4. Dark Matter Candidates

Dark Matter Candidates

- Carry masses
- Stable
- Neutral
- Non-Baryonic
- Cold

Dark Matter Candidates

a) MACHO X
  - Baryonic

b) Neutrinos X
  - Too small masses
  - Against structure formation

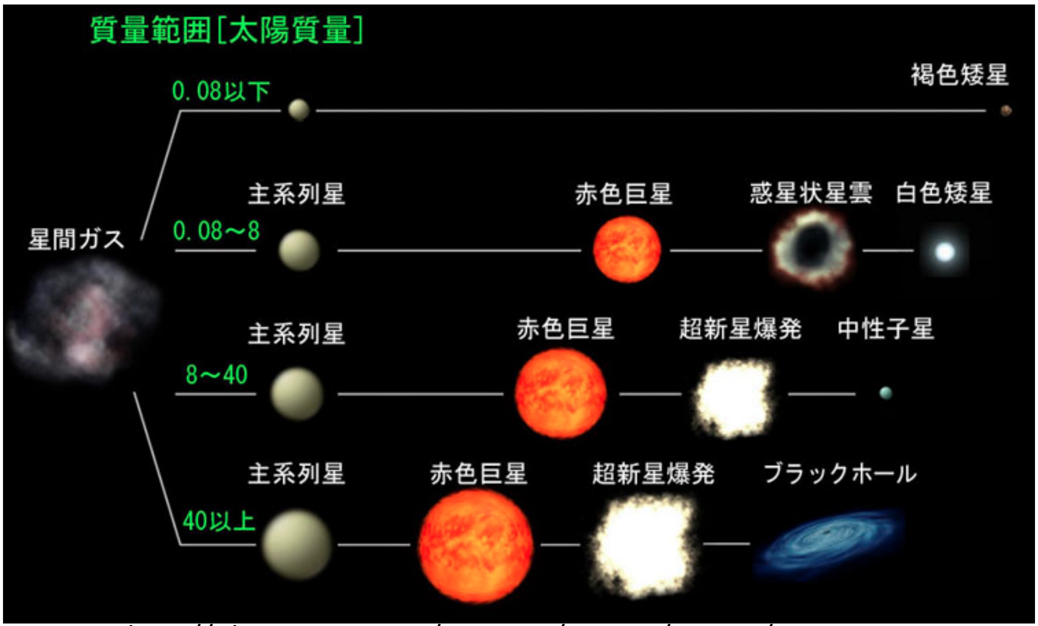
c) Axion ○
  - Satisfy all the conditions
  - Produced non-thermally in the early universe

d) WIMPs ○
  - Satisfy all the conditions
  - Produced thermally in the early universe
  - Cross section ⇔ DM density
    - Weak scale interaction

e) Others

MACHO

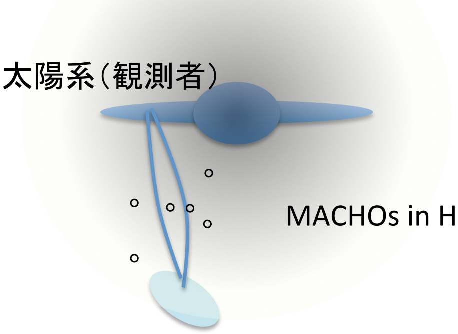
- invisible starts in halo (Baryonic)



http://rikanet2.jst.go.jp/contents/cp0320/images/F05010001.jpg

MACHO

- Historical ref) ‘Gravitational Micro-lensing by the Galactic Halo’, B. Paczynski, Astop.J, 304, 1 (1986).
- マゼラン星雲の～100万個の星（背景天体）を観測すると、その前を通過する銀河haloのMACHOのMicro-lensing効果で、星の増光が観測可能



太陽系（観測者）

MACHOs in Halo

大（小）マゼラン雲(背景天体）

Remember the garavitational Lensing
(Gravitational Lensingの続き）

- For point source/mass

$$\beta = \theta - \alpha(\theta) = \theta - \frac{D_{ds}}{D_s}\frac{4GM(\theta)}{D_d c^2 \theta}$$

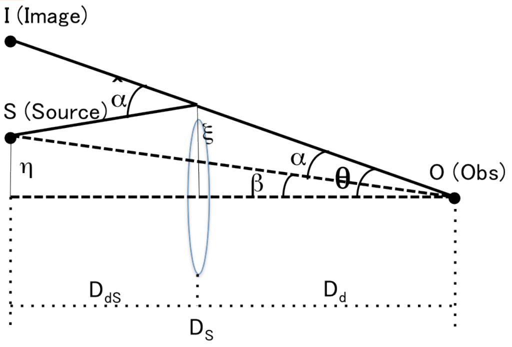
$$\theta_E = \sqrt{\frac{D_{ds}}{D_s D_d}\frac{4GM(\theta_E)}{c^2}}$$

M(θ) → M

$$\beta = \theta - \frac{\theta_E^2}{\theta}$$

θ_E: Einstein Radius

$$\theta^2 - \beta\theta - \theta_E^2 = 0$$



two solutions,

$$\theta_{\pm} = \frac{1}{2}(\beta \pm \sqrt{\beta^2 + 4\theta_E^2})$$

One inside of θ_E
One outside of θ_E

Amplification(増光)

- Solid Angleの差 ➔ 明るさの差, 増幅率（Amplification: μ）を作る → $\mu = \frac{\theta d\theta}{\beta d\beta}$

$\beta = \theta - \frac{\theta_E^2}{\theta}$ から $d\beta = (1 + (\theta_E/\theta)^2)d\theta$

$$\mu = \frac{\theta d\theta}{\beta d\beta} = \frac{\theta d\theta}{\theta(1-(\theta_E/\theta)^2)(1+(\theta_E/\theta)^2))d\theta}$$

$$\therefore \quad \mu_\pm = \left[1 - \left(\frac{\theta_E}{\theta_\pm}\right)^4\right]^{-1}$$

$$\theta_\pm = \frac{1}{2}(\beta \pm \sqrt{\beta^2 + 4\theta_E^2})$$

$u = \beta\theta_E^{-1}$ とおく。（θ_E単位で、βを測ったもの）

$$\frac{\theta_\pm}{\theta_E} = \frac{1}{2}(u \pm \sqrt{4+u^2})$$

Amplification(増光)

$$\left(\frac{\theta_E}{\theta_\pm}\right)^4 = \left(\frac{2}{u \pm \sqrt{4+u^2}}\right)^4$$

$$\frac{\theta_\pm}{\theta_E} = \frac{1}{2}(u \pm \sqrt{4+u^2})$$

$$\begin{aligned}
\mu_\pm &= \left[1 - \left(\tfrac{\theta_E}{\theta_\pm}\right)^4\right]^{-1} \\
&= \frac{(u\pm\sqrt{u^2+4})^4}{(u\pm\sqrt{u^2+4})^4 - 2^4} \\
&= \frac{(u\pm\sqrt{u^2+4})^4}{2u(u\pm\sqrt{u^2+4})\cdot 2\sqrt{u^2+4}(u\pm\sqrt{u^2+4})} \\
&= \frac{(u\pm\sqrt{u^2+4})^2}{2u\cdot 2\sqrt{u^2+4}} \\
&= \frac{2u^2+4\pm 2u\sqrt{u^2+4}}{4u\sqrt{u^2+4}}
\end{aligned}$$

$$\mu_\pm = \frac{u^2+2}{2u\sqrt{u^2+4}} \pm \frac{1}{2}$$

Amplification(増光)

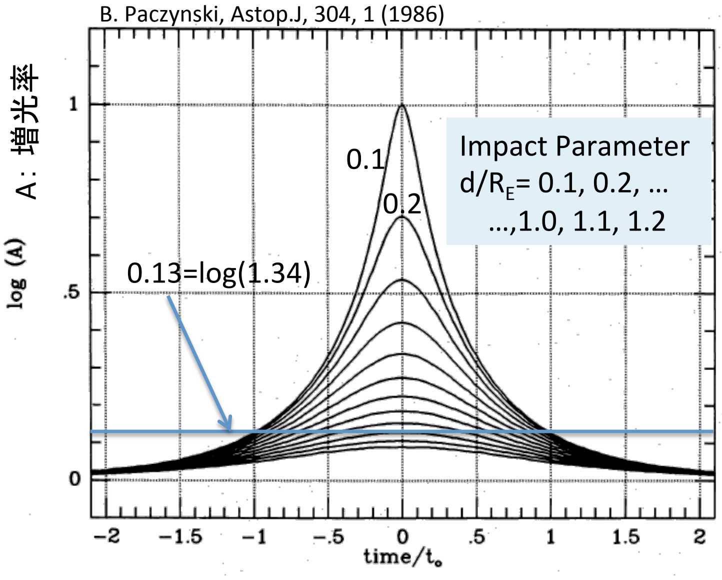
- Amplification μ

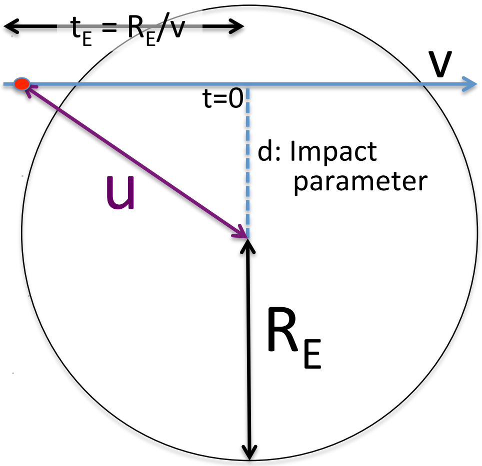
$$\mu = |\mu_+| + |\mu_-| = \frac{u^2+2}{u\sqrt{u^4+1}} \qquad u=\beta/\theta_E$$

- SourceがEinstein Radius にあるとき($\beta=\theta_E$) ➔ u=1
  μ = 1.17 + 0.17 = 1.34
- SourceがEinstein Radius 内にあるときには、増光率は、1.34より大きくなる。

Amplification

B. Paczynski, Astop.J, 304, 1 (1986)





- Time variation of the amplification
- $t_0 = t_E = R_E/v$
  - v is the relative tangential velocity

$$\theta_E = \sqrt{\frac{D_{ds}}{D_s D_d}\frac{4GM}{c^2}}$$

$$R_E = \theta_E D_d = \sqrt{\frac{D_d D_{ds}}{D_s}\frac{4GM}{c^2}}$$

Amplification

$$R_E = \theta_E D_d = \sqrt{\frac{4GM}{c^2}\frac{D_d D_{ds}}{D_s}} = \sqrt{\frac{4GM}{c^2} D_s a(1-a)}, \quad a = \frac{D_d}{D_s}$$

$$\Rightarrow \sqrt{\frac{4\times6.67\times10^{-11}}{(3\times10^8)^2}\left(\frac{M\times1.99\times10^{30}}{M_\odot}\right)\left(\frac{D_s\times50\times10^3\times3.26\times3.15\times10^7\times3\times10^8}{50kpc}\right)\left(\frac{a(1-a)\times0.25}{0.25}\right)}$$

$$R_E = 10\mathrm{AU}\left(\frac{M}{M_\odot}\right)^{1/2}\left(\frac{D_s}{50\mathrm{kpc}}\right)^{1/2}\left(\frac{a(1-a)}{0.25}\right)^{1/2}$$

$$t_E = \frac{R_E}{v} = \frac{10\times1.5\times10^8(\mathrm{km})}{232(\mathrm{km/s})}\left(\frac{M}{M_\odot}\right)^{1/2}\left(\frac{D_s}{50\mathrm{kpc}}\right)^{1/2}\left(\frac{a(1-a)}{0.25}\right)^{1/2}$$

$$t_E = 75\mathrm{days}\left(\frac{M}{M_\odot}\right)^{1/2}\left(\frac{D_s}{50\mathrm{kpc}}\right)^{1/2}\left(\frac{a(1-a)}{0.25}\right)^{1/2}$$

質量 ($M_\odot$)	増光時間
10	237 d
1	75 d
0.1	24 d
10^{-2}	7.5 d
10^{-4}	18 hr
10^{-6}	1.8 hr
10^{-8}	10.8 min

Optical Depth

- A chance seeing a micro-lensing event

$$\tau = \frac{1}{\delta\omega}\int dV n(D_d)\pi\theta_E^2$$

δω : solid angle
dV= δω D_d^2 dD_d
N(D_d): number density
Solid AngleにEinstein Ringがいくつはいるか。

$$\theta_E = \sqrt{\frac{D_{ds}}{D_s D_d}\frac{4GM}{c^2}}$$

$$\tau = \int dD_d D_d^2 n(D_d)\frac{4\pi GM}{c^2}\frac{D_s - D_d}{D_s D_d}$$

$$= \frac{4\pi G}{c^2}\int dx\rho(x)D_s^2\frac{D_d}{D_s}\left(1-\frac{D_d}{D_s}\right)$$

X = D_d/D_s
dx = dD_d/D_s

$$\tau = \frac{4\pi G D_s^2}{c^2}\int_0^1 \rho(x)x(1-x)dx$$

Optical depth toward LMC

- Depend on the halo model
- $\tau_{LMC} = 4.7 \times 10^{-7}$

MACHO探索

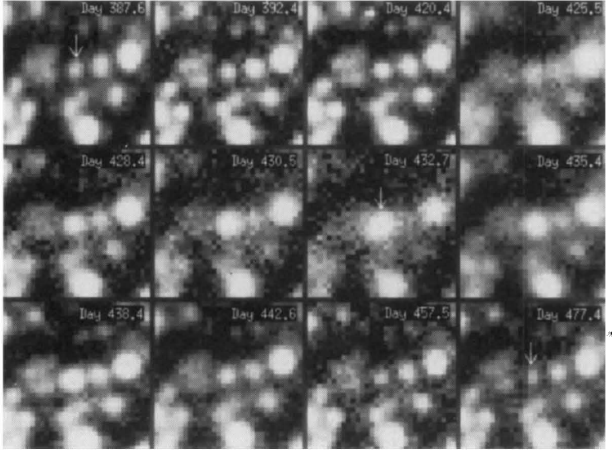
- 増光は波長によらない
- 増・減光曲線は時間に対して対称
- 同じ背景天体に対して一度しか増光しない！

- 主なバックグランド事象
  - 長周期、不規則な変光星
  - 新星、超新星

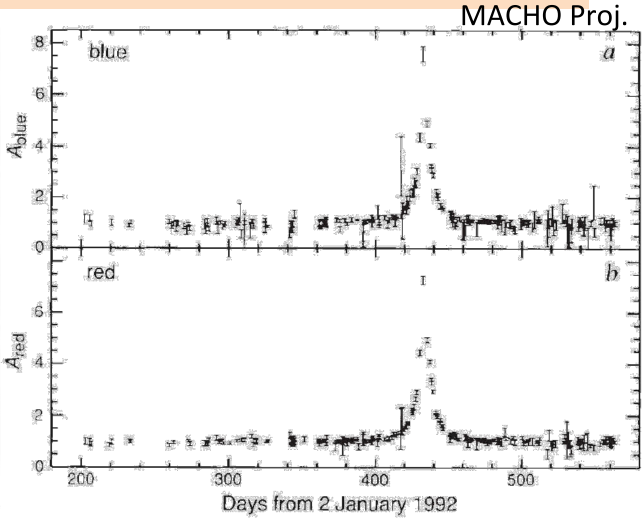
Early Results from MACHO Proj.

