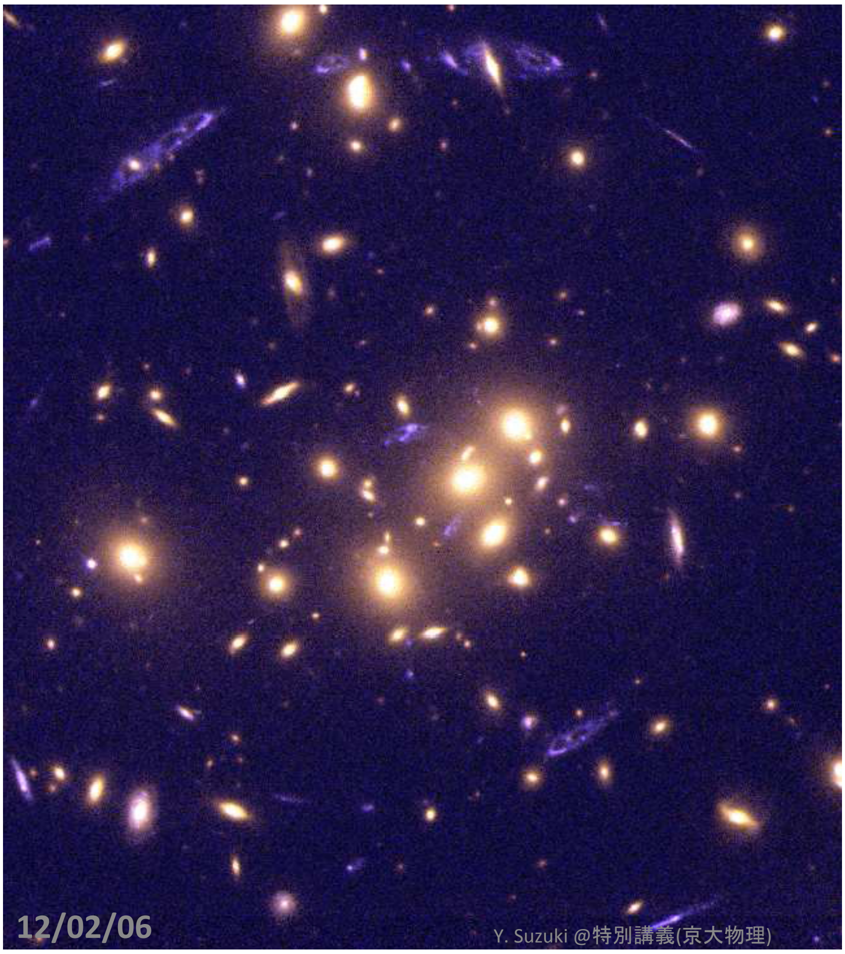
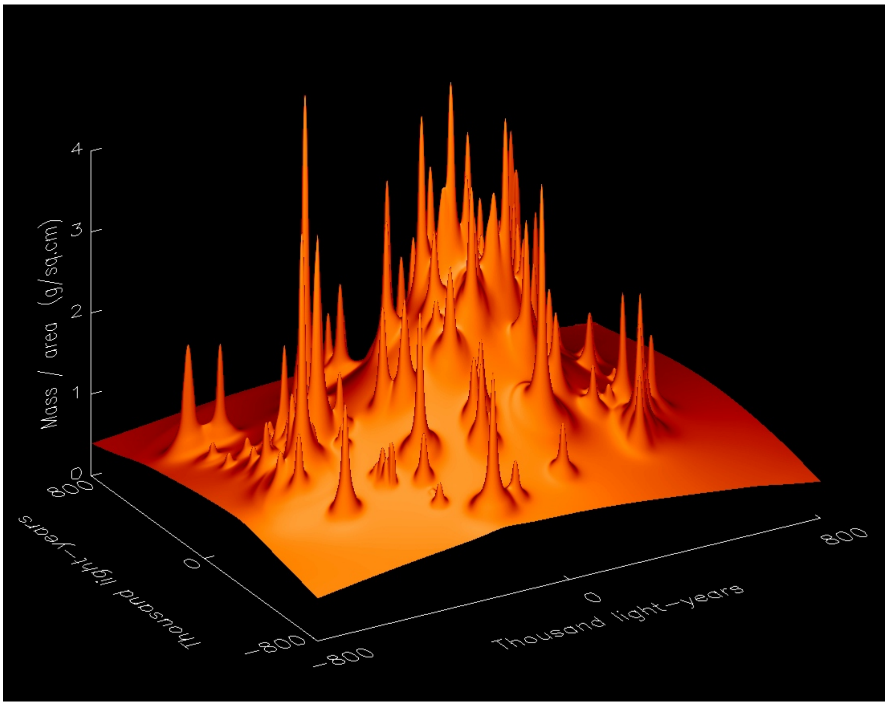


Image distorsion by intervening gravitational potential.
Sensitive to the total mass.



Galaxy Cluster
CL0024+1654

Reconstructed Matter Distribution in CL0024-1654



[Tyson et al., Ap. J. **498** L107-110, 1998]

1-c)-2. X-ray from clusters of galaxies

- X-ray 🡸 by thermal bremsstrahlung

  ← hot inter cluster gas (ICG)

  (move in gravitational field): $10^7 \sim 10^8$ K
- r-dependent measurement
- Assume thermal equilibrium and spherical symmetry

Hydrostatic Equilibrium

- Balance : gravitational infall ⇔ pressure
- Hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{GM(r)\rho_{gas}(r)}{r^2}$$

- P: Pressure of the gas, ρ: density of the gas
- M(r): total mass < r

$$PV = NkT \quad \Rightarrow \quad P = \frac{\rho_{gas} kT}{\mu m_p}$$

- k: Boltzman constant, T: temperature of the gas,
- μ: mean molecular weight of the gas,
- m_p: proton mass

Total mass

- 全ページの２式から $\frac{dP}{dr} = -\frac{GM(r)\rho_{gas}(r)}{r^2}$ $P = \frac{\rho_{gas} kT}{\mu m_p}$

$$M_T(r) = -\frac{kT(r)r}{G\mu m_p}\left(\frac{d(\ln\rho_{\rm gas})}{d(\ln r)} + \frac{d(\ln T)}{d(\ln r)}\right)$$

r d(ln r) = dr を用いて、それぞれから
d(ln P)/d(ln r)を作ればよい。

- Isothermal & Sphericalを仮定すると、（T: 一定）

$$M_T(R) = -\frac{kTR}{G\mu m_p}\left(\frac{d(\ln\rho_{gas}(r))}{d(\ln r)}\right)$$

- R: outer radius, μ: constant
- X線の観測から ➔ T, $\rho_{\rm gas}$(r)を求める ➔ M_T(R)

計算

$\frac{dP}{dr} = -\frac{GM(r)\rho_{gas}(r)}{r^2}$ $P = \frac{\rho_{gas} kT}{\mu m_p}$

