mu-e conversion theory

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Contents of this talk

- Distinguishing various LFV interactions
- Updated calculation of the mu-e conversion rate in various nuclei

  V. Cirigliano, R.Kitano, Y.Okada, and P.Tuzon, arXiv: 0904.0957 , PRD 80, 013002 (2009)
- A few new physics examples

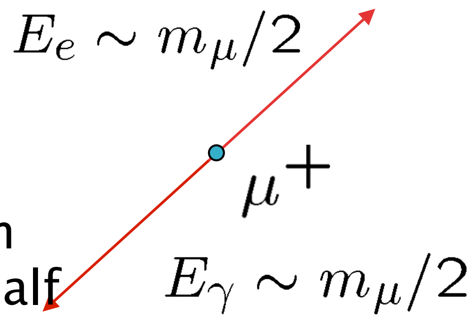
[1]Distinguishing various LFV interactions

- Comparison of three muon LFV processes.
  (μ-> eγ, μ->eee, μ-e conversion)
- Angular distribution of polarized muon decays in μ-> eγ, μ->eee.
- Atomic number dependence of the mu-e conversion rate.

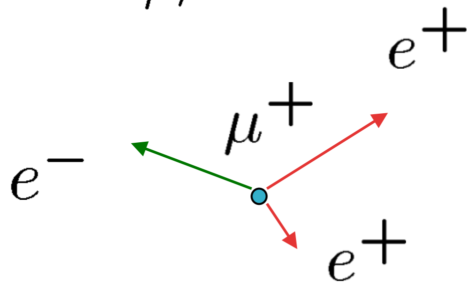
Three muon LFV processes

(1) $\mu^+ \to e^+\gamma$

Back to back emission of a positron and a photon with an energy of a half of the muon mass.



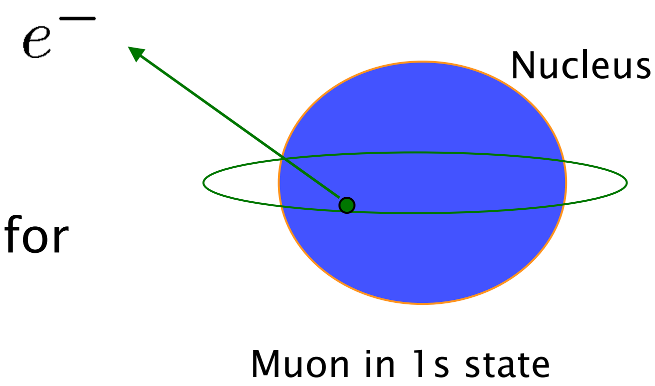
(2) $\mu^+ \to e^+e^+e^-$



(3) $\mu^- A \to e^- A$

$$E_e \sim m_\mu - E_{1sBinding}$$

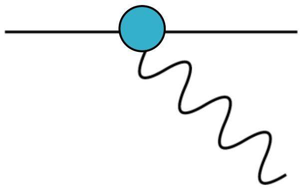
A monochromatic energy electron emission for the coherent mu-e transition.