Ref) MACHO Proj., C.Alcok et al., Nature 365, 621 (1993)

MACHO Proj.

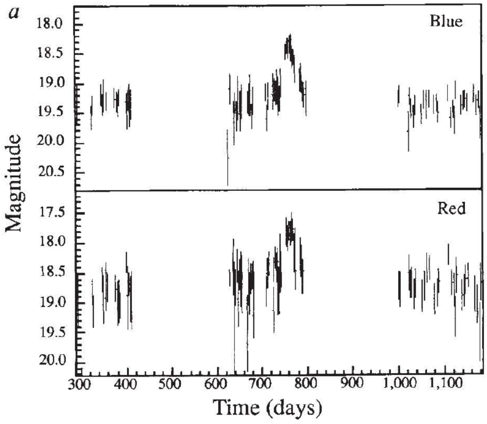


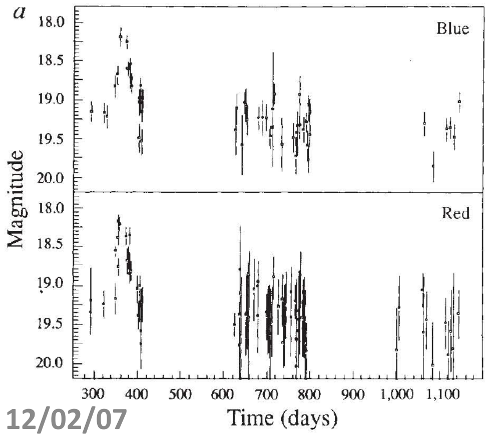
NATURE · VOL 365 · 14 OCTOBER 1993



- 1.27 m telescope at Mt. Stromlo in Australia (0.5 deg^2 of F.o.V.)
- Monitoring 1.8 million stars in LMC (12 k images); Optical depth of LMA: 5×10^{-7}
- One candidate: Amax = 6.86, t = 33.9±0.26 days
  - ➔ Mass: $0.03\ M_\odot \sim 0.5\ M_\odot$

Ref) EROS, E.Aubourg et al., Nature 365, 623 (1993)





Early Results from EROS

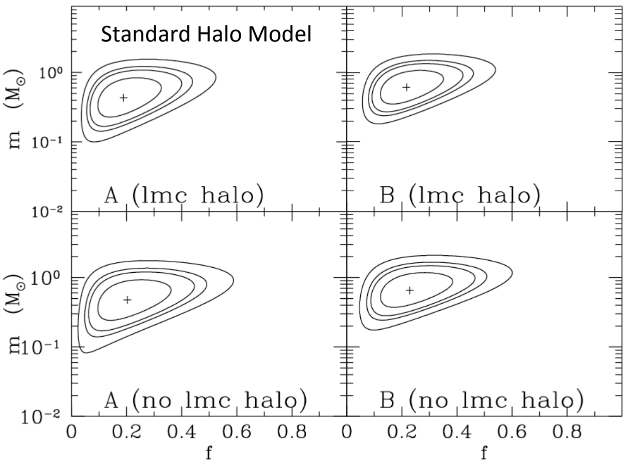
- EROS La silla in Chile (ESO)

1) $5^{\circ} \times 5^{\circ}$ Schmidt plate
2) $1^{\circ} \times 0.4^{\circ}$ CCD

- Monitored 3 million stars in 3 years
- Two candidates:
  - t=27±2 d; Amp(mag) 1.0±0.1
  - t=30±3 d; Amp(mag) 1.2±0.2

$A = 2.5^{\Delta \text{mag}}$

MACHO Project



- 5.7 years obseration
- 11.9 Million stars
- Found 13 (Category A) to 17 candidates
- Time scale 34-230 days
- Optical depth for 2<t<400
  - $\tau_2^{400} = 1.2^{+0.4}_{-0.1} \times 10^{-7}$

EROS (Experience de Resherche d’Objets Sombres)

http://en.wikipedia.org/wiki/
File:Observatoire_de_LaSilla_depuis_la_route_de_Las_Campanas.jpg

- Marly 1m telescope at La Silla (Chille) [ESO: European Southern Observatory]
- 2 Cameras, 2x8 CCDs Wide field (1 deg^2)
- EROS-II (July 1996- Feb 2003)

EROS-II

- 850 k images processed
- 55 M stars monitored in LMC and SMC (55 kpc)
- 7 M Bright starts are analyzed

Signal and Backgrounds

Candidate	EROS-2 star	R_{eros}	$(B-R)_{\mathrm{eros}}$	Original ref.	Status
EROS1-LMC-1	lm058-2k-21915	18.75	0.34	Aubourg et al. (1993)	2nd variation (Tisserand 2004) (Figure 9)
EROS1-LMC-2	lm043-6m- 9377 B-S	19.32	–0.04	Aubourg et al. (1993)	2nd variation (Lasserre et al. 2000)
EROS2-LMC-3	lm034-6l-20493	20.90	0.61	Lasserre et al. (2000)	2nd variation (Tisserand 2004)
EROS2-LMC-4	lm018-6n-23236	19.10	1.87	Lasserre et al. (2000)	2nd variation (Milsztajn et al. 2001)
EROS2-LMC-5	lm015-3n-22431 B-S	19.17	0.14	Milsztajn et al. (2001)	Supernova (Tisserand 2004)
EROS2-LMC-6	lm067-5m-14700	21.01	0.63	Milsztajn et al. (2001)	Supernova (Tisserand 2004)
EROS2-LMC-7	lm070-3n-23389	21.00	0.76	Milsztajn et al. (2001)	Supernova (Tisserand 2004)
EROS2-SMC-1	sm005-4m-5761 B-S	18.13	–0.13	Palanque-D. et al. (1998)	Fig. 7
EROS2-SMC-2	sm001-6l-13221 B-S	19.56	0.44	Afonso et al. (2003a)	long period variable (Tisserand 2004)
EROS2-SMC-3	sm001-6n-16904 B-S	19.31	0.59	Afonso et al. (2003a)	long period variable (Tisserand 2004)
EROS2-SMC-4	sm002-7m-21331 B-S	19.48	0.32	Afonso et al. (2003a)	long period variable (Tisserand 2004)

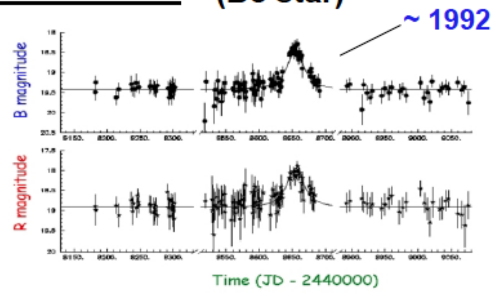
- The 11 events of EROS in the past and present. All candidates except EROS2-SMC-1 have been eliminated as variable stars or as supernovae.

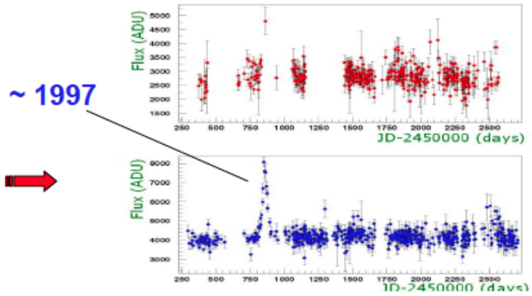
Candidates follow-up : longer baseline (+ 3 yrs)

3 candidates show a new bump a few years later !!
⇨ Variable Stars = **Background**

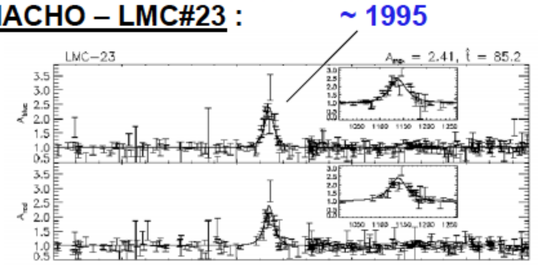
Withdrawn !

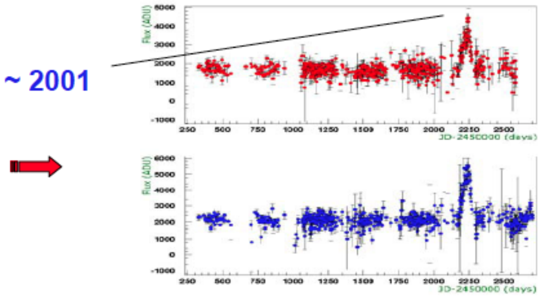
- **EROS 1 – LMC#1** : **(Be star)**





- **MACHO – LMC#23** :

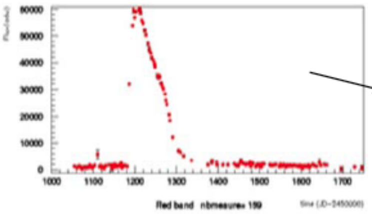




Supernovae :

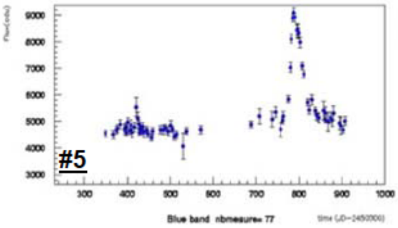
SN behind the LMC

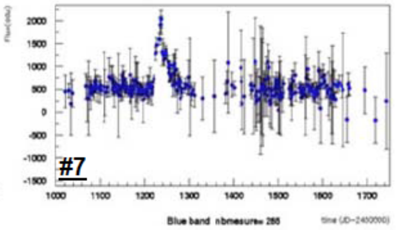
Serious background !



- ~ 20 SN found with low S/N,
with *asymmetric* light curves

- Better photometry → refined cut :
Elimination of the 3 remaining EROS2-LMC candidates

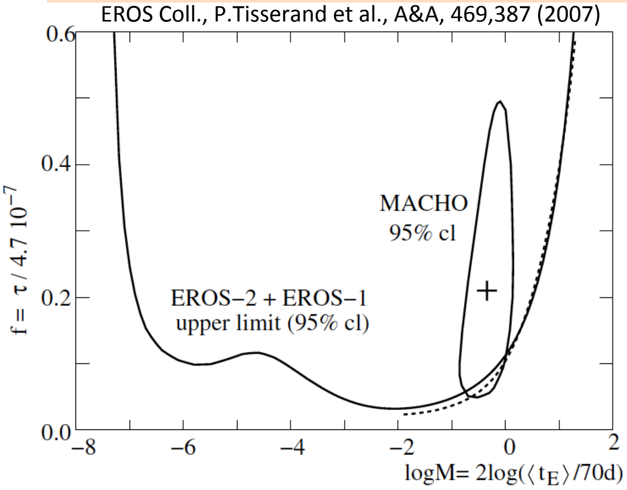




Variable stars or SN

All former EROS LMC candidates have been eliminated !

Results (MACHO & EROS)



EROS Coll., P.Tisserand et al., A&A, 469,387 (2007)

- Limit on f (LMC)
  - – 2% at 10^{-2} M
  - – 5% at 0.4 M
  - – 8% at 1 M

- Solid line: $f = \tau_{lmc}/4.7x10^{-7}$ @95% C.L.
  for no observed events in EROS LMC data.
- Dashed line: $f = \tau_{lmc}/4.7x10^{-7}$ @95% C.L.
  for one observed event in EROS SMC data.
  by assuming $\tau_{smc\text{-}halo} = 1.4\ \tau_{lmc}$

NeutrinoはDark Matterか

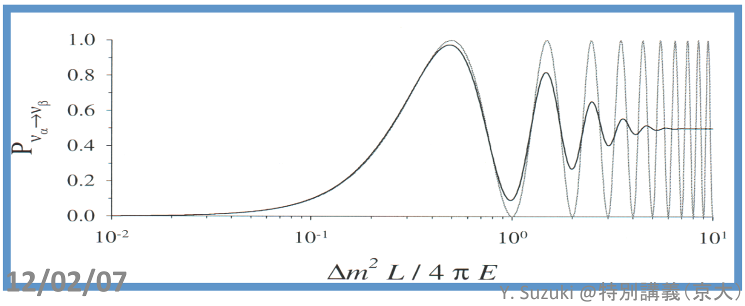
Neutrinos

- ニュートリノ振動の結果
  → 少なくとも1種類のνの質量は 0.05 eV以上

Two neutrino case

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2 2\theta \sin^2(1.27 \Delta m^2 L/E)$$



$\Delta m^2 = m_2^2 - m_1^2$ (eV^2)
L (km): Neutrino flight length
E (GeV): Neutrino energy

測定できるのは2種のニュートリノの質量差の2乗

Three Neutrinos

$$\text{mixing:} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

flavor eigenstates

mass eigenstates

- *$\Delta m_{23} >> \Delta m_{12}$*
- *Small θ_{13}*

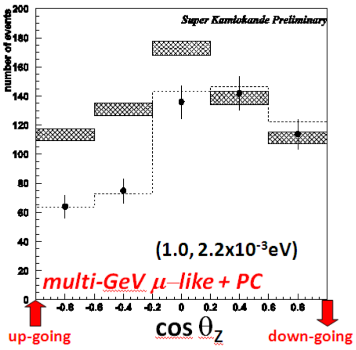
➔ 12-mixing と 23-mixingがほぼdecoupleしている。

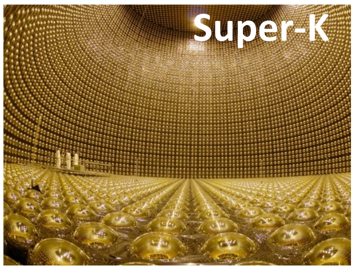
$$U_{\alpha i} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

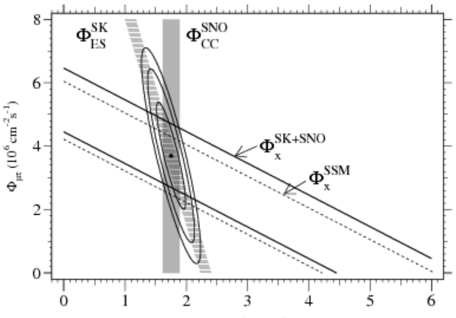
Atmospheric ν Long Base Line Ex (θ_{23}: maximal?)	Reactor Long Base Line Ex (θ_{13}: indication)	Solar ν Reactor LBLE (θ_{12}: large)

Discovery of Neutrino Oscillation

- **1998**: **Atmospheric Neutrino Oscillation** (***Super-Kamiokande***)
  - Asymmetry in zenith angle distribution
  - ν_μ deficits (up-going)
- **2001**: **Solar Neutrino Oscillation** (***SNO +Super-Kamiokande***)
  - SNO: charged current $\rightarrow \nu_e$
  - SK: Electron Scattering $\rightarrow \nu_e + 0.15(\nu_\mu + \nu_\tau)$