第１式から

$$\frac{d(\ln P)}{d(\ln r)} = \frac{r}{P}\frac{dP}{dr} = -\frac{GM(r)\rho_{gas}(r)}{Pr} = -\frac{G\mu m_p M(r)}{kTr}$$

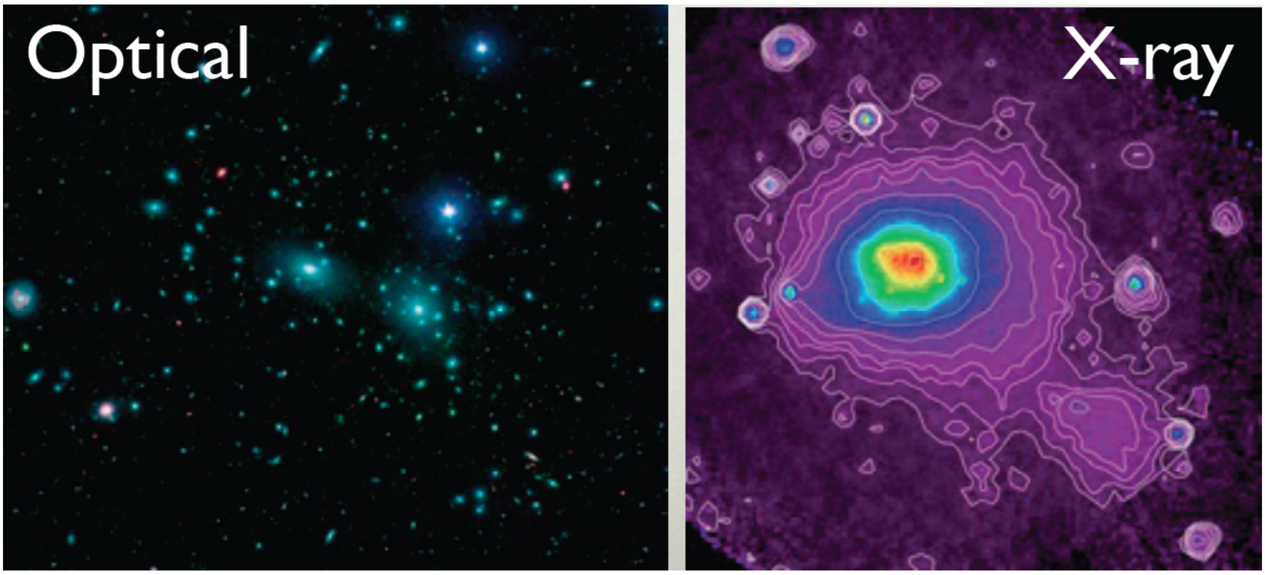
第2式から

$$\frac{d(\ln P)}{d(\ln r)} = \left(\frac{d(\ln\rho_{\rm gas})}{d(\ln r)} + \frac{d(\ln T)}{d(\ln r)}\right)$$

比較して

$$M_T(r) = -\frac{kT(r)r}{G\mu m_p}\left(\frac{d(\ln\rho_{\rm gas})}{d(\ln r)} + \frac{d(\ln T)}{d(\ln r)}\right)$$

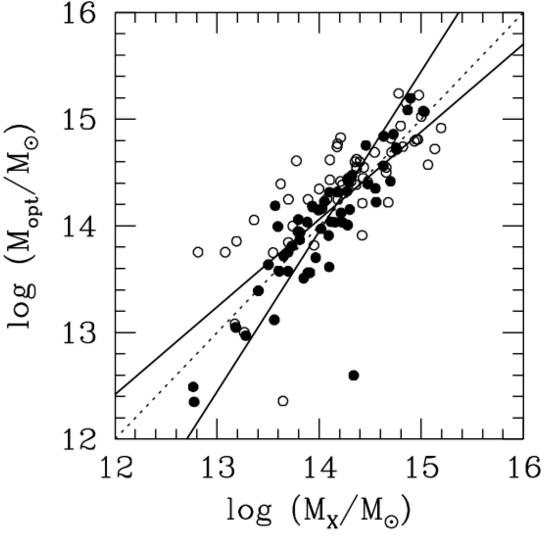
Example



- 1~2% starts, 5~15% gas, rests: dark matter

Optical Virial Mass vs X-ray Mass

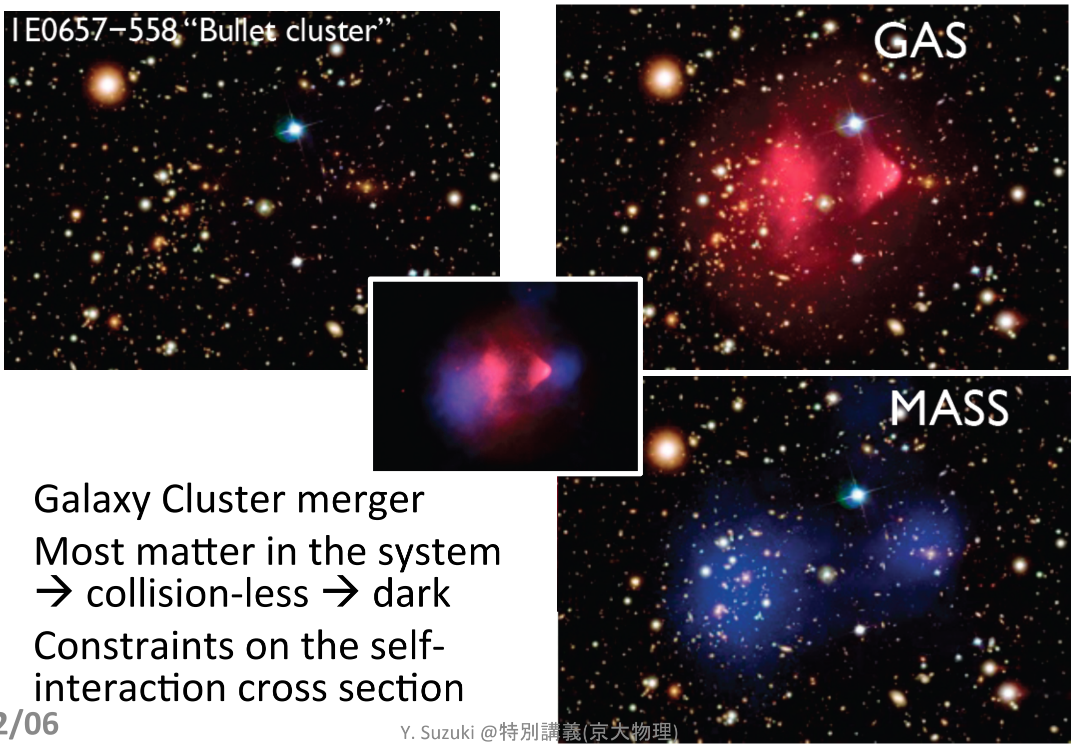
M.Girardi et al., astro-ph/9804187



- Optical Virial Mass vs those derived from X-ray analysis.
- Scaled to the same radius
- Good agreement!
- The solid points: from White et al. (1997).