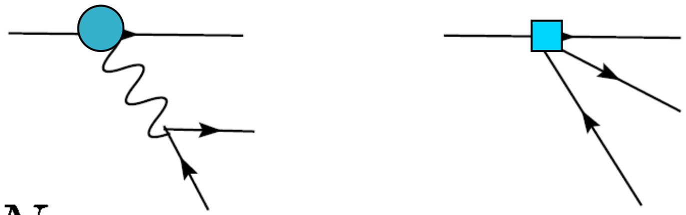


Comparison of three processes

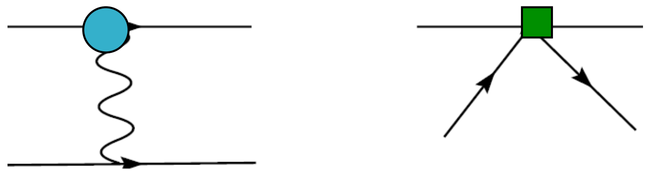
$\mu^+ \to e^+ e^+ e^-$

$\mu^+ \to e^+ \gamma$





$\mu^- N \to e^- N$



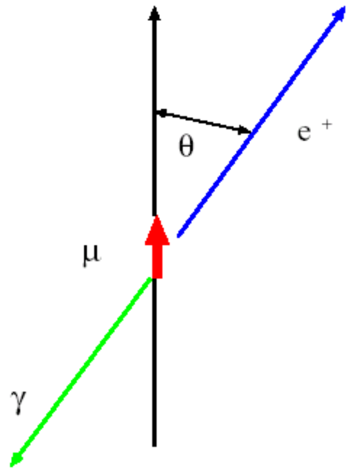
If the photon penguin process is dominated, there are simple relations amon
these branching ratios.

$$
\boxed{
\begin{aligned}
B(\mu \to 3e) &\sim 6.1 \times 10^{-3} B(\mu \to e\gamma) \\
B(\mu Ti \to e Ti) &\sim 4.0 \times 10^{-3} B(\mu \to e\gamma) \\
B(\mu Al \to e Al) &\sim 2.6 \times 10^{-3} B(\mu \to e\gamma)
\end{aligned}
}
$$

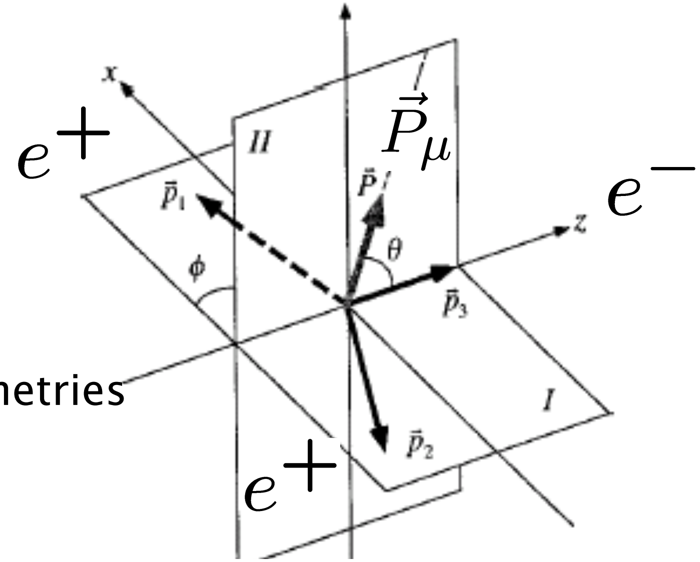
In many case of SUSY modes, this is true, but there is an important case
In which these relations do not hold.

Muon polarization and LFV processes

- If the muon is polarized, we can define a P-odd asymmetry for $\mu \to e\gamma$ and T-odd and P-odd asymmetries for $\mu \to 3e$. These asymmetries are useful to discriminate different models.



μ-> 3e



Two P-odd and
one T-odd asymmetries

$$\frac{dB(\mu^+ \to e^+\gamma)}{d\cos\theta} \propto 1 + A_{\mu\to e\gamma} P_\mu \cos\theta$$