Atmospheric and Solar neutrinos

- **大気ニュートリノの振動：$\nu_\mu - \nu_\tau$ 間の振動**

  $\Delta m^2 = 2.4\times10^{-3}$ eV^2, $\sin^2 2\theta=1.0$

  λosc= ~300km @E_ν=1GeV

- **太陽ニュートリノの振動：$\nu_e - \nu_{\mu,\tau}$ 間の振動**

  $\Delta m^2=8\times10^{-5}$ eV^2, $\sin^2 2\theta=0.88$

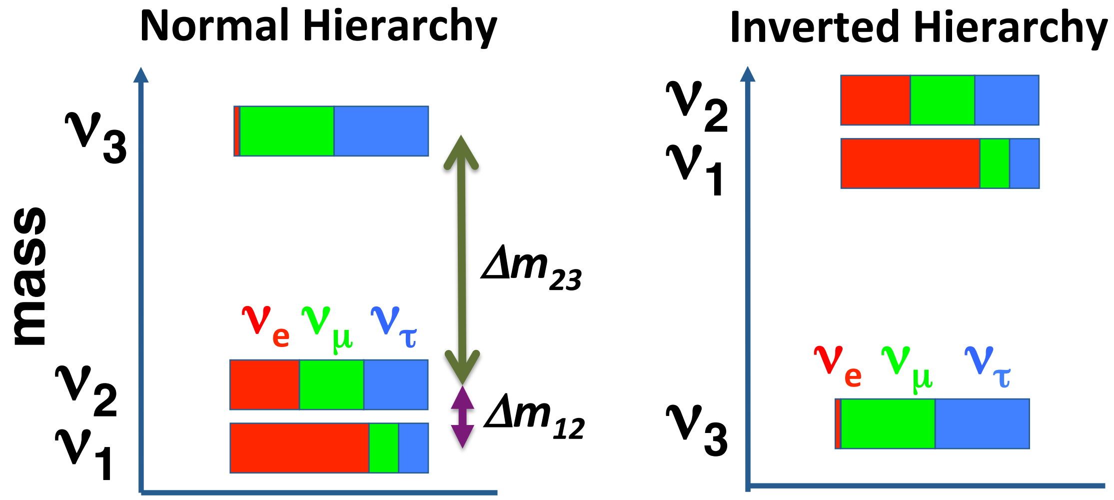
  Losc= ~10^2km @E_ν=0.01GeV

(高エネルギー側の太陽ニュートリノは物質振動）

Mass difference and mixing
> 10 years after the discovery

$\Delta m_{12}^2 = 7.58^{+0.21}_{-0.20} \times 10^{-5}$ eV^2	(~ 3%の精度)	[KamLAND]
$\sin^2\theta_{12} = 0.31^{+0.02}_{-0.02}$	(~ 7%の精度)	[全太陽ν実験]
$\Delta m_{23}^2 = 2.43^{+0.13}_{-0.13} \times 10^{-3}$ eV^2	(~ 5%の精度)	[MINOS]
$= 2.19^{+0.14}_{-0.13} \times 10^{-3}$ eV^2	(~ 6%の精度)	[SuperK]
$\sin^2\theta_{23} = 0.51^{+0.05}_{-0.07}$	(~14%の精度)	[SuperK]
$0.01 < \sin^2\theta_{13} < 0.04$		[T2K]

階層



$\Delta m_{23} >> \Delta m_{12}$

- 少なくとも一つは $m_\nu > 0.05$ eV ($\Delta m^2 \sim 0.003$eV^2)
- 3νが縮退していれば、質量は質量差よりもかなり大きくてもよい。

Neutrinos

- 電子ニュートリノの上限質量は、トリチウムのベータ崩壊から制限がついている。<3 eV
- DM全てがニュートリノであるためには、質量の総和が 47 eV/c^2 必要である。
  - 質量の総和がぜんぜん足りない。
  - ダークマターのほんの一部である。
- Hot dark matterになる。

END

5. Axion and Axion Like Particles

References

- The Strong CP Problem and Axions, R.D. Peccei, hep-ph/0607268v1(2006)
- CAST results and Axion Review, T. Geralis
- AXIONS: RECENT SEARCHES AND NEW LIMITS, G. G. Rafelt, arXiv:hep-ph/0504152v2 (2005)
- Microwave cavity searches for dark-matter axions, R. Brandley et at., Rev. Mod. Phys. 75, 777 (2003)

Strong CP Problem

1) QCD allows CP violating interaction

$$\mathcal{L}_{QCD} = ... + \frac{g_s^2\overline{\theta}}{32\pi^2} G^{\alpha}_{\mu\nu}\tilde{G}^{\alpha\mu\nu}$$

E-M counterpart: $F_{\mu\nu}\tilde{F}^{\mu\nu}$
➔ E･B
🡐 E: vector, B: axial vector

2) Contradict by stringent limit on the electric dipole moment

$$d_n = \frac{e}{m_n}\bar{\theta}\frac{m_u m_d}{m_u + m_d}\frac{1}{\Lambda_{QCD}} \sim 10^{-16}\bar{\theta}e \cdot cm$$

Experimental limit (neutron EDM):
$d_n < 2.9 \times 10^{-26}\ e \cdot cm$ (最新は—27乗？）
➔ |θ| < 10^{-11}

Why θ term is so small ➔ Strong CP Problem !!

Resolve Strong CP Problem

- One solution
  - Introduce global $U_{PQ}(1)$ symmetry, broken at some large energy scale, f_a (Pecci-Quinn)
  - ➔ Strong CP problem→ dynamically resolved
  - Axion must exist: corresponding to pseudo Mambu-Goldstone Boson (Weinberg, Wilczek)

$$m_a = \frac{f_\pi m_\pi}{f_a}\frac{\sqrt{z}}{1+z} = 6\mu\mathrm{eV}\left(\frac{10^{12}\mathrm{GeV}}{f_a}\right)$$

f_a: axion decay constant or PQ scale
f_π= 93MeV: π decay constant
$z = m_u/m_d \approx 0.56$ (but, 0.3~0.7)

Axion-photon Interaction

$$\mathcal{L}_{a\gamma} = -\frac{1}{4} g_{a\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu} = g_{a\gamma} a \overrightarrow{E} \cdot \overrightarrow{B}$$

1.95 (typical)

$$g_{a\gamma} = \frac{\alpha}{2\pi f_a} \left(\frac{E}{N} - \frac{2}{3} \frac{4+z}{1+z} \right)$$

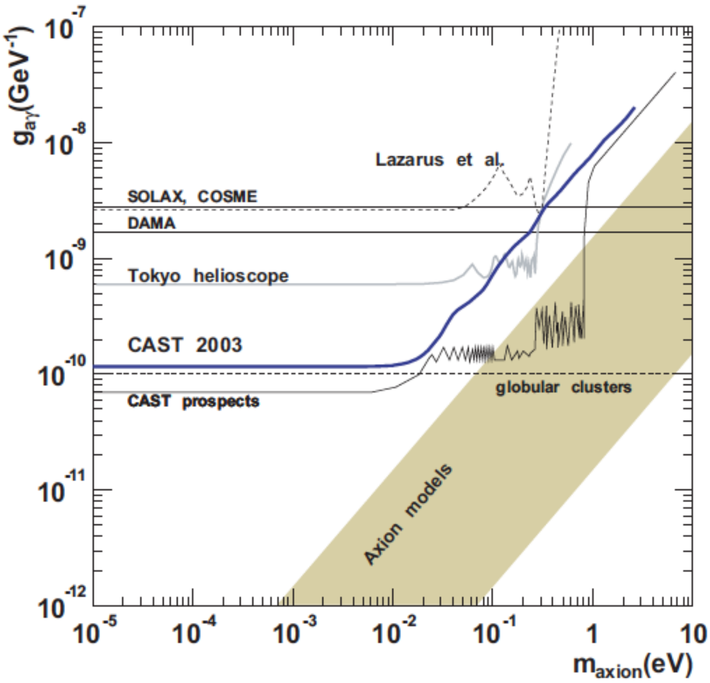
E,N model dependent value
- For DFSZ model → E/N = 8/3
- But any value, not known

Free parameters, m_a and $g_{\gamma\gamma a}$: linear, but 'broad band'

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_x & 0 \end{pmatrix}$$

$$\tilde{F_{\mu\nu}} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^{\sigma\rho}$$

m_a vs $g_{a\gamma}$



- A typical range (best-motivated) for $g_{a\gamma}$ for a given m_a
- In principle, $g_{a\gamma}$ can take any value.

Invisible Axion

$$m_a = 6\mu\text{eV}\left(\frac{10^{12}\text{GeV}}{f_a}\right)$$

- Symmetry breaking @ Electro-Weak scale (100 GeV) was excluded by
  - Accelerator experiments for no observation of axion with mass ~ 100 keV
  - ~ 100 keV ⇔ f_a ~ 100 GeV
- Invisible axion: DFSZ, KSVZ models
  - Breaking scale much higher
  - Coupling extremely small

Astrophysical Constraints

- Stellar evolution
  - Limits energy loss by axion emission
- Neutrinos from SN1987A
  - Duration of a few seconds
  - Indicates cooling primarily by neutrinos
  - Limits axion coupling and mass

➔ gives upper limits on coupling and mass

Relic Abundance

- Invisible axions
  - Cosmological abundance increases if mass reduces
  - Good candidate for Cold Dark Matter

$$\Omega_a = (0.5 \sim 3.0)\left(\frac{6\mu\mathrm{eV}}{m_a}\right)^{7/6}\left(\frac{0.7}{h}\right)^2$$

$$\Omega_a = (0.5 \sim 3.0)\left(\frac{f_a}{10^{12}\mathrm{GeV}}\right)^{7/6}\left(\frac{0.7}{h}\right)^2$$

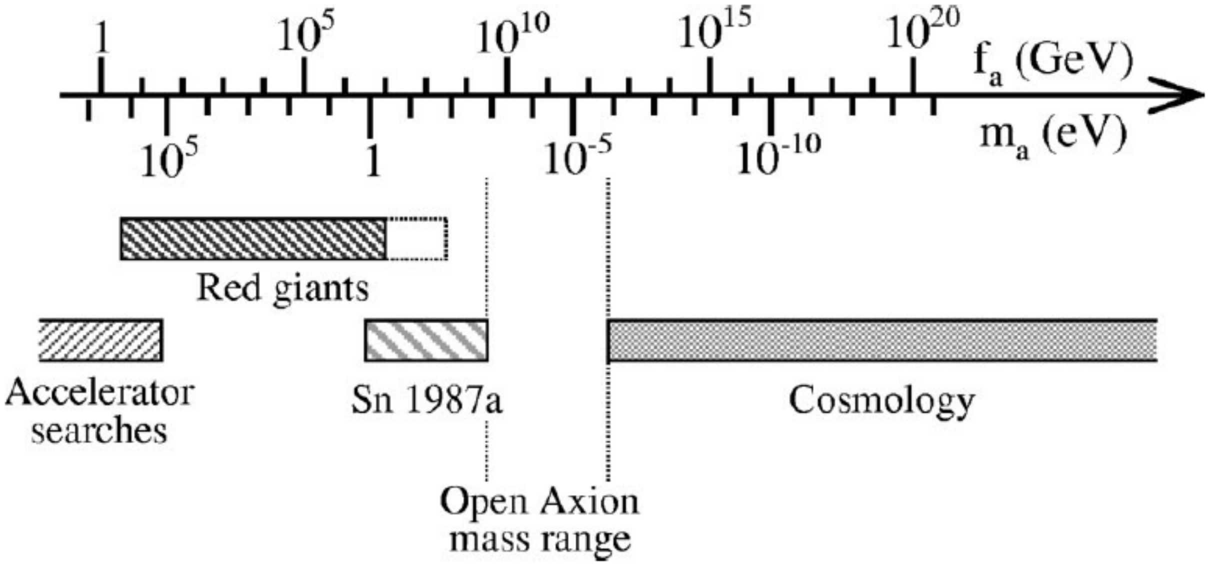
- Gives lower bounds on coupling and mass
  - $\Omega_{CDM} \sim 0.22$
  - $m_a < 10^{-6}$ eV ➔ Over closure of the Universe
  - $m_a \sim 10^{-6}$ eV: lower bound
  - For example, axions with $m_a \sim 10^{-5}$ eV is a good candidate for CDM

Cold Axions?

- DM need to be non-relativistic before the structure formation
  - $T_{st} > 2.7\mathrm{K} = 2\mathrm{x}10^{-4}$ eV
  - Thermal axions with $m_a \sim 10^{-5}$ eV is relativistic
- Axions produced at the QCD phase transition
  - Axion momenta: $P_a = 10^{-8}$ eV
  - Surrounding plasma temperature: $T_1 = 1$ GeV
  - Ref) Ipser, J., and P. Sikivie, 1983, Phys. Rev. Lett. **50**, 925.

Open Window
Astrophysical Constraints

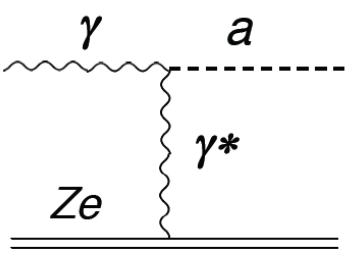
R. Brandley et at., Rev. Mod. Phys. 75, 777 (2003)



$10^{-(6\text{ to }5)}$ eV < m_a < $10^{-(3\text{ to }2)}$ eV

Production and detection

- Production: Primakoff effect



$$P_{\gamma\gamma\to a} = 4\frac{(g_{a\gamma}\omega B)^2}{m_a^4}\sin^2\left(\frac{m_a^2 L}{4\omega}\right) = \left(\frac{g_{a\gamma}B}{q}\right)^2\sin^2\left(\frac{qL}{2}\right)$$

$q = k_\gamma - k_a \approx m_a^2/2\omega$

If $qL \le 1$, then $P_{\gamma\gamma\to a} = (g_{a\gamma}BL/2)^2$

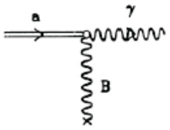
- Detection : Conversion in B-field (Inverse process)

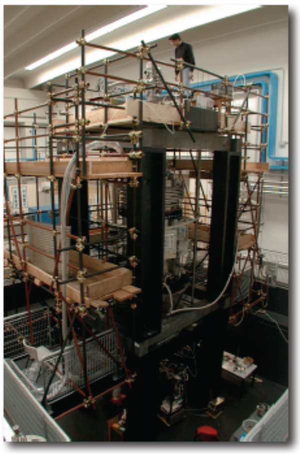
E=m_aの光子に変換: (1eV=1.2μm^{-1})
1 μeV = 1.2 m^{-1} → λ=O(1m) → O(100MHz)
1meV = → λ=O(1mm) → O(100GHz)

Axion Search Experiments

Slide form: L.J. Rosenberg, TAUP2009

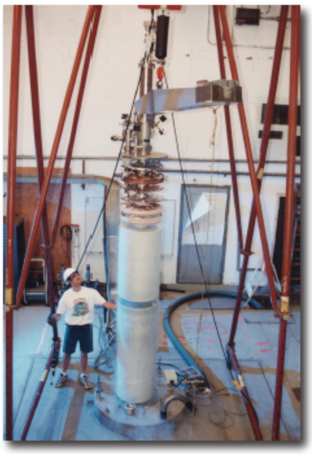
$$L_{\text{int}} = a g_{a\gamma\gamma} E \cdot B$$





Laboratory
("laser")