Others

Bullet Cluster



- Galaxy Cluster merger
- Most matter in the system → collision-less → dark
- Constraints on the self-interaction cross section

2.Dark matter and Cosmology

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2. Dark Matter and Cosmology

a) Big Bang Nucleosynthesis

b) Cosmic Microwave Background

c) Large Scale Structure

Summary

- Big Bang Nucleosynthesis
  - $\Omega_B h^2 \sim 0.023$, non-Baryonic Dark Matter
- CMB +…
  - $\Omega_B h^2 = 0.0224 \pm 0.009$, $\Omega_M h^2 = 0.135 \pm 0.009$, $\Omega_\Lambda = 0.73 \pm 0.04$
  - $\Omega_{DM} h^2 = 0.113 \pm 0.009 = \Omega_M h^2 - \Omega_B h^2$
    - non-Baryonic Dark Matter
- Structure formation
  - Cold Dark Matter
    - Exclude neutrinos → $\Omega_\nu h^2 < 0.0076$ (WMAP + …)

$\rho_{crit} = 3H_0^2/(8\pi G_N) = h^2 \times 1.9 \times 10^{-29}$ g/cm^3
h= 0.71±0.05 (100 kms^{-1}Mpc^{-1})

2–a) Big Bang Nucleosynthesis

BBN → $\Omega_b << \Omega_{matter}$

Story

- Free Parameter of BBN:
  - Cross sections
  - $\eta_B = n_B/n_\gamma$ (number density of baryons/photons)
- BBN depends only on $\eta_B = n_B/n_\gamma$, which can be determined by the observations

  $\eta_B = 6.1^{+0.3}_{-0.2} \times 10^{-10}$
- $T_\gamma = 2.725 \pm 0.001$ determines

  $n_\gamma = (2/\pi)\zeta(3)T^3 = 410.4 \pm 0.5/\mathrm{cm}^3$

  ➔ n_B ($\Omega_B = \rho_B/\rho_c$)

h: $0.71^{+0.04}_{-0.03}$

Nucleosynthesis

- ~100 s after the Big Bang: ~MeV
  - The first three minutes: S. Weinberg「最初の3分」
- Light elements (D, ^{3}He, ^{4}He) from

  $p + n \rightarrow D + \gamma$ (first step)

  t, ^{3}Heを中間状態として　^{4}Heの生成までゆく

  $D + p \rightarrow {}^3He + \gamma$,　$D + D \rightarrow {}^3He + n$,

  $D + D \rightarrow t + p$,　${}^3He + n \rightarrow t + p$

  $t + D \rightarrow {}^4He + n$,　${}^3He + d \rightarrow {}^4He + p$
- 量数5、8の安定原子核が存在しない。^{4}He以上の重元素の生成は困難。

  $^7Li/H$, $^7Be/H \sim O(10^{-10})$

^{4}He/Hの比

合成開始直前にあった中性子がほとんど全てヘリウムになる（その時，$n_p : n_n = 7 : 1$）

- 質量比：$$\frac{m_{He}n_{He}}{m_{He}n_{He}+m_H n_H} \sim \frac{4(n_n/2)}{(n_p-n_n)+4(n_n/2)} \sim \frac{1}{4}$$
- 密度：空気程度
- 星の内部：高密度　➔ヘリウム以上が作られる

Energy density

- 直前のn_p/n_n比
  - 🡄 expansion rate Hで決まる
  - 🡄 $n_\gamma + n_\nu + n_e$
- At T~100MeV:
  - Energy and number density 🡄 relativistic and effectively massless particles (leptons + photons)
  - Neutrons and protons (~10^{-9}) → no contribution to the total energy density
  - All in thermal equilibrium

At 100 MeV

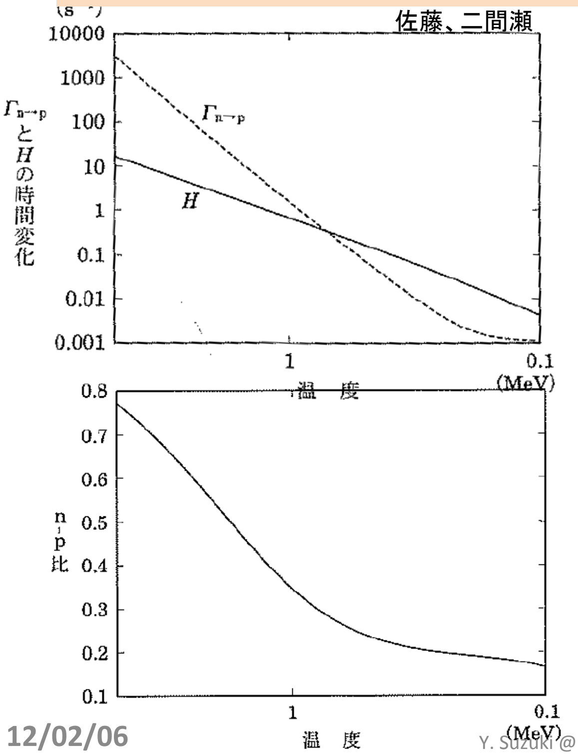
$$\nu_e + n \leftrightarrow p + e^-$$
$$e^+ + n \leftrightarrow p + \bar{\nu_e}$$
$$\text{and}$$
$$n \leftrightarrow p + e^- + \bar{\nu_e}$$

- Neutron and proton number density:

$$\frac{n_n(T)}{n_p(T)} = \exp\left(\frac{-\Delta m}{T}\right)$$
$$\Delta m = m_n - m_p \simeq 1.29MeV$$

- At 100 MeV, n_n/n_p is close to unity.