$$A_{P_1} = \frac{N(P_z > 0) - N(P_z < 0)}{N(P_z > 0) + N(P_z < 0)}$$

$$A_{P_2} = \frac{N(P_x > 0) - N(P_x < 0)}{N(P_x > 0) + N(P_x < 0)}$$

$$A_T = \frac{N(P_y > 0) - N(P_y < 0)}{N(P_y > 0) + N(P_y < 0)}$$

Example :A= -1 for the SUSY seesaw model
Left-handed slepton mixing =>

$$\mu^+ \to e^+\gamma_R \Rightarrow (1 - \cos\theta) \text{ distribution}$$

$\mu \to e\,\gamma$ and $\mu \to 3e$ asymmetries in SUSY models

P and T-odd asymmetries in minimal SUSY GUT models

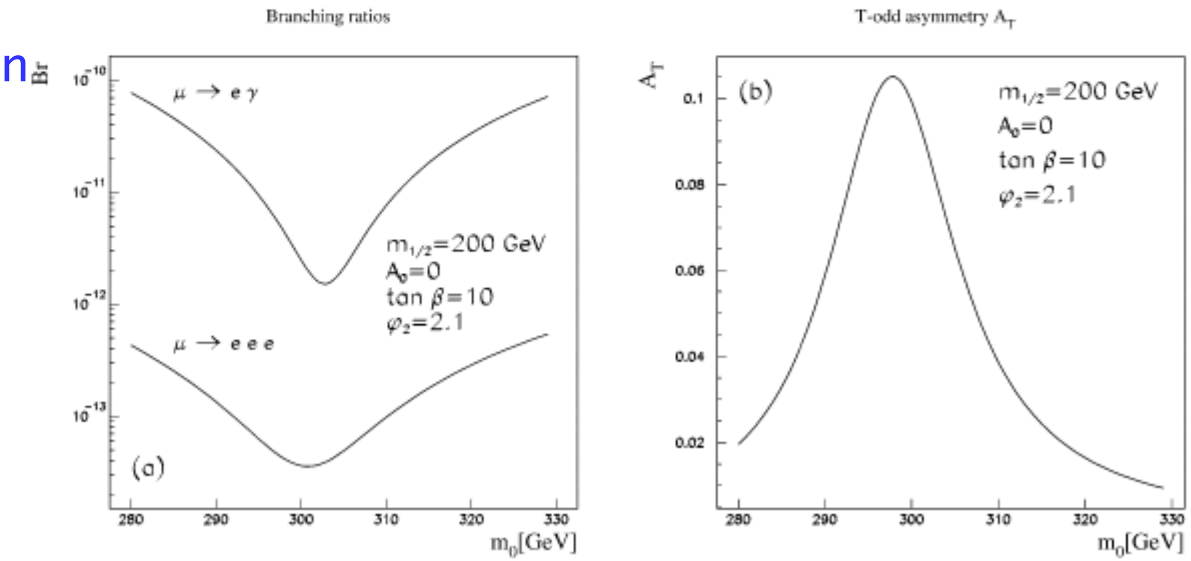
	SU (5)	SO (10)
$A_{\mu\to e\gamma}$	$+100\%$	-100% $-$ $+100\%$
A_{P_1}	-30% $-$ $+40$ %	$\simeq -A_{\mu\to e\gamma}/10$
A_{P_2}	-20% $-$ $+20$ %	$\simeq -A_{\mu\to e\gamma}/6$
$\lvert A_T\rvert$	$\lesssim 15\%$	$\lesssim 0.01\%$

Y.Okada,K.Okumura,and Y.Shimizu, 2000

The T-odd asymmetry can be 10 % level for some parameter space of the SU(5) SUSY GUT and the SUSY seesaw model.

Information on lepton sector CP violation

T-odd asymmetry in the SUSY seesaw model



J.Ellis,J.Hisano,S.Lola, and M.Raidal, 2001

[2] Calculation of the mu-e conversion rate

- The coherent mu-e conversion rate in a wide range of nuclei was calculated for all types of LFV interactions.

R.Kitano, M.Koike, and Y.Okada, PRD 66 (2002)

- The calculation has been updated with estimation of uncertainty associated with the s-quark scalar density in a nucleon.

V. Cirigliano, R.Kitano, Y.Okada, and P.Tuson, PRD 80 (2009)

Operators relevant to the coherent mu-e conversion

$$
\begin{aligned}
\mathcal{L}_{\rm eff}^{(q')} \;=\; & -\frac{1}{\Lambda^2}\Big[\left(C_{DR} m_\mu\, \bar{e}\sigma^{\rho\nu} P_L \mu + C_{DL} m_\mu\, \bar{e}\sigma^{\rho\nu} P_R \mu\right) F_{\rho\nu} && \text{Photonic dipole} \\
& + \sum_{q=u,d,s} \left(C_{VR}^{(q)}\, \bar{e}\gamma^\rho P_R \mu + C_{VL}^{(q)}\, \bar{e}\gamma^\rho P_L \mu \right) \bar{q}\gamma_\rho q && \text{Vector} \\
& + \sum_{q=u,d,s} \left(C_{SR}^{(q)} m_\mu m_q G_F\, \bar{e} P_L \mu + C_{SL}^{(q)} m_\mu m_q G_F\, \bar{e} P_R \mu \right) \bar{q} q && \text{Scalar} \\
& + \left(C_{GQR} m_\mu G_F\, \bar{e} P_L \mu + C_{GQL} m_\mu G_F\, \bar{e} P_R \mu \right) \frac{\beta_L}{2 g_s^3} G_a^{\rho\nu} G_{\rho\nu}^a + h.c. \Big] && \text{gluonic}
\end{aligned}
$$