Solar



Dark matter

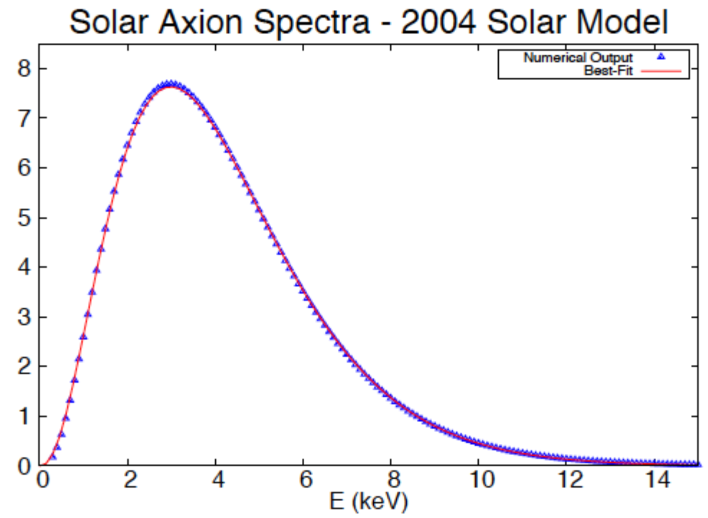
Solar Axion

- Search for keV axions produced in the sun (not DM)
- Primakoff conversion of thermal photons + fluctuating electro/magnetic field+Solar Model
- Flux at the earth

$$\phi_a = 3.67 \times 10^{11} \left(\frac{g_{a\gamma}}{[10^{10} GeV]^{-1}} \right)^2 cm^{-2} s^{-1}$$

- Average energy: 4.2 keV ~ <E_{sun}>

Solar Axion Spectra



$$\frac{d\Phi_a}{dE_a} = 3.821 \times 10^{10} \left(\frac{g_{a\gamma}}{[10^{10} GeV]^{-1}} \right)^2 cm^{-2} s^{-1} keV^{-1} \frac{(E_a/keV)^3}{(e^{E_a/1.103keV} - 1)}$$

CAST

- History
  - BNL(1992)
  - Tokyo Axion Helioscope (2002)
  - CERN Axion Solar Telescope (CAST)

- CERN Axion Solar Telescope (Helioscope)
  - (ref) hep-ex/0411033v2 (July 27, 2011)
- Detector:
  - Convert to X-rays in 9.5 Tesla field
  - Decommissioned LHC superconducting magnet at 1.8 K
  - 9.26 m long magnet (two bores)
  - D= 6cm
  - Cross sectional area: 2 x 14.5 cm^2

Conversion Probability in a Laboratory

- In B-field in vacuum

$$P_{a\to\gamma} = (g_{a\gamma}B/q)^2 sin^2(qL/2)$$

- L: path length, $q=m_a^2/2E_a$: axion-photon momentum difference
- For qL << 1 : axion-photon oscillation length exceeds L

➔ $$P_{a\to\gamma} = (g_{a\gamma}BL/2)^2$$

Expected X-ray flux

- By using the axion flux from the sun and the conversion probability:

$$\Phi_\gamma = 0.51 cm^{-2} d^{-1} \left(\frac{g_{a\gamma}}{[10^{10}GeV]^{-1}}\right)^4 \left(\frac{L}{9.26m}\right)^2 \left(\frac{B}{9.0T}\right)^2$$

Higher mass

- For qL >1 → momentum mismatch

→Add a buffer gas → refractive (effective) photon mass m_γ (plasma frequency)

$$q = |m_\gamma^2 - m_a^2|/2E_a$$

- For $m_a \sim m_\gamma$ restore the rate: coherent conversion
- $0.1\ \mathrm{eV} \leq m_a \leq 5\ \mathrm{eV}$ (0.1 – 300 atm)
- Counting rate: $10^{-5} \sim 10\ \mathrm{sec}^{-1}$

CAST

- Rotating plat form(V: ±8°, H: ±40°)
- Observation 1.5 hours at sunset and sunrise each nearly all the year
- Rest of the time: background measurements
- Tracking accuracy: 1 arc minutes

X-Ray detector

- Detectors at both ends of magnet
  - Sunrise side: MicroMEGAS (Micro Mesh Gaseous Structure), pnCCD (fully depleted) and forcusing device (ABRIXAS: S/N improve by factor 200)
  - Sunset size: TPC (Phase I), MicroMegas (Phase II)

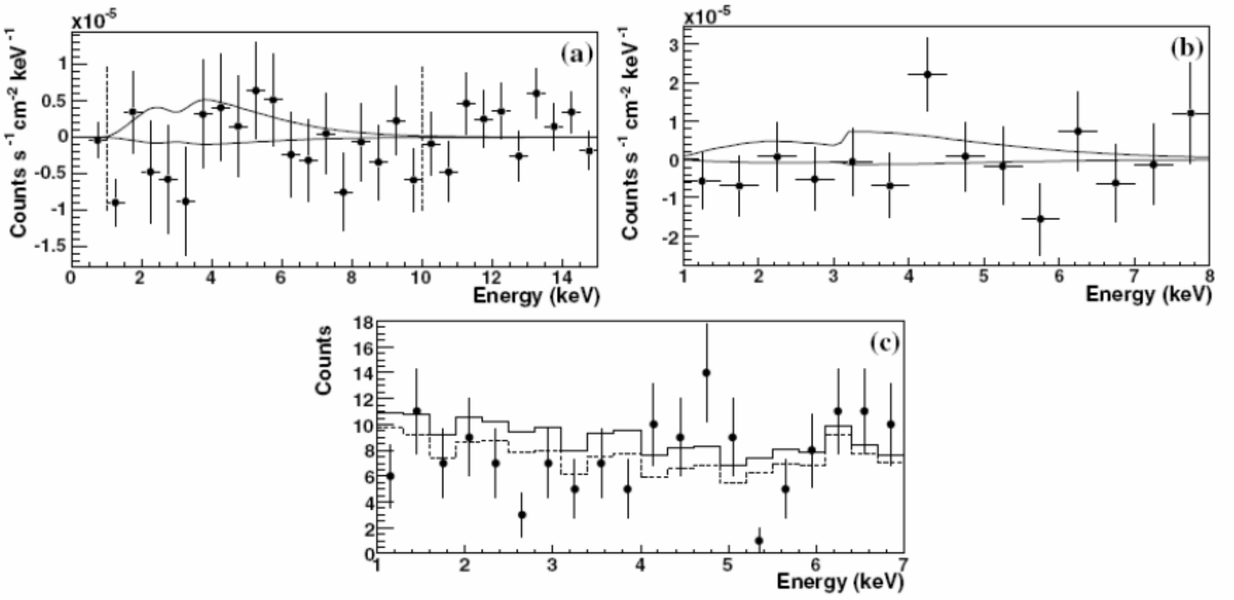
Results

- Phase I (2003-2004)
  - m_a : up to 0.02 eV (best sensitivity)
  - $g_{a\gamma\gamma} < 8.8 \times 10^{-11}$ GeV^{-1} @ 95% C.L.
  - Surpassing the astrophysical limit of the Horizontal Branch stars evolution
- Phase II (2005-2006)
  - Ma > 0.02: axion-photon coherence is lost in vacuum.
  - Insert refractive gas to restore:
    - ^{4}He up to 0.39 eV to 13.4 mbar
    - ^{3}He up to 1.2 eV to 135.6 mbar
- July27's paper

  $g_{a\gamma\gamma} < 1.16 \times 10^{-10}$ GeV^{-1} @ 95% C.L.

  $m_a < 0.02$ eV

Phase-I results

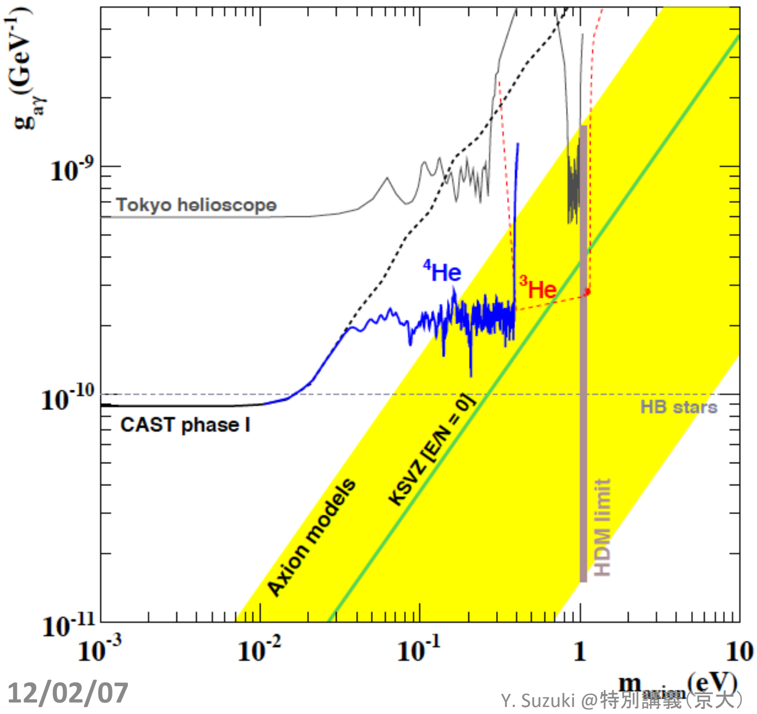


Time Projection Chamber (TPC)

Smaller MICROMEGAS gaseous chamber

X-ray mirror system with CCD

Axion-photon coupling vs axion mass



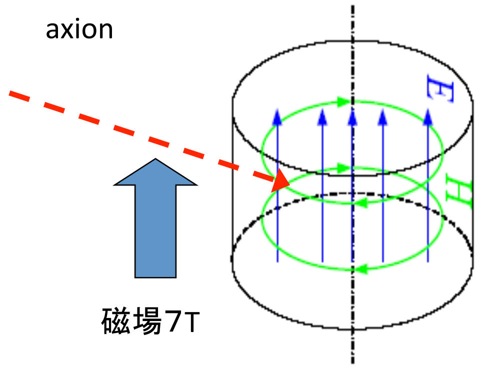
- Limit achieved by the CAST experiment (combined result of the CAST-I and 4He part of CAST-II)
- The yellow band represents typical theoretical models with |E/N – 1.95| in the range 0.07–7 while the green solid line corresponds to the case when E/N = 0 is assumed.
- The red dashed line shows prospects for the 3He run

Tokyo/SUMICO

- Best limit
  - $g_{a\gamma\gamma} < 5.6\text{-}13.4 \times 10^{-10}\ \mathrm{GeV}^{-1}$ @ 95% C.L.
    for $0.84\ \mathrm{eV} < m_a < 1\mathrm{eV}$

Axion Dark Matter Detection

- Massの範囲：10^{-6}~10^{-3} eV



(P.Sikivie)

- Micro Wave Cavity に磁場と振動電場をかける
- Axionによりcavity内のEnergyが増加
  - $E_a = m_a + m_a \beta^2/2$
    $\beta = 10^{-3}$ for halo axions

Micro Wave Cavity

$$P_a=\left(\frac{\alpha}{\pi}\frac{g_\gamma}{f_a}\right)^2 V\, B_0^2\rho_a C\frac{1}{m_a}\mathrm{Min}(Q_L,Q_a)$$

$$=0.5\times10^{-26}\ \mathrm{W}\left(\frac{V}{500\ \mathrm{liter}}\right)\left(\frac{B_0}{7\ \mathrm{T}}\right)^2 C\left(\frac{g_\gamma}{0.36}\right)^2$$

$$\times\left(\frac{\rho_a}{\frac{1}{2}\times10^{-24}\ \mathrm{g/cm^3}}\right)$$

$$\times\left(\frac{m_a}{2\pi\ (\mathrm{GHz})}\right)\mathrm{Min}(Q_L,Q_a),$$

$$C=\frac{\left|\int_V d^3x\,\vec{E}_\omega\cdot\vec{B}_0\right|^2}{B_0^2 V\int_V d^3x\,\epsilon|\vec{E}_\omega|^2}$$

- v: volume of the cavity
- B_0: magnetic-field strength
- Q_L: cavity's loaded quality factor (Q)
- Q_a: quality factor of axion signal (the ratio of axion energy to energy spread)
- ρ_a: density of galactic axions on the earth
- C: mode dependent form factor
- B_0: static magnetic field,
- $E_\omega(x)\exp(i\omega t)$: oscillating electric field of the cavity mode
- ε: dielectric constant

ADMX実験のパラメータ($Q_L=10^5$)をいれると
$P_a \sim 3\times10^{-22}$ W @ $m_a=3\mu$eV

ADMX Experiment

- Axion Dark Matter Experiment
- Several cavities; Different and tunable resonant frequencies (axion masses)
- Look for $m_a = \sim \mu$eV

ADMX Experiment

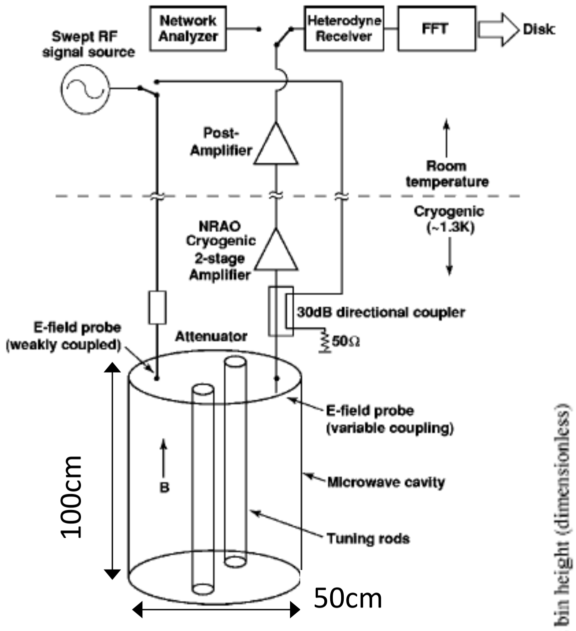
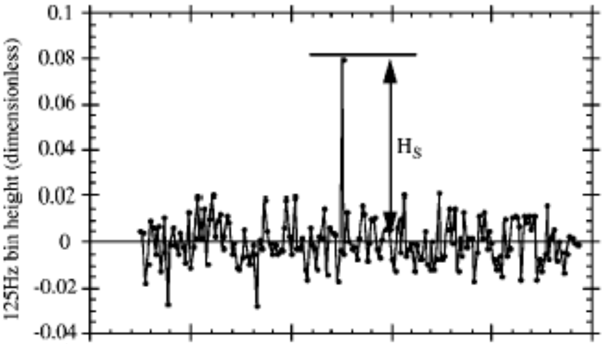


FIG. 4. Schematic of the U.S. search apparatus. From Aszt *et al.*, 2001.

S.Astalos et al, PRD64(2001)092003

- B=7.6T, Physical temp. 1.3K
- V=200litre, Cavity Q=2×10^5
- Cavity form factor C =0.69
- HFET amp+Heterodyne
- Noise power ~10^{-13}W/125Hz
  （10^{-14} W/ √Hz 相当 ）

入力検出感度 >3~9×10^{-23} W



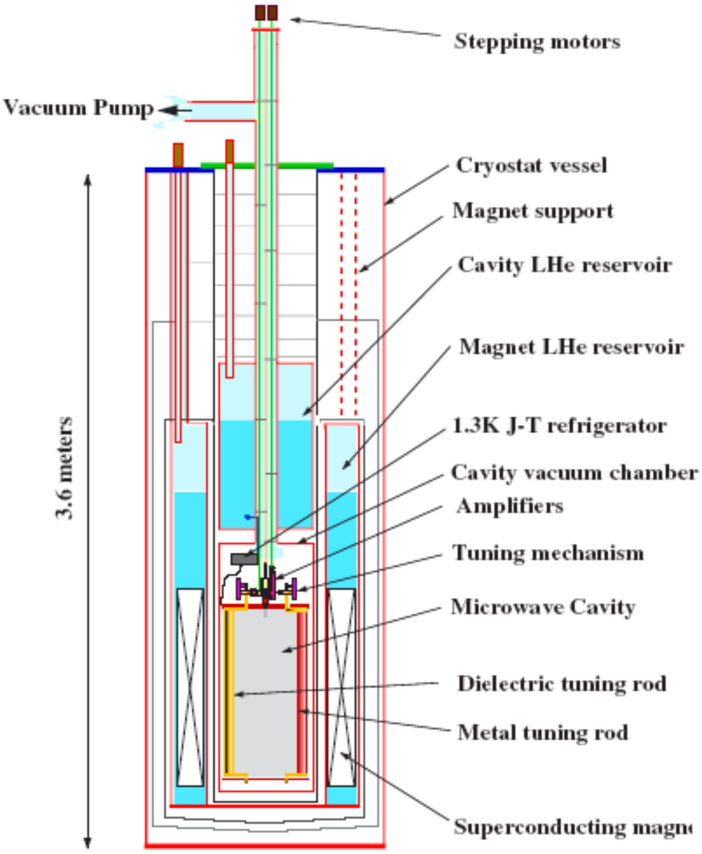


FIG. 2 (color online). Sketch of the ADMX detector.