At 0.7 MeV



佐藤、二間瀬

$$\frac{n_n(T)}{n_p(T)} = \exp\left(\frac{-\Delta m}{T}\right)$$
$$\Delta m = m_n - m_p \simeq 1.29MeV$$

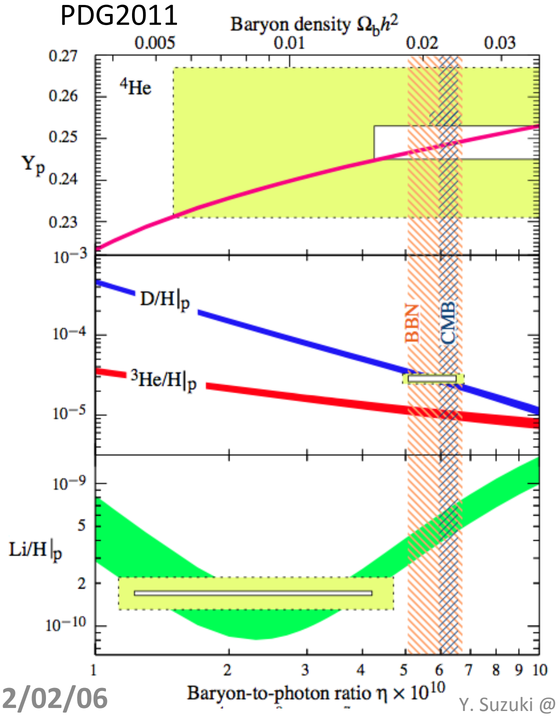
- $\Gamma_{n\rightarrow p} < H$ になった時（~0.7MeV）
  - n_n/n_p比はほぼ固定
  - exp(-1.293/0.7) → $n_n : n_p = 1 : 6$

At 0.1 MeV

- n + p → D + γ Q=2.22 MeV
  - しかし、逆反応 D + γ → n + p が起きない為には、γ のエネルギーが2.22 MeVより低くなる必要
  - しかし、大量のphotonがある：　$n_\gamma \sim n_B \times 10^{10}$
  - プランク分布で、エネルギーがε_γ > Tとなる光子の数が10^{-10}となる温度は、
    - $\exp(-\varepsilon_\gamma/T) = 10^{-10}$
    - $T_D = 2.2\mathrm{MeV}/\ln(10^{10}) \sim 0.1$ *MeV*　になると、Dの生成が始まる。→ $n_n : n_p = 1 : 7$　（0.7から0.1 MeVになるの間の neutron decayにより neutronの数が減る)

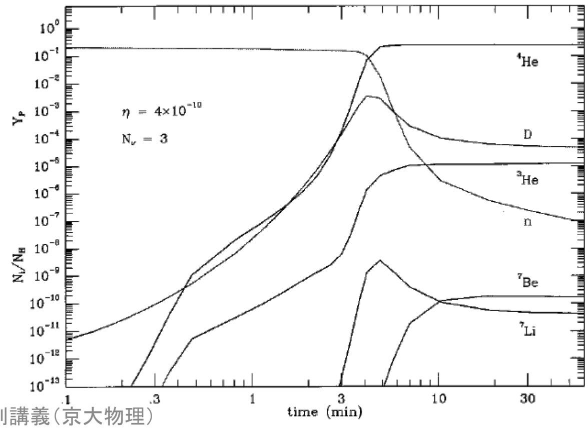
Concordance

PDG2011



Concordance:

- η_B = 5.1 – 6.5x 10^{-10}
  + T_γ = 2.725
  → ρ_B = 3.5-4.5 x 10^{-31}g/cm^3
- $\Omega_B h^2$= 0.019 – 0.024



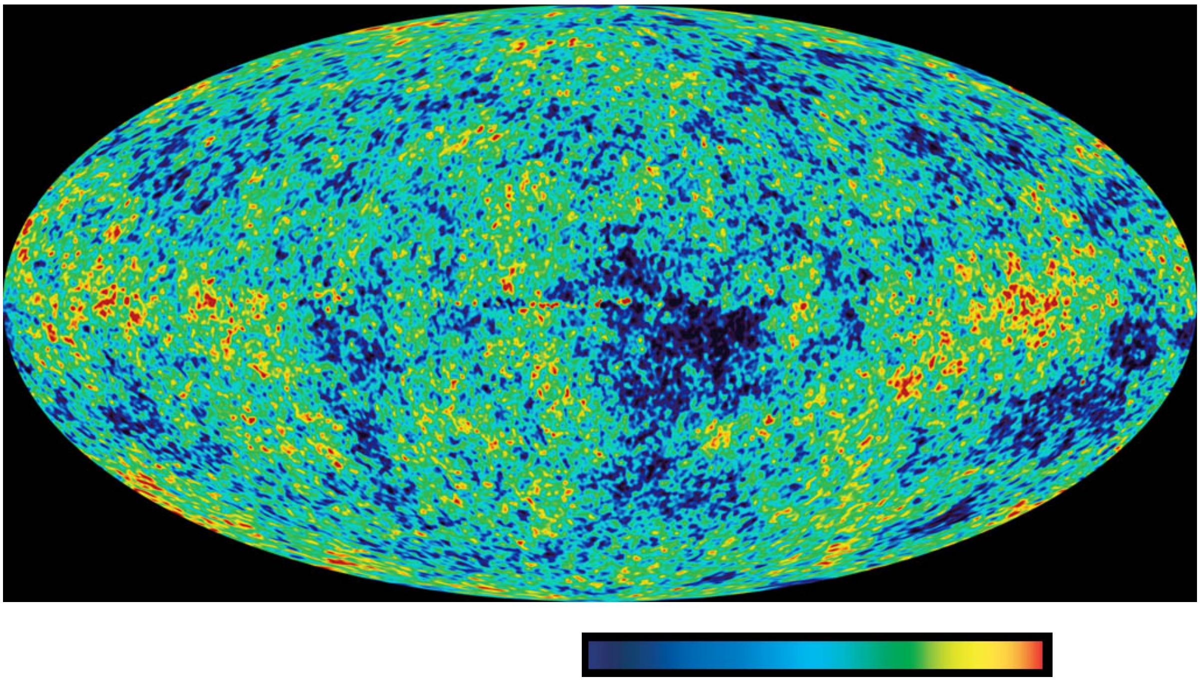
Non-Baryonic Dark Matter

- Constraints on baryon density (Galaxies+ Gas):
  - $\Omega_B h^2 = 0.0224 \pm 0.009$
    - Ω_B ~ few % of the critical density ($\Omega = \rho/\rho_0$)
    - Agree with CMB estimate of Baryon density
  - $\Omega_M h^2 = 0.135 \pm 0.009$
    - Most matter in the Universe is non-Baryonic

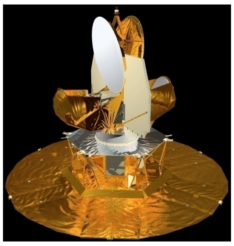
- BBN → Non-baryonic dark matter

最新の数値はチェックしてません。
整合性はとってません。

2-b) Cosmic Microwave Backgrounds



Cosmic Microwave Backgrounds



- Universe
  - Isotropic, Homogeneous
- Matter and Radiation decoupled
  - Protons and electrons form neutral hydrogen: recombination
  - Become transparent for photons
  - ~380,000 years after the Big Bang: ~eV
- Very small temperature fluctuations (CMB)
  - But too small to evolve into structure observed
- Today: T=2.715K
  - $\Delta T \sim 20\ \mu K$ ($\sim 10^{-5}$)
- ➔ require additional matter to start forming structure earlier