The gluonic operator can arise by heavy quark loop diagrams.
The gluonic coupling to a nucleon can be expressed by scalar quark densities in a nucleon.

The μ–e conversion rate is defined

$$B \equiv \frac{\omega(\mu N(A,Z) \rightarrow eN(A,Z))}{\omega(\mu N(A,Z) \rightarrow capture)}$$

Schematically,

$$\omega(\mu N(A,Z) \rightarrow N(A,Z))$$
$$\sim |\sum \int d^3x \bar{\psi}_e \Gamma \psi_\mu^{(1S)} \rho^{(p,n)}|^2$$

Calculation goes in the following steps:

(1) Take a matrix element of quark operators in a proton/a neutron state.

(2) Sum over all the protons and neutrons in a nucleus coherently.

(3) Evaluate overlap integrals of the above type.

Theoretical uncertainty depends on a type of operators

(1) Photonic dipole case: Almost no uncertainty
The calculation only depends on the charge distribution in a nucleus, which is precisely known by electron scattering.

(2) Vector case:
The main uncertainty comes from the neutron density.
Little uncertainty for light nuclei.
Uncertainty is 5% level for heavy nuclei if the proton scattering data is available (ex. Pb).

(3) Scalar case:
An addition source of uncertainty is scalar quark densities in a nucleon.

The new lattice QCD estimation of strange quark scalar density.
H. Ohki et.al. (JLQCD) PRD 78, 054502

$$y = \frac{2\langle p|\bar{s}s|p\rangle}{\langle p|\bar{u}u + \bar{d}d|p\rangle} = 0.030 \pm 0.016(\mathrm{stat})^{+0.006}_{-0.008}(\mathrm{extrap})^{+0.001}_{-0.002}(m_s).$$

compared with the previous phenomenological estimate $y = 0.21 \pm 0.2$

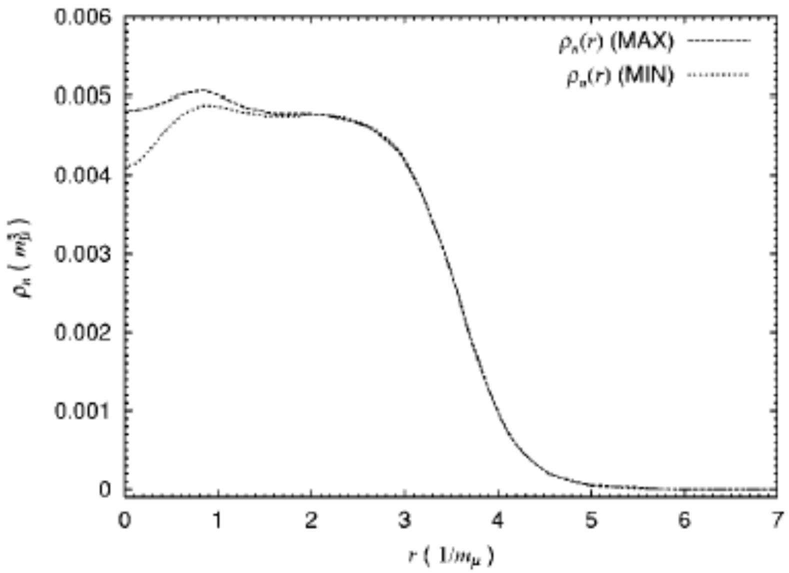
Example:
Uncertainty of overlapping integrals for Pb due to neutron distribution .