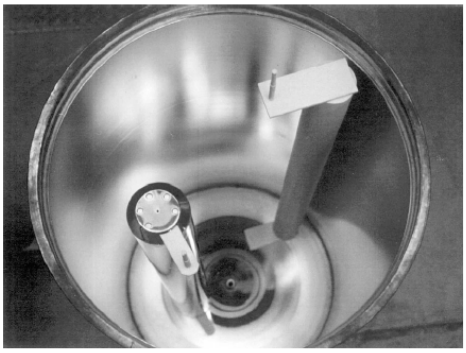
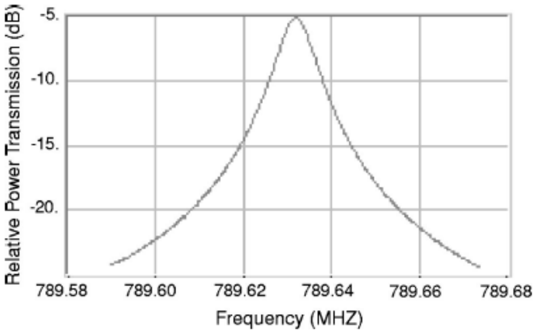
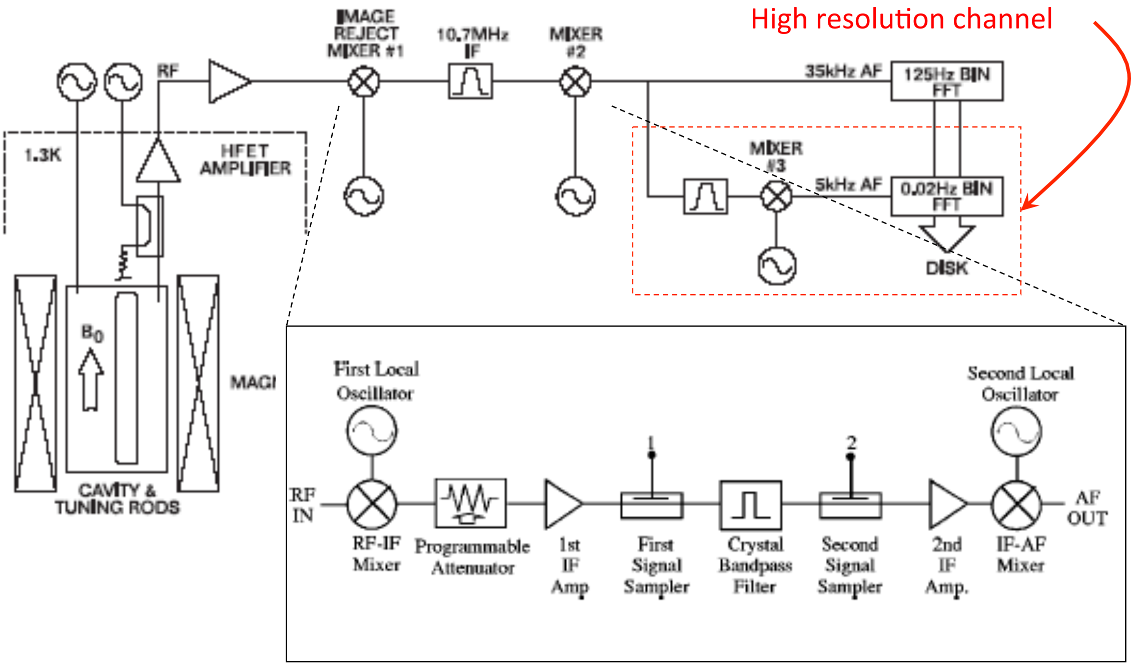


FIG. 6. The resonant cavity viewed from above with the top flange removed. The cavity is a right circular cylinder of diameter 50 cm and depth 1 m. An alumina tuning rod is at the upper right, a copper tuning rod is at the lower left. From Asztalos *et al.*, 2001.



ADMX Experiment



High resolution channel

Spectrum data

Data

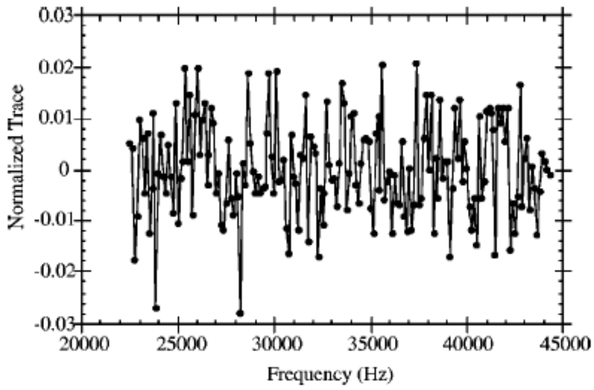


FIG. 18. A single trace after correcting for the receiver response by the equivalent circuit model of the amplifier, transmission line, and cavity interaction.

Data with artificial axion peak

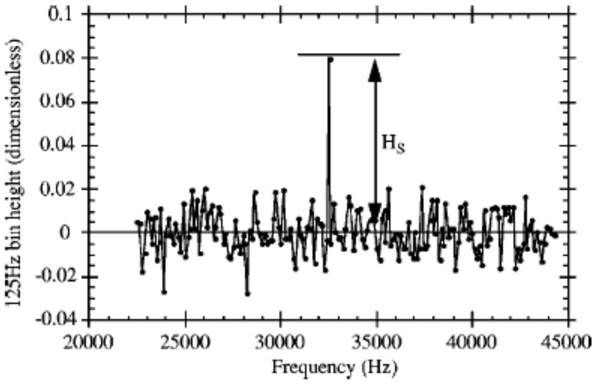
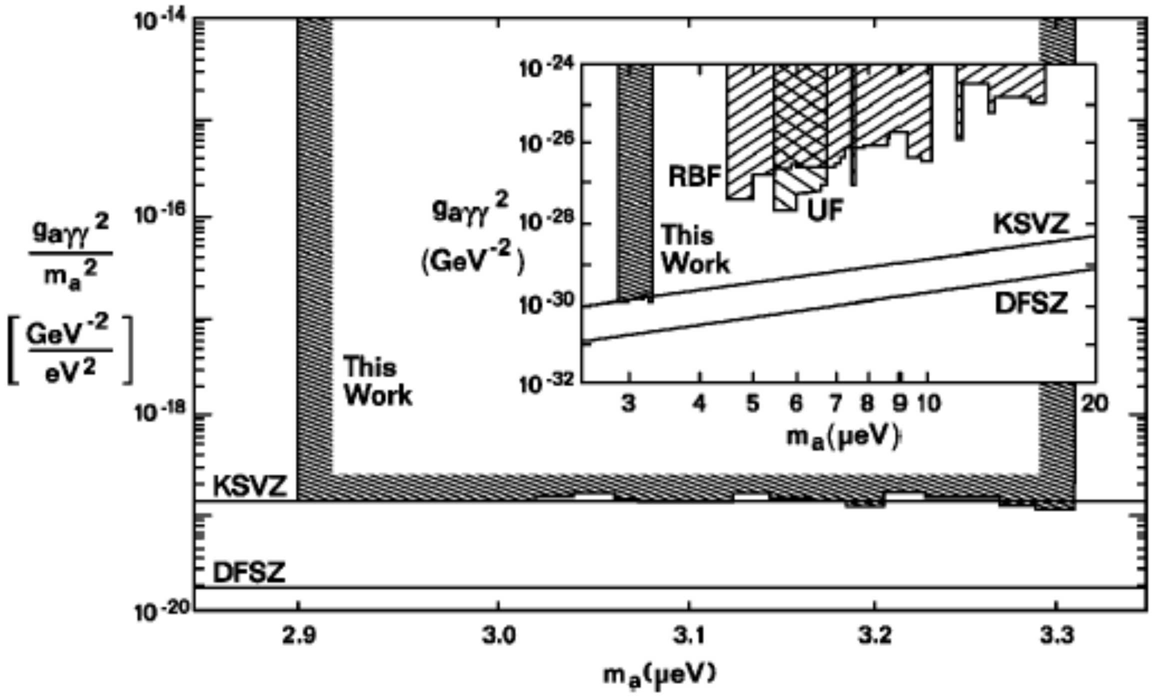


FIG. 20. A single trace from the raw data with an overlaid artificial single bin axion peak.

ADMX (Medium Resolution)の結果



END

6. WIMPs Dark Matter

6. WIMPs Dark Matter

a) Overview
   1. Relic abundance

b) Direct Interactions
   1. Scattering Cross section
   2. Spectrum
   3. Seasonal Variation

c) Annihilation processes

6-a) Overview

- WIMPs Dark Matter
  - Satisfies all the requirements
    - Carry masses
    - Stable
    - Neutral
    - Non-Baryonic
    - Cold
  - Mχ = 0.01 ~ 1 TeV
  - DMのRelic abundanceがweak scaleだと適量になる。
  - WIMPsの候補としてのSUSYが後ろにある。
    - 素粒子論的にも魅力

Relic Abundance

- Ref) SUSY Dark Matter, G.Jungman et al., Phys. Reports, 267, 195 (1996).
- Ref) Cal. of Relic Densities in the early universe, Nucl. Phys. B310, 693 (1988)

Relic abundance (Essenceのみ）

- n_χ^{eq}: number density of χ in thermal equilibrium

$$n_\chi^{eq} = \frac{g}{(2\pi)^3}\int f(\mathbf{p})d^3p$$

  - *g*: internal freedom
  - *f*(**p**): FD or BE distribution [exp((ε-μ)/T)±1]$^{-1}$, ε=(p^2+m^2)$^{1/2}$

T >> m_χ (相対論的、高温時）

$$n_\chi^{eq} \simeq g\frac{\zeta(3)}{\pi^2}T^3 \times \left[\begin{array}{ll} 1 & : \text{Bose-Einstein} \\ 3/4 & : \text{Fermi-Dirac} \end{array}\right]$$

ζ(3) = 1.202…. : ゼーター関数

T << m_χ (非相対論的）

$$n_\chi^{eq} \simeq g\left(\frac{m_\chi T}{2\pi}\right)^{3/2} e^{-\frac{m_\chi}{T}}$$

Boltzman suppressed:
ずっと、thermal equilibriumなら、
(宇宙の膨張がslowの時）
n_χ^{eq} → 0 → no relic WIMPs

参考

$$n_\chi^{eq} = \frac{g}{(2\pi)^3}\int f(\mathbf{p})d^3p$$

- 運動量に関する積分(相対論的: m=0)

$$\int d\mathbf{p} \rightarrow \int 4\pi E^2 dE \longrightarrow n = \int_0^\infty \frac{E^2 \mathrm{d}E}{2\pi^2} f(E)$$

Bose 粒子の場合： *x=E/(kT)*として

$$\int_0^\infty \frac{x^2 dx}{e^x - 1} = 2!\,\zeta(3) \qquad \zeta(3) = 1.202056903....$$

$$\int_0^\infty \frac{x^3 dx}{e^x - 1} = \frac{\pi^4}{15} \longrightarrow$$ Energy densityの時に使う

$$n_{boson} = \frac{\zeta(3)}{\pi^2}(kT)^3, \qquad u_{boson} = \frac{\pi^2}{30}(kT)^4$$

参考

Fermi 粒子の場合 :

$$\frac{x^n}{e^x-1}-\frac{x^n}{e^x+1}=\frac{2x^n}{e^{2x}-1}$$

の両辺を積分する

$$\int_0^\infty \frac{x^n \mathrm{d}x}{e^x-1}-\int_0^\infty \frac{x^n \mathrm{d}x}{e^x+1}=\int_0^\infty \frac{2x^n \mathrm{d}x}{e^{2x}-1}=\frac{1}{2^n}\int_0^\infty \frac{t^n \mathrm{d}t}{e^t-1}$$

左辺一項目と右辺は同じ積分

$$\int_0^\infty \frac{x^n \mathrm{d}x}{e^x+1}=\left(1-\frac{1}{2^n}\right)\int_0^\infty \frac{x^n \mathrm{d}x}{e^x-1}$$

従って、係数 $\left(1-\frac{1}{2^n}\right)$ からFeremi粒子の場合が求まる。

$$n_{fermion}=\frac{3}{4}n_{boson}, \qquad u_{fermion}=\frac{7}{8}u_{boson}$$

Freeze out

$$\chi\chi \longrightarrow l\bar{l} \Leftrightarrow l\bar{l} \longrightarrow \chi\chi$$

- Equilibrium in high temperature

T > m_χ

T < m_χ

- Number density of χ drops exponentially, then
- The rate for the annihilation, Γ=<σ_Av>n_χ drops below the expansion rate: Γ < H
- Then χ's cease to annihilate, fall out of equilibrium.
- Dark Matter freeze out !

