

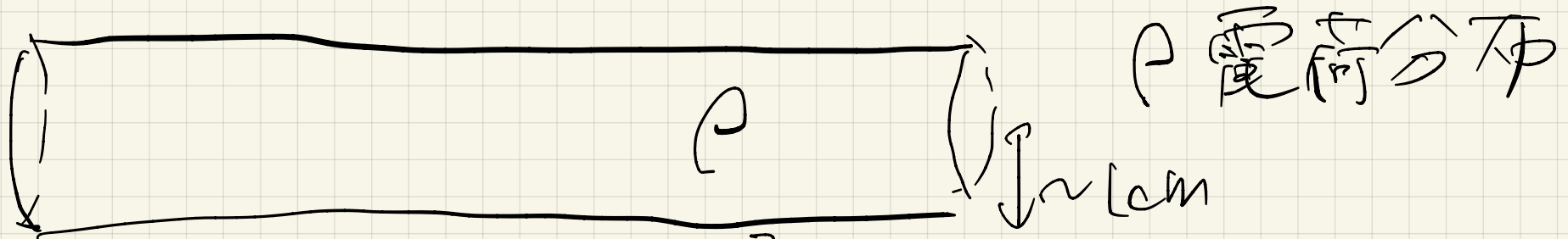
空間的電荷効果

(陽子数 (バンク内) が大きくなる)

陽子 \leftrightarrow 陽子

← 無視できない

進行方向に充分長い \Rightarrow 一様な陽子分布



J-PARC MR

$$E|_{\sigma} = 5\pi \cdot 4\pi \cdot 10^9 \cdot d \quad \beta = 20$$

$$\sqrt{\beta} E_{cr} = 1 \text{ cm}$$

束の長さ l

$$\operatorname{div} \vec{F} = \frac{\rho}{\epsilon_0}$$

$$\int \operatorname{div} \vec{F} \cdot dV = \frac{\rho \pi r^2}{\epsilon_0} \Delta l$$

$$= \int \vec{F} \cdot d\vec{S} = 2\pi r \Delta l \cdot E_r$$

$$F_r = \frac{\rho r}{2\epsilon_0}$$

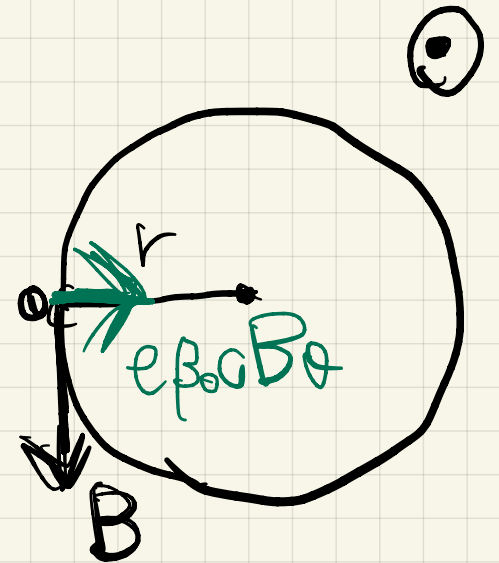
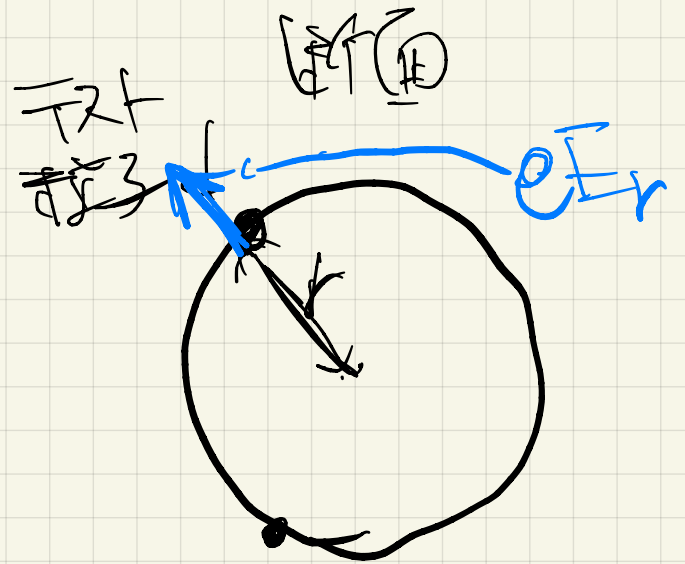
$$\operatorname{rot} \vec{B} = \beta_0 c \rho \cdot \mu_0$$