R.Kitano, M.Koike, and Y.Okada,2002

			Proton		Neutron	
		dipole	scalar	vector	scalar	vector
		D	$S^{(p)}$	$V^{(p)}$	$S^{(n)}$	$V^{(n)}$
$^{208}_{82}$Pb	Minimum	0.163	0.0493	0.0845	0.0675	0.119
	Maximum	0.163	0.0493	0.0845	0.0697	0.121

Ambiguity from neutron distribution is a few %.

Neutron distribution from proton scattering.



V.E. Staodubsky and N.M.Hintz, Phys. Rev C,1994

Atomic number dependence for several LFV operators

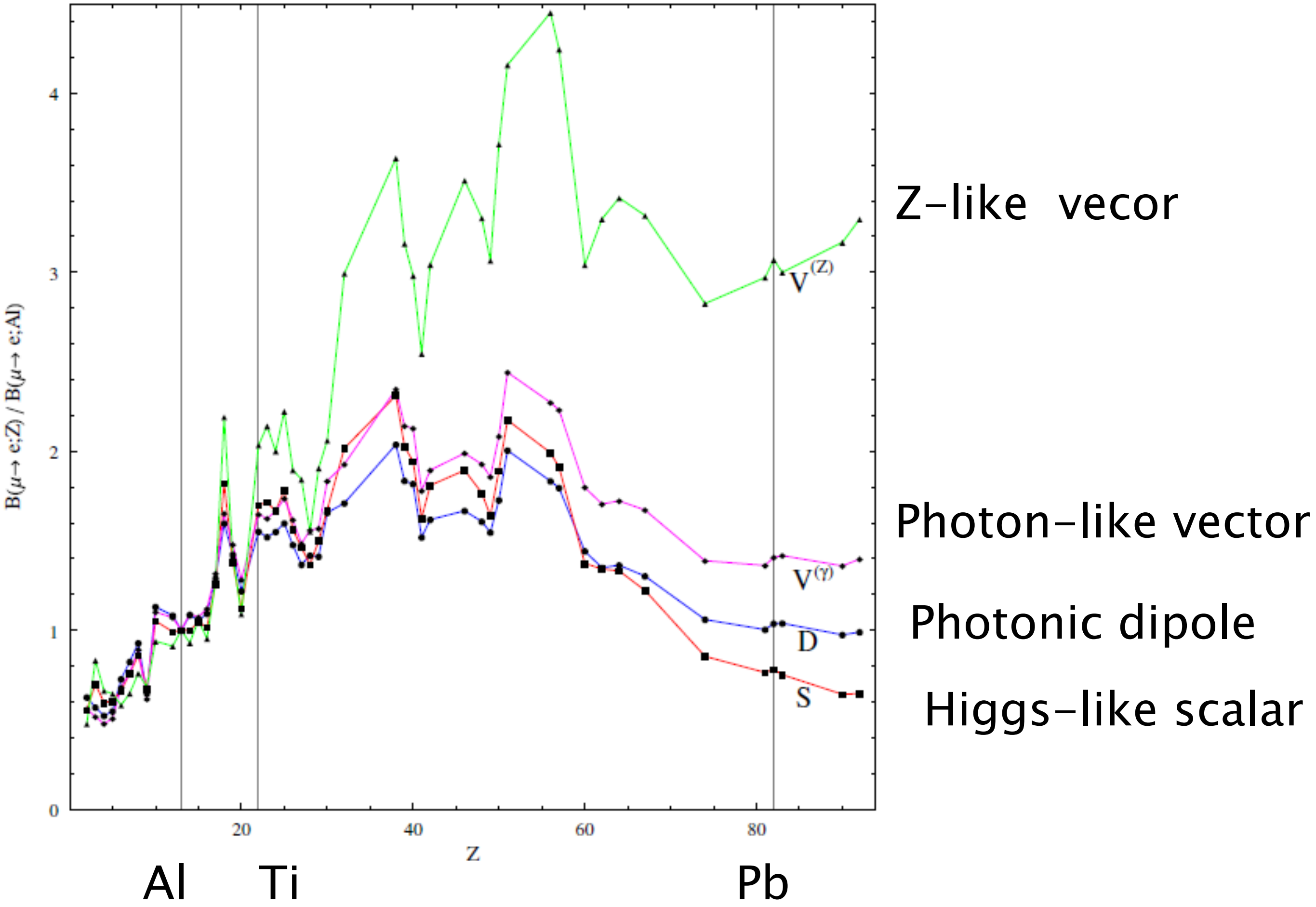
(1) Dipole model: “D”
Photonic dipole coupling