粒子数の膨張宇宙における発展

$$\frac{dn_\chi}{dt} + 3Hn_\chi = - < \sigma_A v > [(n_\chi)^2 - (n_\chi^{eq})^2]$$

$H = \dot{a}/a$: Hubble expansion rate

a: scale factor of the Universe

$<\sigma_A v>$: thermal average of the annihilation cross section; v: average velocity

n_χ : actual number density of χ

n_χ^{eq} : number density in the thermal equilibrium

粒子数の膨張宇宙における発展

$$\frac{dn_\chi}{dt} + 3Hn_\chi = - < \sigma_A v > [(n_\chi)^2 - (n_\chi^{eq})^2]$$

- Early times: $H \propto T^2, n_\chi \propto T^3$
  - Right hand side is dominant
  - Expansion is slow and thermal equilibrium

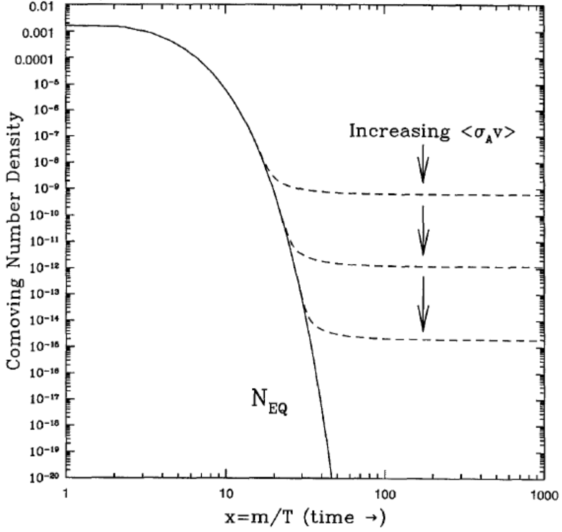
$$\left(\frac{\overset{H}{\dot{a}}}{a}\right)^2 + \frac{K}{a} = \frac{8\pi G}{3}\overset{T^4}{\rho}$$

- Γ < H:
  - Right hand side ≈ 0
    - ➔ $n_\chi \sim a^{-3}$: freeze out

$n_\chi = Aa^{-3}$ とおく、
$dn_\chi/dt = -3Aa^{-4}\mathring{a} = -3Hn_\chi$

Freeze out

Fig. from Kolb and Turner, The early Universe



The dashed curves are the actual abundance, and the solid curve is the equilibrium abundance

- $\Gamma(T_f) = H(T_f)$
  - After T_f
    ➔ n_χ=constant in co-moving volume
  - $T_f \sim m_\chi/20$
- $\Gamma=n_\chi<\sigma_A v>=H$ から、relic densityと$<\sigma_A v>$の関係が決まる。
- If $<\sigma_A v>$ is larger, WIMPs stay in equilibrium longer
  ➔ a smaller relic abundance when freeze out

Weak Scale

Approximate Solution

$$\Omega_\chi = \frac{m_\chi n_\chi}{\rho_c h^2} \simeq \left(\frac{3\times 10^{27}\mathrm{cm}^3\mathrm{sec}^{-1}}{< \sigma_A v >} \right)$$

- Independent of the WIMP mass, but by the annihilation cross section
- *v* at freeze-out ~ appreciable fraction of c
- ➔ $\sigma \sim 10^{-8}\ \mathrm{GeV}^{-2}$

$$\frac{3\times 10^{-27}(\mathrm{cm}^3\mathrm{s}^{-1})}{4\times 10^{-28}(\mathrm{GeV}^2\cdot\mathrm{cm}^2)\times 3\times 10^{10}(\mathrm{cm}\cdot\mathrm{s}^{-1})}$$

dimension合わせ

ħc = 200 MeV･fm
= 2 x 10^{-14} GeV･cm

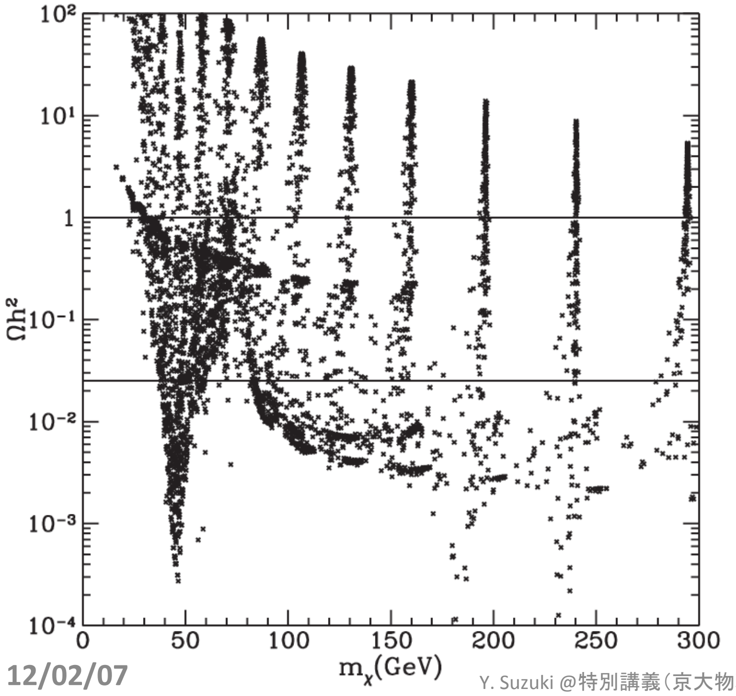
- Weak scale interactions:

$$\sigma_{weak} \simeq \frac{\alpha^2}{m_{weak}^2} \sim 10^{-8}$$

$\alpha \sim O(0.01)$
$m_{weak} \sim O(100\ \mathrm{GeV})$

WIMPs mass and abundance

Jungman et al., Phys. Rep. 267, 195 (1996)



- SUSY modelsによる、WIMP mass vs Cosmological abundance
- Spikeは、パラメータの取り方の人為的なもので、物理的意味はない。

SUSY

- Ref: SUSY
  - – Dark SUSY, P. Gandolo et al., astro-ph/0406204, JCAP0407(2004)008 (J. of Cosmology and Astroparticle physics)
  - – SUSY Dark Matter, Jungman et al., Phys. Rep. 267, 195 (1996)

χ : Neutralino

- Linear combination of the super-partners of the neutral gauge bosons and neutral Higgsinos.
  $$\widetilde{\chi}_i^0 = a_{i1}\widetilde{B} + a_{i2}\widetilde{W}^3 + a_{i3}\widetilde{H}_1^0 + a_{i4}\widetilde{H}_2^0$$
- Lightest(LSP): $\widetilde{\chi}_1^0 = \chi$

  ➔ Dark Matter Candidate

cf. Charginos:
$$\widetilde{\chi}_i^- = U_{i1}\widetilde{W}^- + U_{i2}\widetilde{H}_1^-$$
$$\widetilde{\chi}_i^+ = V_{i1}\widetilde{W}^+ + V_{i2}\widetilde{H}_2^+$$

速度分布、Density

- Maxwell distribution
  - $v_{\mathrm{dispersion}}$ = 270 km
- Density
  - 0.4 GeV/cm^3
- Rotation velocity @around the sun
  - 220 km
- 地球のvelocity
  - (220 + 12 ± 15) km

6-b) Direct Interactions

- Ref) R.W.Schnee, arXiv:1101.5205v1
- Ref) Lewin and Smith, Astropart. Phys. 6,87 (1996)
- Ref) Kurylov and Kamiokowski, Phys. Rev. D69, 063503 (2004)

Scattering Cross Sections

$$\frac{d\sigma_{\chi A}(q)}{dq^2} = \frac{1}{\pi v^2}|\mathcal{M}|^2 = \frac{\sigma_{0\chi A}F^2(q)}{4\mu_A^2 v^2}$$

- $\sigma_{\chi A}$(q): WIMP-Nucleus cross section
- F^2(q): form factor, $\mu_A=M_\chi M_A/(M_\chi+M_A)$: reduced mass

$$\sigma_{0\chi A} = \frac{4\mu_A^2}{\pi}[Zf_p + (A-Z)f_n]^2 + \frac{32G_F^2\mu_A^2}{\pi}\frac{J+1}{J}(a_p < S_p > + a_n < S_n >)^2$$

- see ref) Kurylov and Kamiokowski, Phys. Rev. D69, 063503 (2004)
- At zero momentum transfer ← form factor
- f_p, f_n (a_p, a_n): effective spin independent (dependent) couplings
- $<S_{p,n}>=<A|S_{p,n}|A>$: expectation values of the proton (neutron) spins within nucleus
- $<S_p>=<S_n>=1/2$ for free nucleon
- J: total nuclear spin

Spin Independent (SI) Cross Section

$$\sigma_{0\chi A} = \frac{4\mu_A^2}{\pi}[Zf_p + (A-Z)f_n]^2 + \frac{32G_F^2\mu_A^2}{\pi}\frac{J+1}{J}(a_p < S_p > + a_n < S_n >)^2$$

- For $f_p \approx f_n$ (for many cases), SI WIMP-nucleus σ;

$$\sigma_{0\chi A,SI} = \frac{4\mu_A^2}{\pi} f_n^2 A^2$$

- Introducing SI cross section on a single nucleon; σ_{SI}

$$\sigma_{0\chi A,SI} = \sigma_{SI}\frac{\mu_A^2}{\mu_n^2} A^2$$

$$\sigma_{SI} \equiv \frac{4\mu_n^2 f_n^2}{\pi}$$

- ➢ σ_{SI} is used for comparisons.
- ➢ $\sigma_{0\chi A,SI} \sim (\mu_A^2/\mu_n^2)\, A^2$ ➔ Advantage of larger A
- ➢ For A=50, $\mu_A^2/\mu_n^2 = 625$ → SI WIMP nucleus > WIMP nucleon x~10^5

Spin Dependent (SD) Cross Section

$$\sigma_{0\chi A,SD} = \frac{32G_F^2\mu_A^2}{\pi}\frac{J+1}{J}(a_p < S_p > + a_n < S_n >)^2$$

- SIと同様にして、

$$\sigma_{0\chi A,SDp} = \sigma_{SDp}\frac{\mu_A^2}{\mu_p^2}\frac{4}{3}\frac{J+1}{J} < S_p >^2$$

$$\sigma_{0\chi A,SDn} = \sigma_{SDn}\frac{\mu_A^2}{\mu_n^2}\frac{4}{3}\frac{J+1}{J} < S_n >^2$$

$$\sigma_{SDp} \equiv \frac{24\mu_p^2 G_F^2 a_p^2}{\pi} \qquad \sigma_{SDn} \equiv \frac{24\mu_n^2 G_F^2 a_n^2}{\pi}$$

Spin Dependent (SD) Cross Section

- Contribution of nucleons with opposite spins cancel
  - → Total SD cross section depends on the net spin of the nucleus
- Even numbers of protons in nuclei
  - → no sensitivity to SD interactions on proton
- Even numbers of neutrons in nuclei
  - → no sensitivity to SD interactions on neutron

Sensitivity

R.W.Schnee, arXiv:1101.5205v1

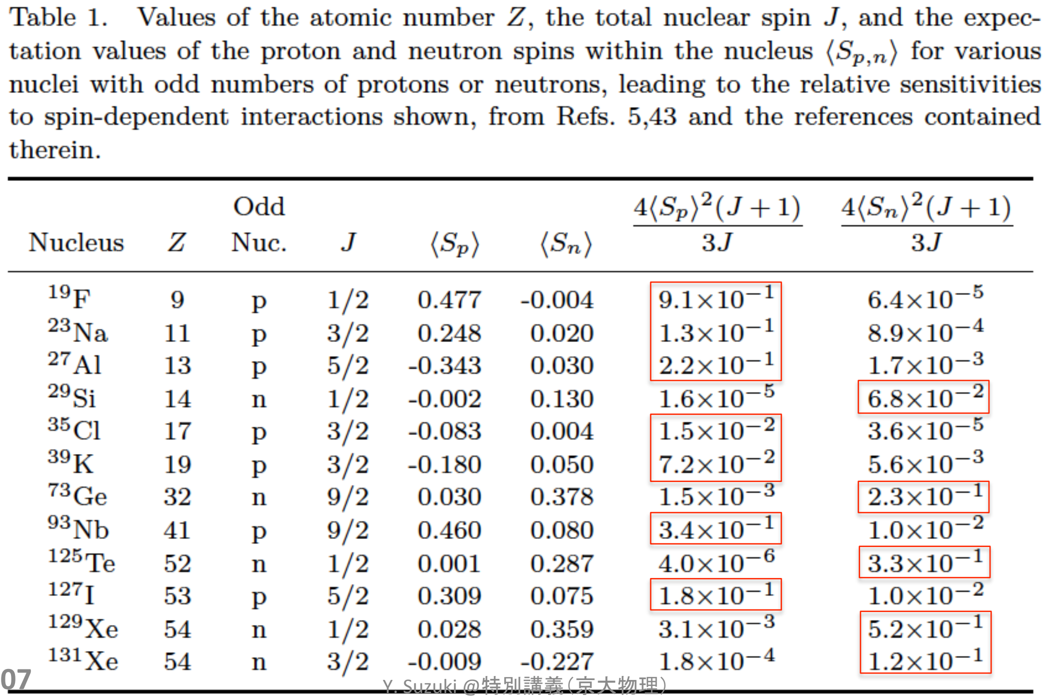


Table 1. Values of the atomic number Z, the total nuclear spin J, and the expectation values of the proton and neutron spins within the nucleus $\langle S_{p,n}\rangle$ for various nuclei with odd numbers of protons or neutrons, leading to the relative sensitivities to spin-dependent interactions shown, from Refs. 5,43 and the references contained therein.

Nucleus	Z	Odd Nuc.	J	$\langle S_p\rangle$	$\langle S_n\rangle$	$\frac{4\langle S_p\rangle^2(J+1)}{3J}$	$\frac{4\langle S_n\rangle^2(J+1)}{3J}$
^{19}F	9	p	1/2	0.477	-0.004	9.1×10^{-1}	6.4×10^{-5}
^{23}Na	11	p	3/2	0.248	0.020	1.3×10^{-1}	8.9×10^{-4}
^{27}Al	13	p	5/2	-0.343	0.030	2.2×10^{-1}	1.7×10^{-3}
^{29}Si	14	n	1/2	-0.002	0.130	1.6×10^{-5}	6.8×10^{-2}
^{35}Cl	17	p	3/2	-0.083	0.004	1.5×10^{-2}	3.6×10^{-5}
^{39}K	19	p	3/2	-0.180	0.050	7.2×10^{-2}	5.6×10^{-3}
^{73}Ge	32	n	9/2	0.030	0.378	1.5×10^{-3}	2.3×10^{-1}
^{93}Nb	41	p	9/2	0.460	0.080	3.4×10^{-1}	1.0×10^{-2}
^{125}Te	52	n	1/2	0.001	0.287	4.0×10^{-6}	3.3×10^{-1}
^{127}I	53	p	5/2	0.309	0.075	1.8×10^{-1}	1.0×10^{-2}
^{129}Xe	54	n	1/2	0.028	0.359	3.1×10^{-3}	5.2×10^{-1}
^{131}Xe	54	n	3/2	-0.009	-0.227	1.8×10^{-4}	1.2×10^{-1}

Recoil Spectrum

(Initially consider the case of zero momentum transfer)

- E_R : recoil energy of mass A (Lab. Frame)
- $E_i = M_\chi v^2/2$: WIMPの運動エネルギー(Lab. Frame)

$$E_R = E_i r \frac{(1-\cos\theta)}{2}$$

- θ: scattering angle of WIMP (CM frame)

$$r \equiv \frac{4{\mu_A}^2}{M_\chi M_A} = \frac{4M_\chi M_A}{(M_\chi + M_A)^2}$$

$$\mu_A \quad \text{(reduced mass)} \quad \equiv \frac{M_\chi M_A}{(M_\chi + M_A)}$$

Recoil Spectrum

$E_R = E_i r \frac{(1-\cos\theta)}{2}$ の証明

- Lab系での散乱後の速度： 重心系における散乱後の粒子の速度にLab系から見た重心系の速度を加える。

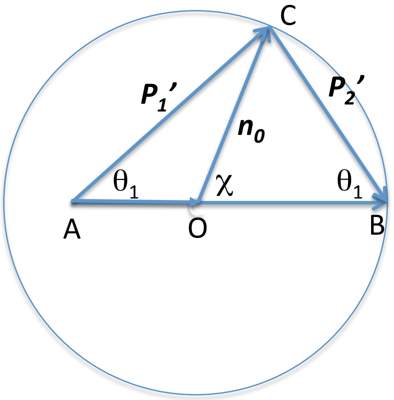
$$\mathbf{v_1}' = \frac{m_2}{m_1+m_2} v\mathbf{n}_0 + \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1+m_2}$$
$$\mathbf{v_2}' = -\frac{m_1}{m_1+m_2} v\mathbf{n}_0 + \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1+m_2}$$