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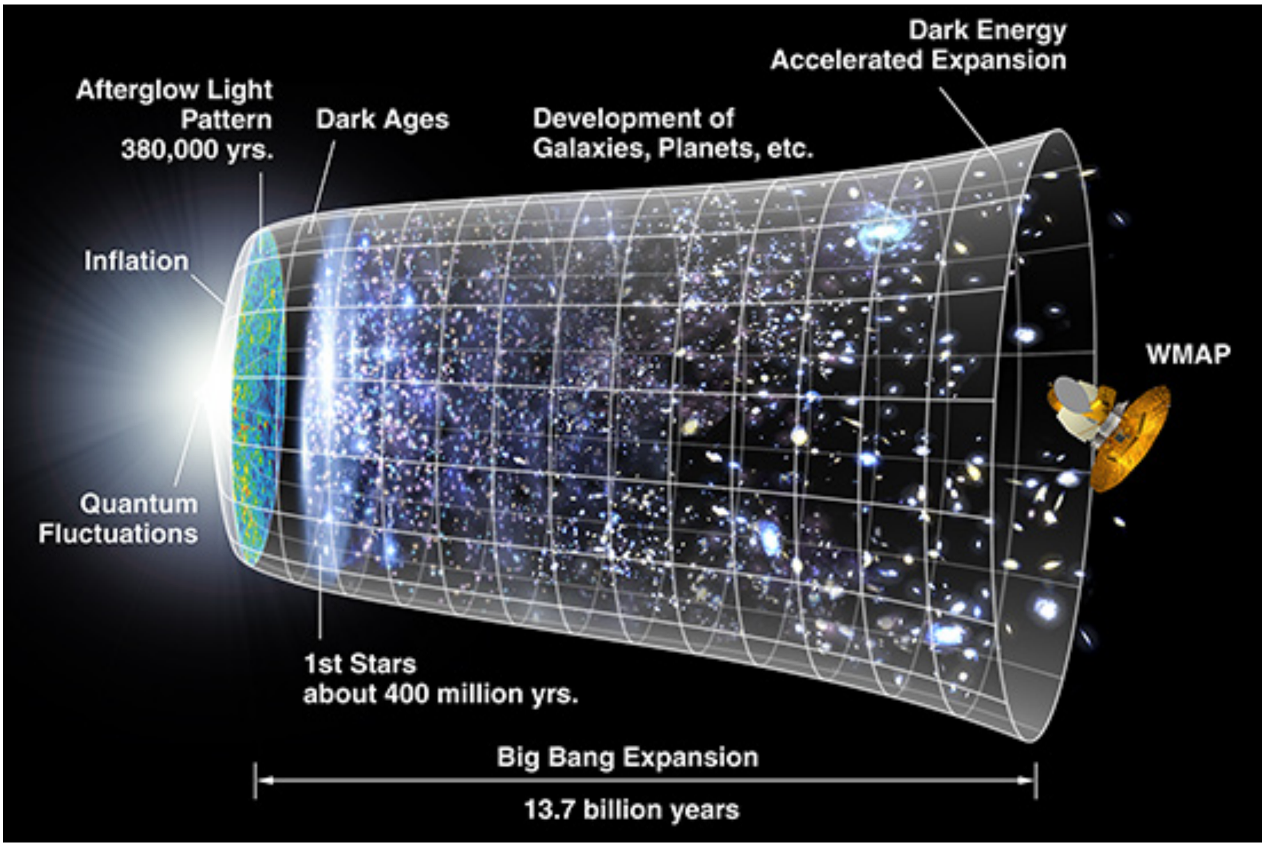


Cosmic Microwave Backgrounds

- Decompose temperature field into spherical harmonics

$$\frac{\delta T}{T}(\theta,\phi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} a_{T,lm}Y_{lm}(\theta,\phi)$$

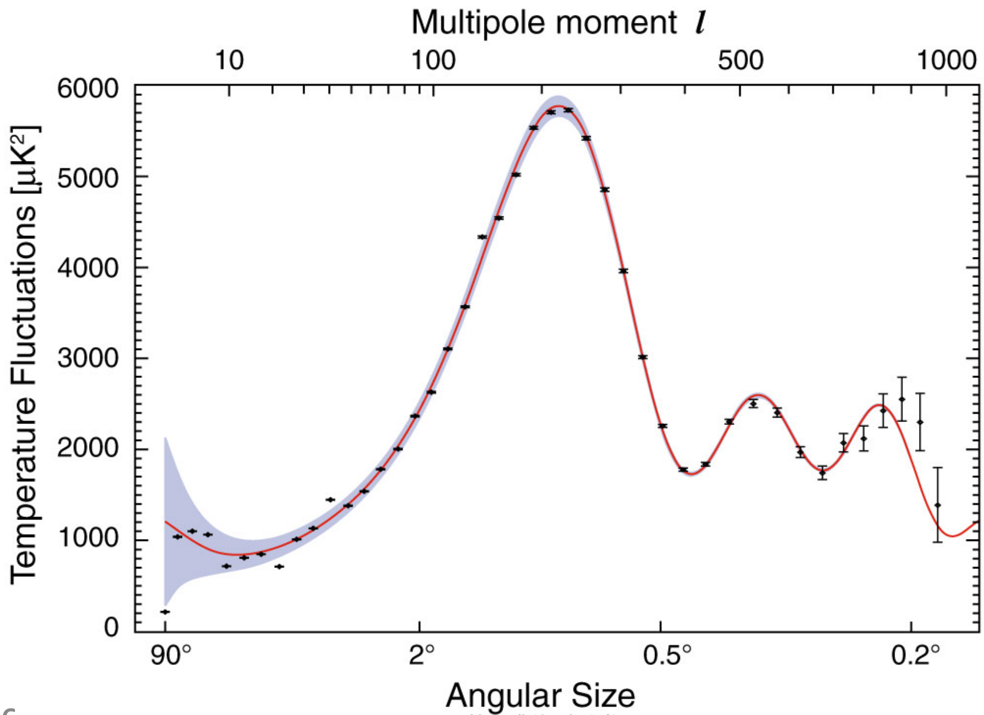
$$C_l^{TT} = \frac{1}{2l+1}\sum_{m=-l}^{l}|a_{T,lm}|^2$$