(2-1) Vector model 1: “$V^{(\gamma)}$”
Couple to u/d quarks with charges

(2-2) Vector model 2: “$V^{(Z)}$”
Couple to u/d quarks like Z boson.
Neutron coupling is dominant.

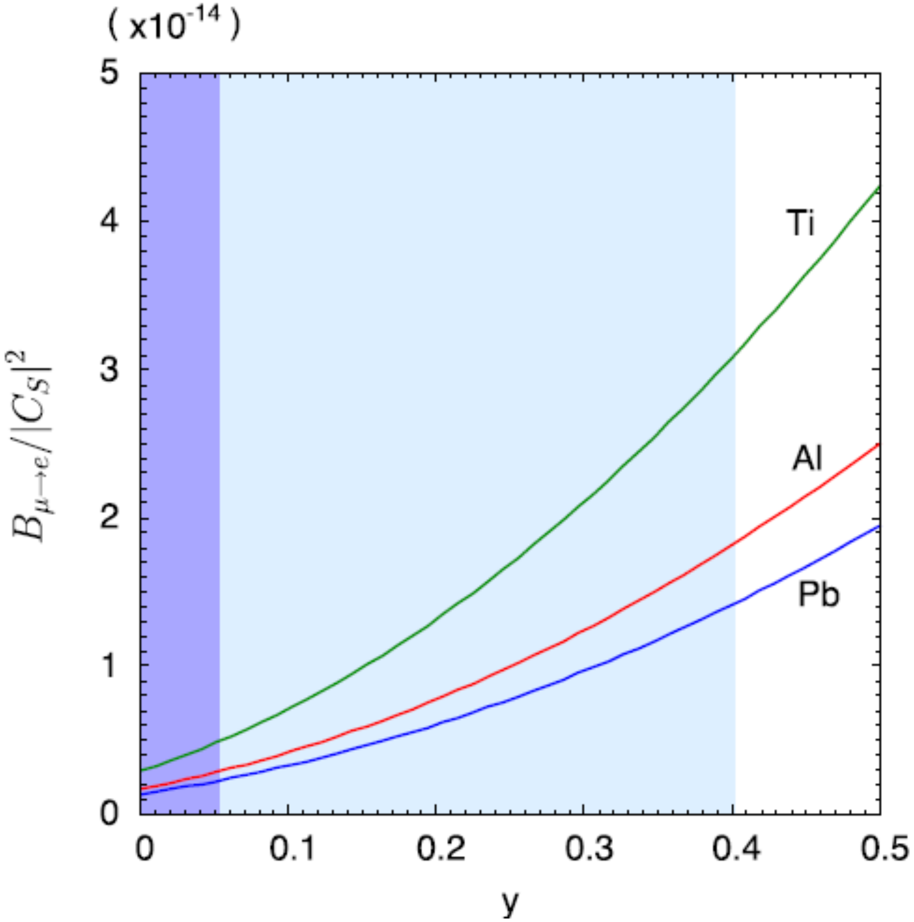
(3) Scalar model : “S”
Couple to down-type quarks like the Higgs coupling.

Atomic number dependence of the mu-e conversion rate for various LFV operators



- Maximal in the intermediate nuclei.
- Different Z dependence for heavy nuclei.
- Large enhancement in the Z-like vector case (neutron-rich for heavy nuclei).