運動量に変換：μは換算質量

$$\mathbf{p}_2' = \mu v\mathbf{n}_0 + \frac{m_2}{m_1+m_2}(\mathbf{p}_1 + \mathbf{p}_2)$$
$$\mathbf{p}_2' = -\mu v\mathbf{n}_0 + \frac{m_2}{m_1+m_2}(\mathbf{p}_1 + \mathbf{p}_2)$$

Recoil Spectrum

$E_R = E_i r \frac{(1-\cos\theta)}{2}$ の証明



$$|\mathbf{p}_2'|^2 = |\mu v|^2 + |\mu v|^2 - 2|\mu v||\mu v|\cos\chi$$

$$(m_A v_R)^2 = 2|\mu v|^2(1-\cos\chi)$$

$$\frac{1}{2}m_A v_R^2 = \frac{4\mu^2}{m_A m_\chi}\frac{1}{2}m_\chi v^2\frac{(1-\cos\chi)}{2}$$

$$r \equiv \frac{4\mu^2}{m_\chi m_A} = \frac{4m_\chi m_A}{(m_\chi+m_A)^2}$$

$$\overrightarrow{AO} = \frac{m_1}{m_1+m_2}(\mathbf{p}_1+\mathbf{p}_2) \rightarrow \frac{m_1^2\mathbf{v}_1}{m_1+m_2}$$

$$\overrightarrow{OB} = \frac{m_2}{m_1+m_2}(\mathbf{p}_1+\mathbf{p}_2) \rightarrow \mu\mathbf{v}_1$$

Target 静止 ***P₂***=0 ➔ $|\mathbf{v}_1|$ = v : Bは円周上

Recoil Spectrum

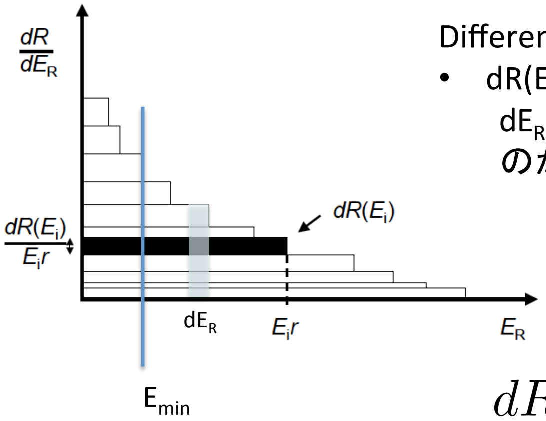
$E_R = E_i r \frac{(1-\cos\theta)}{2}$ $\qquad r \equiv \frac{4\mu_A{}^2}{M_\chi M_A} = \frac{4M_\chi M_A}{(M_\chi+M_A)^2}$

- r ≤ 1 [$\because (M_\chi+M_A)^2 - 4M_\chi M_A = (M_\chi - M_A)^2 \geq 0$]
- r=1 for $M_\chi = M_A$
  - 質量が同じ時recoil energyは最大をとる。
- Energy Range E_R: 0 ~ rE_i
  - E_{max} ← Galactic escape velocity: $E_{max} = M_\chi v_{esc}{}^2/2$
  - E_Rを作りだす為の、最小energy, velocity :
    - $E_{min} = E_R/r$ (head on ($\theta=\pi$))
    - $v_{min} = (2E_{min}/M_\chi)^{1/2} = (2E_R/(rM_\chi))^{1/2}$

Rate

- Differential rate: $$d\left(\frac{dR}{dE_R}(E_R)\right) = \frac{dR(E_i)}{E_i r}$$

図： R.W.Schnee, arXiv:1101.5205v1



Differential rate dR/dE$_R$ への contribution::

- dR(E$_i$) は、0~rE$_i$ に一様に寄与
  dE$_R$へは、dR(E$_i$)[Area]をrE$_i$[長さ]で割ったものが寄与:dR(E$_i$)/(rE$_i$)

Recoil → Initial に変換

$$\frac{dR}{dE_R}(E_R) = \int_{E_{min}}^{E_{max}} \frac{dR(E_i)}{E_i r}$$

Rate

$$dR = \frac{N_{avo}}{A} n_0 \sigma v f(\overrightarrow{v}) d^3\overrightarrow{v}$$

- N_{avo}/A : target number in unit volume
- $n_0 = \rho_\chi/M_\chi$: WIMPs number density
  $\rho_\chi = 0.3$ GeV/cm^3 (typical)
- σv dt: interaction in dV in dt
- WIMP velocities follow Maxwellian distribution
- Ignore earth's velocity and escape velocity (for illustrative purpose)

Rate

$dR = \frac{N_{avo}}{A} n_0 \sigma v f(\overrightarrow{v}) d^3\overrightarrow{v}$

$\frac{dR}{dE_R}(E_R) = \int_{E_{min}}^{E_{max}} \frac{dR(E_i)}{E_i r}$

Maxwellの速さ分布

$$f(v) = \frac{4v^2}{\sqrt{\pi} v_0^3} \exp\left(-\frac{v^2}{v_0^2}\right)$$

$$R_0 \equiv \frac{2}{\sqrt{\pi}} \frac{N_{abo}}{A} n_0 \sigma v_0$$

とおく、

$$\frac{dR}{dE_R}(E_R) = \int_{E_{min}}^{\infty} \frac{dR(E_i)}{E_i r} = \int_{E_{min}}^{\infty} \frac{1}{r(\frac{1}{2}M_\chi v^2)} \cdot R_0 \cdot \frac{\sqrt{\pi}}{2} \frac{v}{v_0} \cdot \frac{4v^2}{\sqrt{\pi} v_0^3} \exp\left(-\frac{v^2}{v_0^2}\right) dv$$

$$\begin{aligned} \frac{dR}{dE_R}(E_R) &= \frac{R_0}{r(\frac{1}{2}M_\chi v_0^2)} \int_{v_{min}}^{\infty} \frac{2}{v_0^2} e^{-v^2/v_0^2} v dv \\ &= \frac{R_0}{E_0 r} e^{-E_R/E_0 r} \end{aligned}$$

次ページ

- $E_0 = M_\chi v_0^2/2$ most probable WIMP incident energy

  [$v_{min} = (2E_{min}/M_\chi)^{1/2} = (2E_R/(rM_\chi))^{1/2}$]

計算

$$\begin{aligned} \int_{v_{min}}^{\infty} v e^{v^2/v_0^2} dv &= -\frac{v_0^2}{2} \left| e^{-v^2/v_0^2} \right|_{v_{min}}^{\infty} \\ &= \frac{v_0^2}{2} e^{-v_{min}^2/v_0^2} \\ &= \frac{v_0^2}{2} e^{-\frac{E_R}{r(\frac{1}{2}M_\chi v_0^2)}} \end{aligned}$$

Rate

$$\frac{dR}{dE_R}(E_R) = \frac{R_0}{E_0 r} e^{-E_R/E_0 r}$$

- 平均 $\langle E_R \rangle = E_0 r$ ← $\int x f(x)dx \,/\!\int f(x)dx$
  - $\langle E_R \rangle = E_0 = (1/2) M\chi v_0^2$ only if $M\chi = M_A$
  - For $M\chi = M_A = 100$GeV,

V_0=220km/s

$\langle E_R \rangle = (1/2) \cdot (100\text{x}10^9/(3\text{x}10^8)^2) \cdot 220\text{x}10^3 \sim 27$ keV

- $R_0 \equiv \frac{2}{\sqrt{\pi}} \frac{N_{abo}}{A} n_0 \sigma v_0$: total rate for $\text{kg}^{-1}\text{day}^{-1}$

$$R_0 \sim 4 \times \left(\frac{1}{A}\right) \left(\frac{\rho_\chi}{0.3GeV/c^2 \cdot cm^3}\right) \left(\frac{100GeV/c^2}{M_\chi}\right) \left(\frac{\sigma}{1pb}\right) \text{ events/kg/day}$$

➔ Per nucleonからは、およそ10^{6}倍

➔ SI(nucleon)を、10^{-44}cm^2とするとSI(nucleus) → $10^{-38}\text{cm}^2 = 10^{-2}$pb

➔ A~100として 10^{-4} events/kg/day

Rate

$$r \equiv \frac{4\mu_A{}^2}{M_\chi M_A} = \frac{4M_\chi M_A}{(M_\chi + M_A)^2}$$

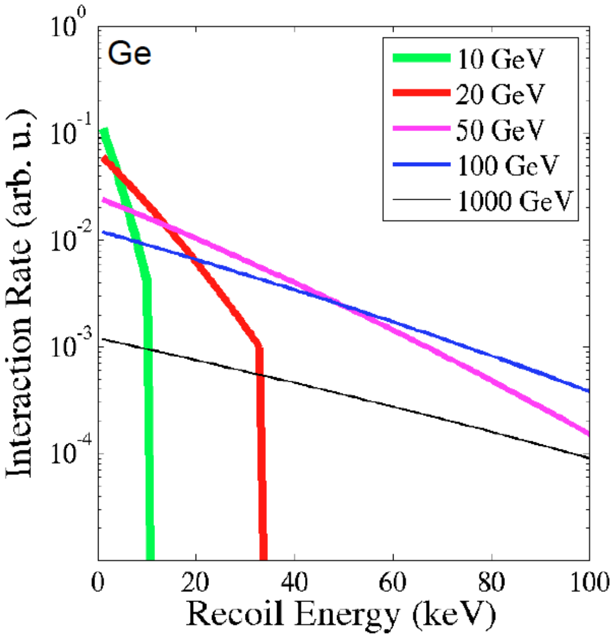
$$\mu_A{}^2 \ \text{(reduced mass)} \ \equiv \frac{M_\chi M_A}{(M_\chi + M_A)}$$

- Effect on $\langle E_R \rangle$ of WIMP mass $M\chi$

$$< E_R >= rE_0 = \frac{2M_A v_0^2}{(1 + M_A/M\chi)^2}$$

  - $M_\chi{}^2$ if $M_\chi << M_A$
  - constant if $M_\chi >> M_A$
- Escape velocity: $v_{esc} \sim 540$ km/s
  - Ref) RAVE servey, M.C.Smith et al., Mon. Not. Roy. Astron. Soc. 379, 755 (2007), astro-ph/1002.1912
  - Cut off energy:
    - $E_{max} = r\,(1/2) M_\chi v_{esc}{}^2 \sim 100$ keV (for 50GeV)
    - If $M\chi$ is smaller, then cut off energy is lower
  - $E_{max} = (v_{esc}/v_0)^2 \langle E_R \rangle \sim 6 \langle E_R \rangle$
    - ← $\langle E_R \rangle = rE_0 = r(1/2)M\chi v_0{}^2$

Spectrum Shape



- Form factor included
- Significant cut off effect in low mass due to escape velocity
- Spectrum shape → similar in high mass
- Number density effect can be clearly be seen between 100 and 1000 GeV

Correct formula with v_E and v_{esc}

- 何かの時に使ってください。
  - Ref) Lewin and Smith, Astropart. Phys. 6,87 (1996)

For $v_{min}(E_R) < v_{esc} - v_E$

$$\frac{\mathrm{d}R}{\mathrm{d}E_R} \approx \frac{R_0}{E_0 r}\left\{\frac{v_0\sqrt{\pi}}{4v_E}\left[\mathrm{erf}\left(\frac{v_{min}+v_E}{v_0}\right) - \mathrm{erf}\left(\frac{v_{min}-v_E}{v_0}\right)\right] - e^{-v_{esc}^2/v_0^2}\right\}$$

For $v_{esc} - v_E < v_{min}(E_R) < v_{esc} + v_E$

$$\frac{\mathrm{d}R}{\mathrm{d}E_R} \approx \frac{R_0}{E_0 r}\left\{\frac{v_0\sqrt{\pi}}{4v_E}\left[\mathrm{erf}\left(\frac{v_{esc}}{v_0}\right) - \mathrm{erf}\left(\frac{v_{min}-v_E}{v_0}\right)\right] - \frac{v_{esc}+v_E-v_{min}}{2v_E}e^{-v_{esc}^2/v_0^2}\right\}$$

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\mathrm{d}t$$

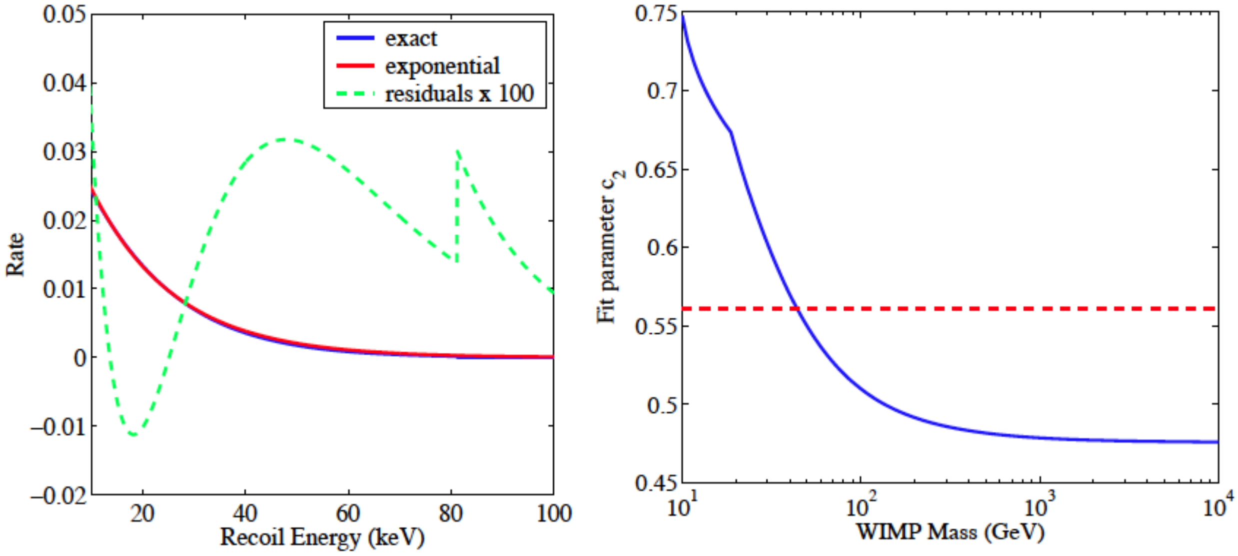
Further approximated (no cut off energy, E_{max}) to the exponential:

$$\frac{\mathrm{d}R}{\mathrm{d}E_R}(E_R) \approx c_1\frac{R_0}{E_0 r}e^{-c_2 E_R/E_0 r}$$

$C_1 \approx 0.75$, $c_2 \approx 0.56$, $c_1/c_2 = 1.3$

30 % increase of the interaction rate by the earth's motion

Accuracy of the exponential approximation



Earth’s Velocity

$$\mathbf{v_E} \approx 233 + 15 \times \cos\left(2\pi \frac{t - 152.5}{365.25}\right) km/s$$

1. 地球の公転による季節変動（15km/s）の測定
2. 地球の運動は Cygnusの方向(<233km/s>)であるから地球の運動によるRecoil角 ψ を計ろうという実験もある

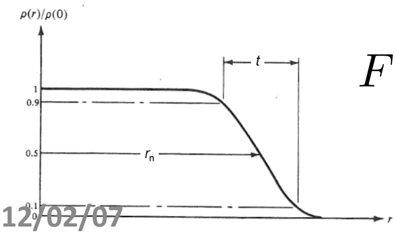
$$\frac{dR}{dRd\cos\psi} \approx \frac{1}{2}\frac{R_0}{E_0 r}\exp\left[-\left(\frac{v_E\cos\psi - v_{min}}{v_0}\right)^2\right]$$