$$l \simeq 180[1^\circ/\theta]$$

- CMB angular power spectrum

  ➔ Depend on Ω_B, Ω_M, Ω_Λ

WMAP、7 yrs data

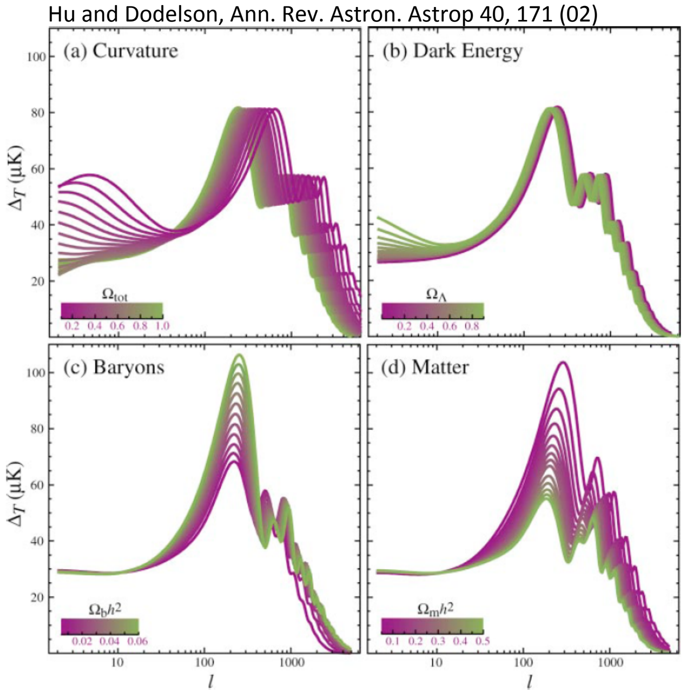


Peaks

- Structure of the CMB Anisotropies
  - Acoustic Oscillations → characteristic peaks
    - Competition in the photon-baryon plasma
    - Pressure of photons → erase anisotropies
    - Gravitational attraction of baryons → collapse to form dense region
- Peaks
  - Resonances when photons decouple when a particular mode is at its peak amplitudes
- First peak:
  - Angular scale → curvature of the universe
- Second Peak:
  - Ratio of the odd and even peaks → reduced baryon density
- Third peak:
  - Information of the DM density

Peaks

Hu and Dodelson, Ann. Rev. Astron. Astrop 40, 171 (02)



- Peak location and heights
- Constraints on the parameters and geometry of the universe

  (flat, Ω_{tot}=1)
  - Ω_B = 0.0449 ±0.0028
  - Ω_M = 0.222 ± 0.026
  - Ω_Λ = 0.734± 0.029

Ref) Jarosik et al (2007), Hu et al. (2002)

DM fluctuation

- CMBのfluctuationでは、１００倍ほど足りない。
- Non-baryonic:
  - Early development of density fluctuation
  - ⇔radiation pressure:
    - Does not slow the early growth of the fluctuation before the recombination

---

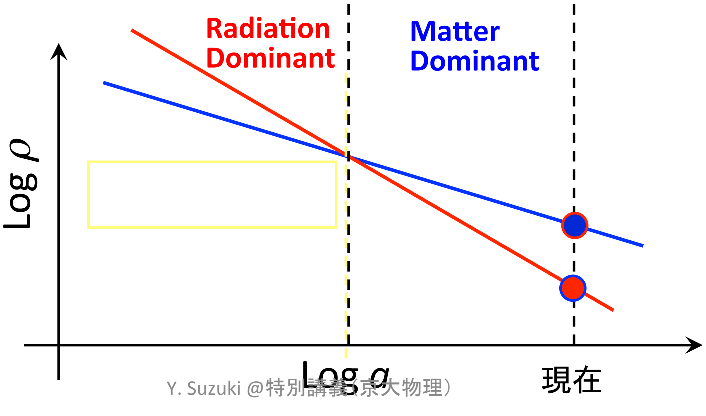
2-c) Large Scale Structure

Structure

- mid-1970 (Observation)
  - Galaxies: chains & filament
  - Void (several tens of Mpc)
- Zeldovich (1970) top → down
- Peeble (1971) bottom → up
- Gravitational Clustering : slow process
  - Density fluctuation must be 1/1000 of the density itself at the time of recombination
  - WMAP: fluctuation
    - Small : 2 orders
    - Luminous (baryonic) matter
  - Early start of the structure formation by Dark Matter

宇宙のエネルギー密度の変化

- 物質 $\rho_m \propto a^{-3}$
- 輻射 $\rho_r \propto a^{-4}$ $(E_r \sim 1/\lambda \sim a^{-1})$
- 現在は、matter dominant: $\rho_m \gg \rho_r$
- 過去に、radiation dominantな時があった。



ゆらぎの成長

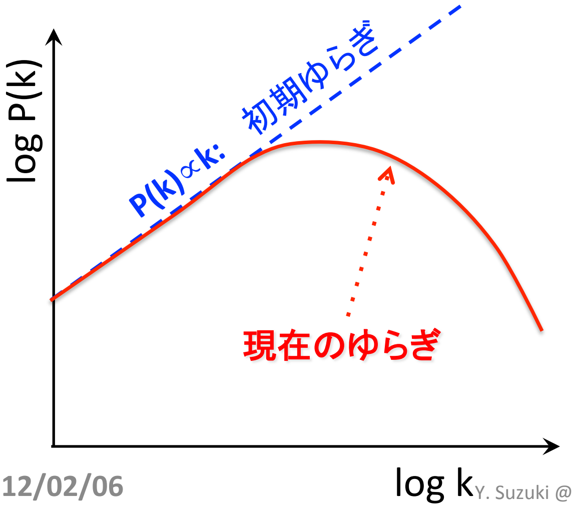
Power Spectrum: P(k)

密度ゆらぎ: $\delta(\mathbf{x},\mathbf{t}) = \rho(\mathbf{x},\mathbf{t})/\overline{\rho}(\mathbf{t}) - \mathbf{1}$

フーリエ分解: $\delta(\mathbf{x}) = \frac{\mathbf{1}}{\sqrt{\mathbf{V}}}\sum_{\mathbf{k}} \delta_{\mathbf{k}} \mathbf{e}^{\mathbf{ik\cdot x}}$

PS: フーリエ係数の平均的な強さ:

$$P(k,t) =< |\delta_k(t)|^2 >, \quad k = |\mathbf{k}|$$

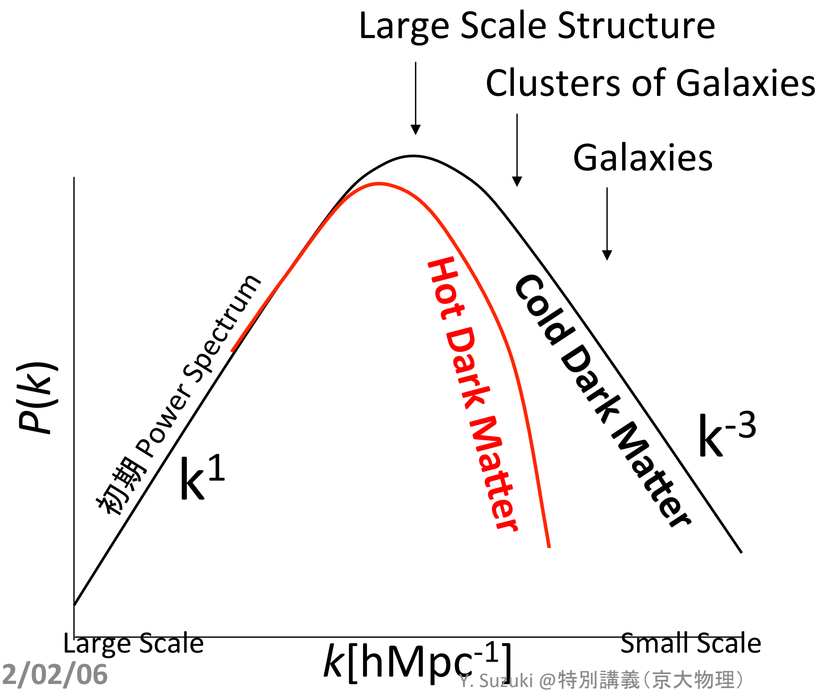


- Radiation Dominantな時は、ゆらぎが成長できない。
- 放射・物質のエネルギー密度が等しい時期（波数）にPower Spectrumに折れ曲がり。