Strange quark content dependence for the Higgs-like scalar case

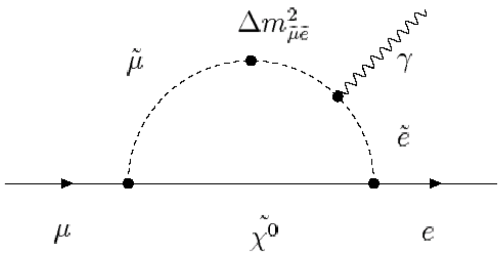


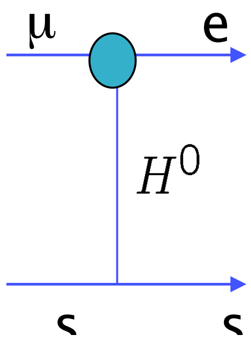
The uncertainty is reduced if the result of the new lattice QCD calculation is confirmed.
The ambiguity is cancelled for the ratio of two nuclei if the only strange quark scalar operator is dominant.

[3] A few new physics examples

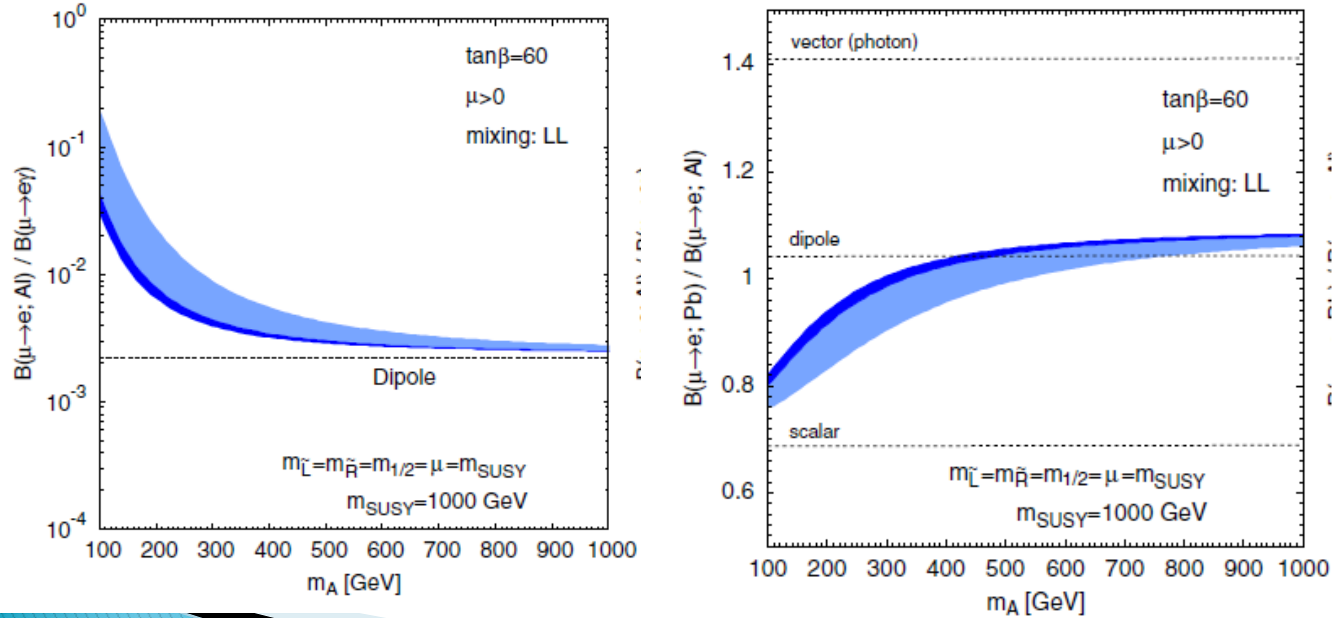
SUSY seesaw model with a large tanβ: The Higgs exchange case