Form factor for Spin Independent (SI) interactions

- Form factor
  - Momentum dependent part of the interaction
  - Fourier transform of the scattering sit positions.

$$F(\overrightarrow{q}) = \int \mathrm{d}^3 x \rho(\overrightarrow{x}) e^{i\overrightarrow{q}\cdot\overrightarrow{x}}$$

- Woods-Saxon form
  - ✓ good approximation for spin-independent interactions
  - ✓ Fourier transform of a solid sphere of radius r_n, with a skin thickness s (t: 10% -90%)



$$F(q) = \frac{3[\sin(qr_n) - qr_n\cos(qr_n)]}{(qr_n)^3} e^{-(qs)^2/s}$$

Form factor for spin dependent (SD) interactions

- Difficult/complicated situation
- First approximation: thin shell of valence nucleons

$$F(q) = \frac{\sin(qr_n)}{qr_n}$$

- Need detailed nuclear-physics calculations

Direct Detection Experiments

- 後ほど

6-c) Annihilation and Propagation Processes for indirect search experiments

- Annihilation : to produce the amounts of matter and antimatter
- But antimatter is scarce in the universe
  - Antimatter ⇔ Dark Matter signal (possibility)
  - Matter and γ ➔ Backgrounds + Dark Matter signal

Relevant processes

- Annihilation → Flux (Many steps)

1. Probability of the pair annihilation
   - $1/2(\rho_\chi(\mathbf{x})/m_\chi)^2\sigma_{ann}v$
2. Production rate of observable particles
   1. Branching ratio of two-body final state
   2. Hadronization and Decay
3. Propagation in the galactic magnetic fields (charged particles)
   → CR flux at the Sun's location
4. Effect of solar modulation (Charged particles)

Charged Particles

Production Processes:

- For $\overline{p}, \overline{D}$

$$\chi\chi \rightarrow q\overline{q}, W^+W^-, Z^0Z^0, W^+H^-, Z^0H^0, H^0H^0$$

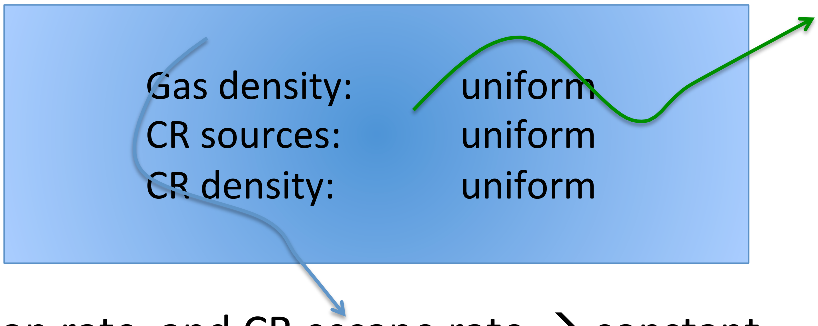
- For e^+

Add l^+l^- in addition to the above channels.

e^+e^- final state gives monochromatic e^+, but negligible and smeared through the propagations

Propagation

Leaky Box Model

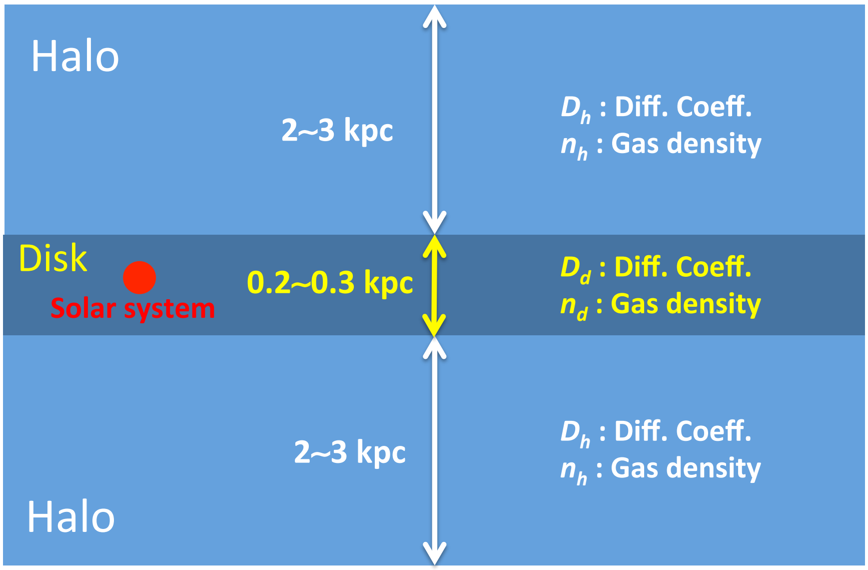


Gas density: uniform
CR sources: uniform
CR density: uniform

- CR production rate and CR escape rate → constant
- CRs are well mixed in the Galaxy: no anisotropy
  - Valid only for stable CR's
- very simple with only one parameter
  - average path length, L, or residence time, τ
    - $\exp(-x/L)$ or $\exp(-t/\tau)$

Propagation

Diffusion Model



Halo

2~3 kpc

D_h : Diff. Coeff.
n_h : Gas density

Disk

Solar system

0.2~0.3 kpc

D_d : Diff. Coeff.
n_d : Gas density

2~3 kpc

D_h : Diff. Coeff.
n_h : Gas density

Halo

Propagation

Energy Loss during the propagation

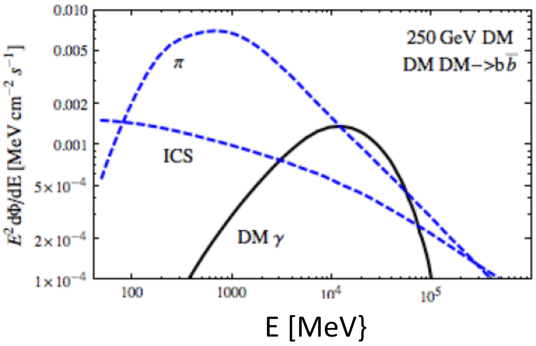
- $\overline{p}, \overline{D}$: no-energy loss
- e^+ : energy loss due to
  - – Inverse Compton on star light and CMB
  - – Synchrotron radiation
- Solar Modulation
  - – Larger effect for low energy particles
  - – Less effect around a few GeV peaks

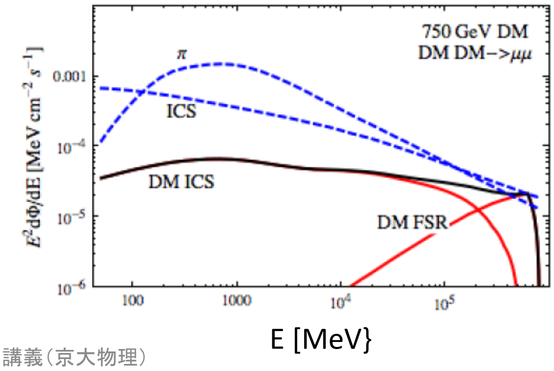
Gamma rays

- Galaxy is transparent for γ-rays
  - – Except:
    - galactic center (massive blackhole)
    - High energy and very distant γ :absorption by starlight and infrared photons
- Source: π^0 in the fragmentation and decay processes
- But π^0 are also come from other astrophysical processes: a limiting factor to identify DM

Continuum energy spectrum

- $\pi^0 \rightarrow 2\gamma$
- Some difference between Dark Matter signal and Backgrounds





Monochromatic γ

- $\chi\chi \rightarrow \gamma\gamma$, $\chi\chi \rightarrow \gamma Z$ production
  - Gives monochromatic γ : because $v_\chi \sim 10^{-3}$
    - $E_\gamma \sim m_\chi$
    - $E_\gamma \sim m_\chi (1 - m_z^2/4m_\chi^2)$
      - $m_\chi > m_z/2$
      - M_χ = 50 GeV → E_γ = 8.6 GeV, 100GeV → 79.2 GeV
  - Lower rate, but
    - Higgsino like neutralino Z_g <0.01 with higer mass ≥ 500 GeV

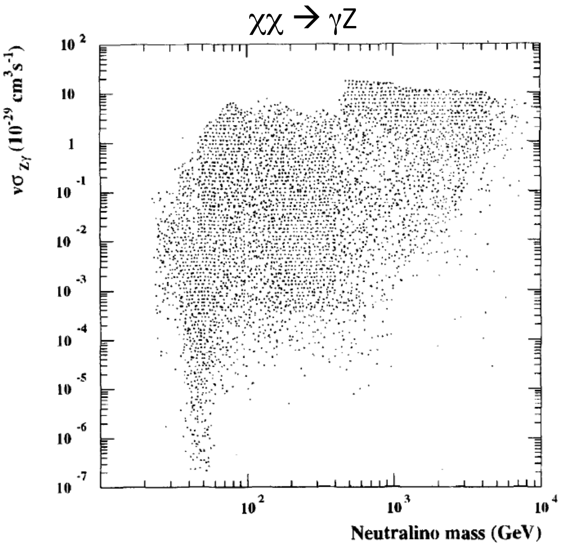
Gaugino fraction: $Z_g = |a_{11}|^2 + |a_{12}|^2$

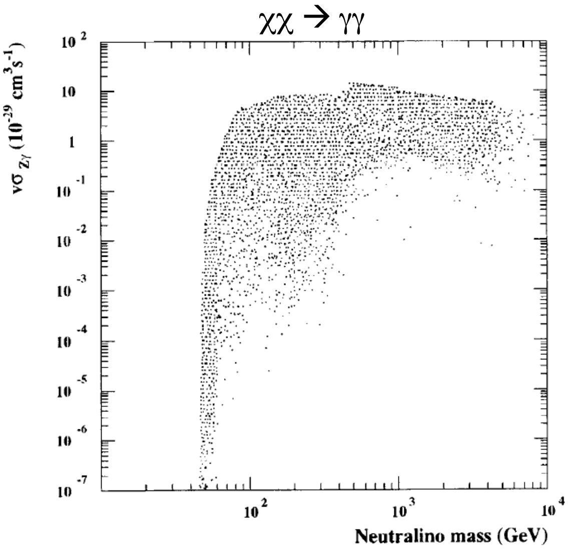
$$\chi = a_{11}\widetilde{B} + a_{12}\widetilde{W}^3 + a_{13}\widetilde{H}_1^0 + a_{14}\widetilde{H}_2^0$$

➔ $\sigma v \sim 10^{-28}$ $cm^3 s^{-1}$ (maximum expectation)

(ref) L. Bergstrom et al., Astroparticle physics 9, 137 (1998)

Calculated cross section (SUSY)





Halo model dependence

$$\rho(r) \propto \frac{1}{(r/a)^{\gamma}[1+(r/a)^{\alpha}]^{(\beta-\gamma)/\alpha}}$$

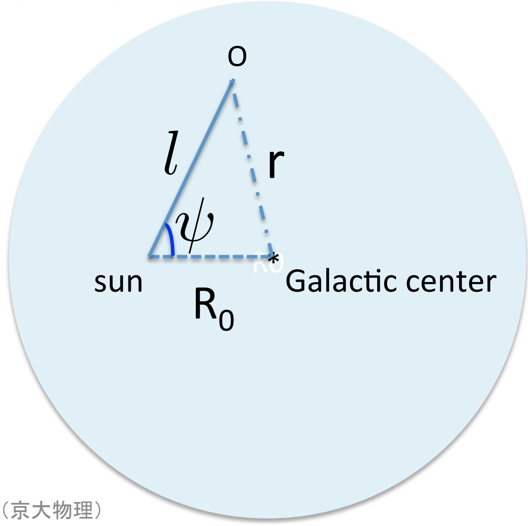
- a: core radius
  - For $\gamma = 0$, scale of the "~ constant density" core of halo
- Need to know
  - R_0: galactocentric distance @ sun
  - ρ_0: halo density @R0

		α	β	γ	references
Isothermal distribution	Sp-profile	2	2	0	
Kravtsov et al.	Ka-profile	2	3	0.2	Astro-ph/p708176
Kravtsov et al.	Kb-profile	2	3	0.4	Astro-ph/p708176
Navarrow et al.	NFW profile (Cuspy)	1	3	1	Astrophys. J. 426, 563 (1996)

Monochromatic γ flux

$$\Phi_\gamma(\psi) = \frac{N_\gamma v\sigma}{4\pi M_\chi^2} \int_{line\ of\ sight} \rho^2(l)dl(\psi)$$

- N_γ = 2 for $\chi\chi \rightarrow \gamma\gamma$
- N_γ = 1 for $\chi\chi \rightarrow Z\gamma$
- ψ : direction to the observation from galactic center



$$r^2 = l^2 + R_0{}^2 - 2lR_0\cos\psi$$

Monochromatic γ flux

$$\Phi_\gamma(\psi) = \frac{N_\gamma v\sigma}{4\pi M_\chi^2} \int_{line\ of\ sight} \rho^2(l)dl(\psi)$$

- Separate ‘cross section + WIMPs mass’ and ‘halo model’

$$\Phi_\gamma(\psi) \simeq 1.87 \times 10^{-11} \left(\frac{N_\gamma v\sigma}{10^{-29} cm^3 s^{-1}}\right) \left(\frac{10GeV}{M_\chi}\right)^2 J(\psi)\ cm^{-2}s^{-1}sr^{-1}$$

$$J(\psi) = \frac{1}{8.5kpc} \left(\frac{1}{0.3GeV/cm^3}\right)^2 \int_{line\ of\ sight} \rho^2(l)dl(\psi)$$

Flux enhancement factor
Halo Model dependent

J(ψ)

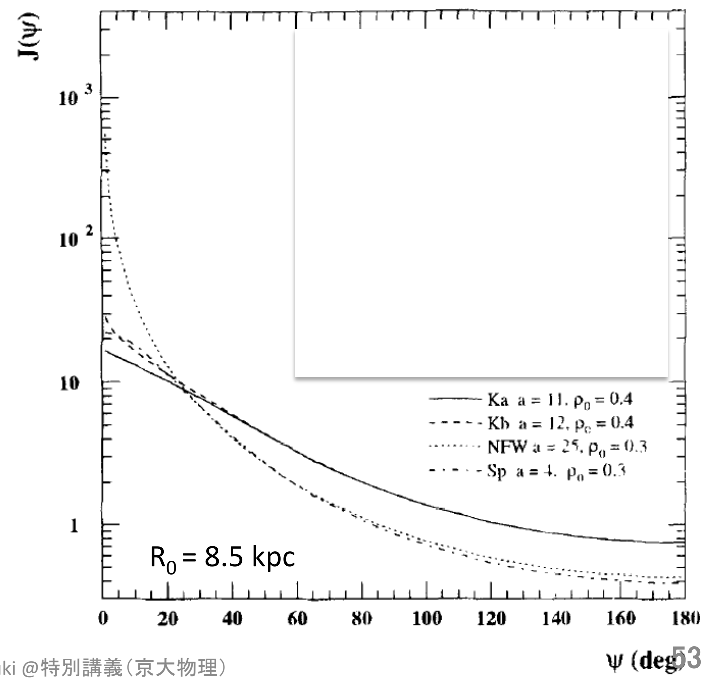
- Maximum flux

➔ Direction of the galactic center

- NWF model (CUSO)

➔Order of magnitude enhancement

L. Bergström et al. / Astroparticle Physics 9 (1998) 137–162



END