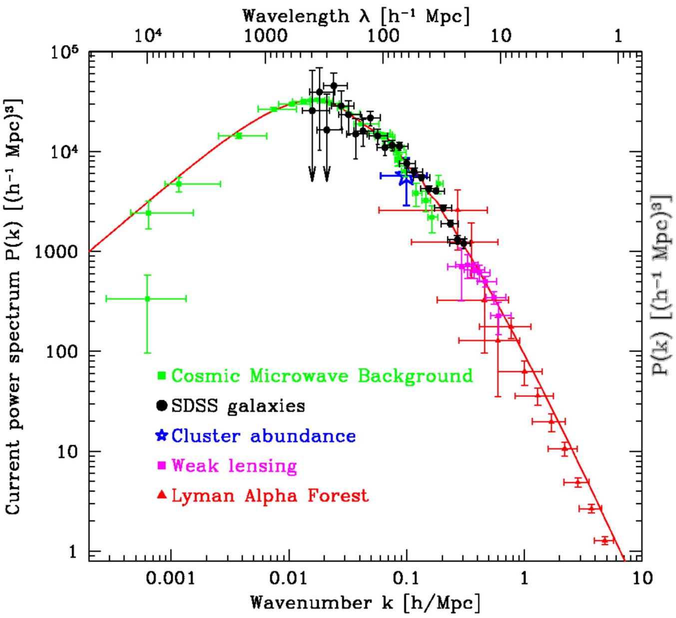
$Z_{eq} = 24000\Omega_M h^2$

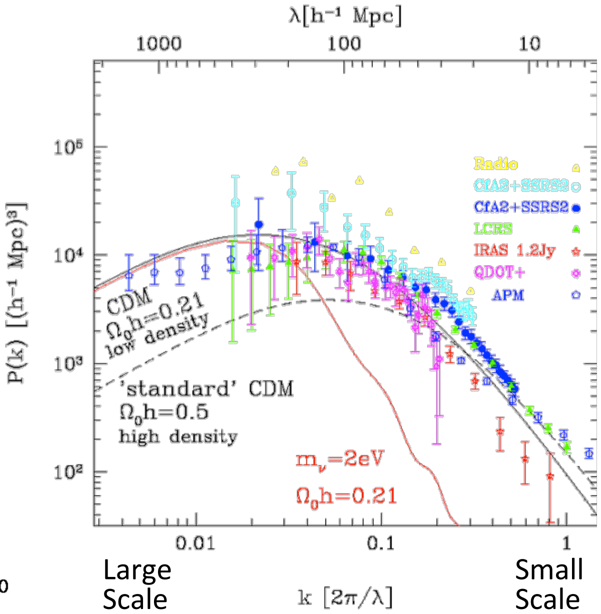
$k=2\pi/\lambda = 2\pi/(cZ/H)$

Power Spectrum



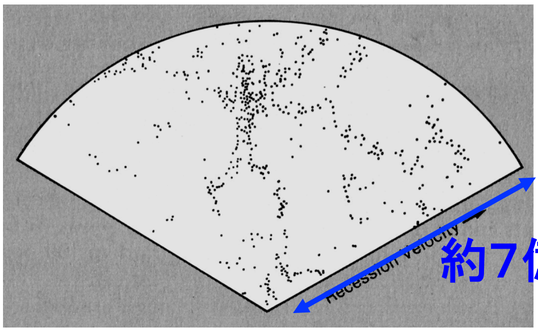
Power Spectrum





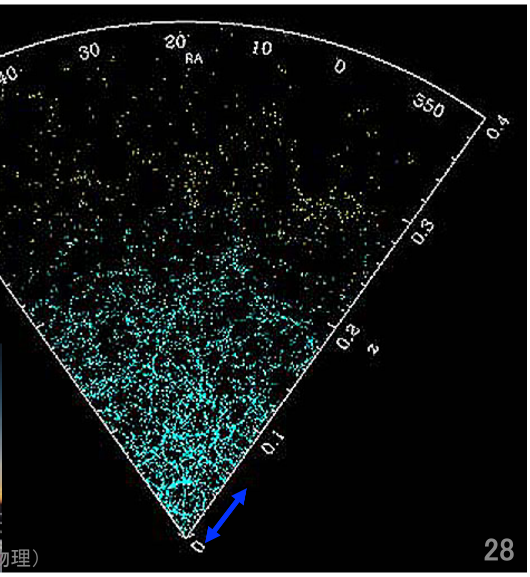
- CDMに一致。Hot dark matterは合わない。

宇宙の大規模構造



宇宙のある方向で15．５等級以上の銀河1000個を表示

約7億光年



スローン･デジタルスカイサーベイによる全天銀河観測

http://www.sdss.org/

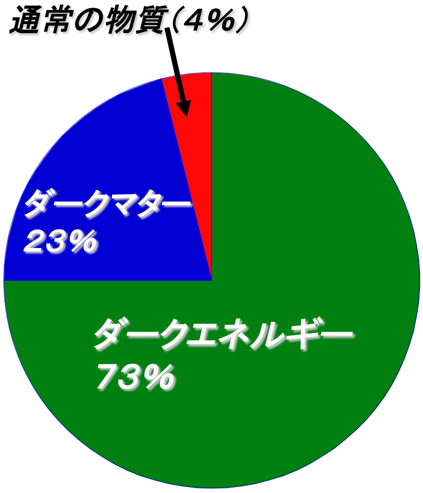
2.5m広視野望遠鏡

まとめ

Dark Matter and Cosmology

- Non-Baryonic Dark Matter
- Cold Dark Matter
- $\Omega_{CDM}h^2 = 0.113 \pm 0.009$