Update from R.Kitano, M.Koike, M.Komine,and Y.Okada, 2003





Blue band :
Uncertainty from y

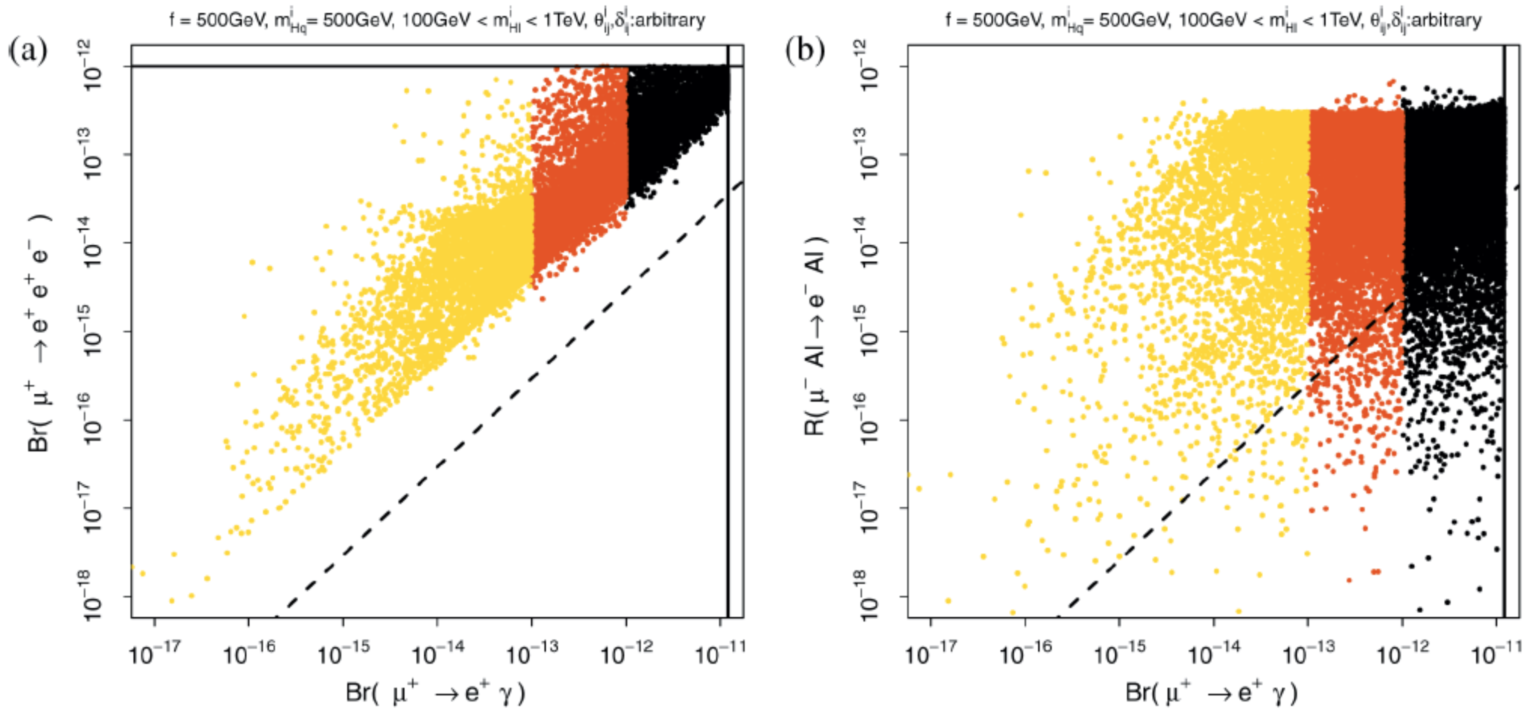


V. Cirigliano, R.Kitano, Y.Okada, and P.Tuson, 2009

Little Higgs Model with T-parity

- Heavy partners (T-odd partners) of quarks and leptons introduce new flavor mixing matrices.
- All three muon LFV processes are generated at one loop level.
- Different pattern from SUSY model.

An example of three LFV branching ratios.



T.Goto, Y.Okada, and Y. Yamamoto, 2011

Summary

- Atomic number dependence of the mu-e conversion is updated with a new result of the strange quark scalar density in a nucleon by JLQCD group.
- Theoretical uncertainty due to this source is reduced, so that the ratio of the mu to e gamma and the mu-e conversion rates and the atomic number dependence of the mu-e conversion rate are useful to distinguish different models, for example, different parameter space in the SUSY seesaw model.