  $(\Omega_M h^2 - \Omega_B h^2)$



3. Galactic Dark Matter

Galactic Dark Matter

a) Isothermal Halo Model

b) Density

c) Velocity

Maxwell-Boltzman distribution

- 速度分布： $f(\mathbf{v}) = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{m|\mathbf{v}|^2}{2kT}\right)$
  - X- 方向成分： $f_1(v_x) = \left(\frac{m}{2\pi kT}\right)^{1/2} \exp\left(-\frac{mv_x^2}{2kT}\right)$

$$f(\mathbf{v}) = f_1(v_x)f_1(v_y)f_1(v_z)$$

- 速さ v の分布： $v = \sqrt{v_x^2 + v_y^2 + v_z^2}, \qquad v_0^2 = \frac{2kT}{m}$

$$\begin{aligned} f(v) &= 4\pi v^2 \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right) \\ &= \frac{4\pi v^2}{(\pi v_0^2)^{3/2}} \exp\left(-\frac{v^2}{v_0^2}\right) \qquad = \frac{4v^2}{\sqrt{\pi} v_0^3} \exp\left(-\frac{v^2}{v_0^2}\right) \end{aligned}$$

ガウス積分

$$I_n(a) = \int_0^\infty x^{2n} \exp(-ax^2)dx = \frac{(2n-1)!!}{2^{n+1}}\sqrt{\frac{\pi}{a^{2n+1}}}$$

$$I_0(a) = \frac{1}{2}\sqrt{\frac{\pi}{a}}, \quad I_1(a) = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}, \quad I_2(a) = \frac{1}{8}\sqrt{\frac{\pi}{a^5}}, \quad \ldots..$$

$$J_n(a) = \int_0^\infty x^{2n+1} \exp(-ax^2)dx = \frac{n!}{2a^{n+1}}$$

$$J_0(a) = \frac{1}{2a^1}, \quad J_1(a) = \frac{1}{2a^2}, \quad J_2(a) = \frac{2}{2a^3}, \quad \ldots..$$

Maxwell-Boltzman distribution

- （例） $<v^2> = (3/2)v_0^2 \quad = 3kT/m$

$$< v^2 >= \frac{\int_0^\infty v^2 f(v) dv}{\int_0^\infty f(v) dv}$$

- （例） $<v> \; = (4/\pi)^{1/2} v_0 = (8kT/\pi m)^{1/2}$

$$< v >= \frac{\int_0^\infty v f(v) dv}{\int_0^\infty f(v) dv}$$

Standard Halo Model

- Dark Halo of the Galaxy (ref) JCAP09(2010)026
  - Single component self-gravitating isothermal sphere (ignore visible matter)
  - Maxwellian velocity distribution (Galaxy rest frame)

$$f(\mathbf{x}, \mathbf{v}) d^3\mathbf{v} = 4\pi\rho(\mathbf{x}) \left(\frac{3}{2\pi < v^2 >} \right)^{3/2} v^2 \exp\left(-\frac{3v^3}{2 < v^2 >} \right) dv$$

$< v^2 >$: velocity dispersion

- Justification of the Maxwellian distribution

(ref) D. Lynden-Bell, *Statistical mechanics of violent relaxation in stellar systems*, *Mon. Not. Roy. Astron. Soc.* 136 (1967) 101.

Standard Halo Model

- Velocity dispersion:

$$< v^2 >^{1/2} = \sqrt{\frac{3}{2}} v_{c,\infty}$$

- $V_{c,\infty}$: asymptotic value of the circular rotation speed of a test particle in the gravitational field of the isothermal sphere
- $V_{c,\infty} \approx V_{c,\odot} \approx 220 \pm 20$ km/s (galactic rotation velocity in the solar neighbor)

  ➔ $<v^2>^{1/2} \approx 270$ km/s

Standard Halo Model

- $\rho_{DM,\odot} \approx 0.3 \pm 0.1$ GeV/cm^3
  - (ref) J.H. Oort, *Note on the determination of K and on the mass density near the Sun*, *Bull. Astr. Inst. Netherlands* 15 (1960) 45.
  - (ref) J.N. Bahcall, *Self-consistent determinations of the total amount of matter near the sun*, Astrophys. *J.* 276 (1984) 169 [SPIRES].

➔

- A kind of guide line, bench mark
- Not accurate
- But, useful standard

Local Dark Matter Density

- Density: $\rho_{DM,\odot}$
  - Oort & Bahcall:
    - $\rho_{DM,\odot} \approx 0.3 \pm 0.1$ GeV/cm^3
  - Model Independent (P.Salucci, F. Nesti et al., arXiv: 1003.3101)
    - w/o assuming galaxy mass distribution
    - $\rho_{DM,\odot} = 0.430 \pm 0.148$ GeV/cm^3
  - Cosmological N-body simulations:
    - Quasi-Maxwellian
      - Drops off at higher velocity
    - $\rho_{DM,\odot} = 0.37$ GeV/cm^3

Halo density models

$$\rho(r) \propto \frac{1}{(r/a)^{\gamma}[1+(r/a)^{\alpha}]^{(\beta-\gamma)/\alpha}}$$

- a: core radius
  - For $\gamma = 0$, scale of the "~ constant density" core of halo
- Need to know
  - R_0: galacto-centric distance @ sun
  - ρ_0: halo density @R_0

		α	β	γ	references
Isothermal distribution		2	2	0	
Kravtsov et al.	Ka	2	3	0.2	Astro-ph/p708176
Kravtsov et al.	Kb	2	3	0.4	Astro-ph/p708176
Navarrow et al.	NFW	1	3	1	Astrophys. J. 426, 563 (1996)

Velocity of DM

- DM Velocity in the rest frame of the Galaxy: $\mathbf{V}_g$
  - Maxwellian distribution
  - $\mathbf{V}_g = \mathbf{V}_D + \mathbf{V}_E$ → $\mathbf{V}_D = \mathbf{V}_g - \mathbf{V}_E$
    - V_D: DM velocity relative to detector (earth)
    - V_E: Velocity of the earth relative to the Galactic rest frame

$$f(\mathbf{v_D} + \mathbf{v_E}) \sim e^{-(\mathbf{v_D}+\mathbf{v_E})^2/\mathbf{v_0}^2}$$

- $\mathbf{V}_E$ = (0, 220, 0) + (10, 13, 7) → $|\mathbf{V}_E|$=233 km/s
  (1st term) Rotational velocity of the neighbor of the sun
  (2nd term) Sun's local motion in the neighbor
- DM wind from Cygnus

Earth's revolution around the sun

$$\mathbf{v_E} \approx 233 + 15 \times \cos\left(2\pi\frac{t-152.5}{365.25}\right) km/s$$

- 2nd term: the earth's revolution
- t: the day of the year
- t= 152.5 days → June 2nd
- Seasonal Variation
  - ±15/233 ~ 6.5 % flux modulation
  - Rate: $dR/dv_E \sim R/2v_E$ → 3~4% rate modulation in total rate
  - Threshold effect → may up to 7~8%