$$\int \vec{B} \cdot d\vec{l} = \pi r^2 c \rho \beta_0 \mu_0$$

$$B \cdot 2\pi r = \pi r^2 c \rho \beta_0 \mu_0$$

$$B = \frac{\beta_0 c \rho \mu_0}{2} r$$

$$c^2 = \frac{1}{\epsilon_0 \mu_0}$$



$$F_r = \frac{epr'}{2\epsilon_0} - e\beta_0 c \frac{\beta_0 c p}{2} \frac{1}{c^2 \epsilon_0 r}$$

$$= \frac{ep}{2\epsilon_0} (1 - \beta_0^2) = \frac{ep}{2\gamma_0^2 \epsilon_0}$$

$\frac{2\pi}{\hbar} \frac{dS}{dt}$ 辐射力

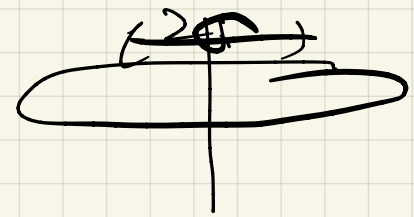
$$\frac{1}{r_0^2}$$

$$\frac{dP_x}{dt} = \frac{ep}{2\gamma_0^2 \epsilon_0} x$$

$$\frac{d}{dt} \frac{dP_x}{ds} = \frac{ep}{2\gamma_0^2 \epsilon_0} x \frac{1}{m\gamma_0 \beta_0 c}$$

$$\beta_0 c \frac{dP_x}{ds} = \frac{ep}{2\gamma_0^2 \beta_0^2 c^2 m p \epsilon_0} x$$

$$N_{ppb} = 3.4 \times 10^{13}$$



$$\rho = \frac{3.4 \times 10^{13} \times e}{\sigma_x^2 \pi \cdot 2\sigma_z}$$

$$d = 1567 \text{ fm}$$

$$\Delta V_{xc} = -\frac{1}{4\pi} \int \beta_x \cdot \frac{e\rho}{2\sigma_0^3 \beta_0^2 c^2 m_p \epsilon_0} dS$$

$$\sigma_x = \sqrt{\beta_x \epsilon_{10}} \quad \sigma_z = 20 \text{ m}$$

$$3 \text{ GeV} \left\{ \begin{array}{l} \beta_0 = 0.99 \\ \gamma_0 = 4.1 \end{array} \right.$$

$$\boxed{\begin{array}{l} H = \frac{1}{2} K x^2 \\ \dot{x} = K x \end{array}}$$

$$\Delta V_x \sim -0.5, \text{ (概算) 無視の打ち}$$

$$\vec{F} = -\text{grad } \phi$$

$$\vec{v} = (0, 0, \beta_0 c)$$

$$\vec{B} = \left(\frac{\partial A_s}{\partial y}, -\frac{\partial A_s}{\partial x}, 0 \right)$$

$$\vec{v} \times \vec{B} = \begin{pmatrix} \frac{\partial A_s}{\partial x} & \frac{\partial A_s}{\partial y} \\ & 0 \end{pmatrix}$$

$$\vec{F} = -e \text{grad } \phi + e (\text{grad } A_s) \beta_0 c \quad \times \beta_0 c$$

$$\Delta \phi = -\frac{\rho}{\epsilon_0}$$

$$\text{rot rot } \vec{A} = \mu_0 \vec{j}$$

$$-\Delta \vec{A} + \text{grad } \underbrace{\text{div } \vec{A}}_{=0} = \mu_0 \vec{j}$$

$$\Delta \vec{A} = \mu_0 \beta_0 c \rho$$

$$\mu_0 = \frac{1}{\epsilon_0 c^2}$$

$$\vec{F} = -e (1 - \beta_0^2) \underline{\underline{\nabla \phi}}$$

$$\frac{d\vec{p}}{ds} = - \frac{e}{mc^2 \beta_0^2 \gamma_0^3} \nabla \phi$$

$$\rho(x, y) = \frac{Ne}{2\pi\sigma_x\sigma_y} e^{-\left(\frac{x^2}{2\sigma_x^2} + \frac{y^2}{2\sigma_y^2}\right)}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \phi = - \frac{1}{\epsilon_0} \rho(x, y)$$

$$\phi(x, y) = \int_0^{\infty} dt \frac{\exp\left(-\frac{x^2}{2\sigma_x^2+t} - \frac{y^2}{2\sigma_y^2+t}\right)}{\sqrt{(2\sigma_x^2+t)(2\sigma_y^2+t)}}$$

$$1 + f_x(t)x^2 + f_y(t)y^2 \dots x^4 \dots x^2y^2 \dots y^4$$

$$\frac{d\vec{p}}{ds} = - \frac{e}{mc^2 \beta \gamma_0^3} \nabla \phi(x, y)$$

↳ 電荷自身が作る電場

空間電荷効果自身が

束線形場とcの共通の原因

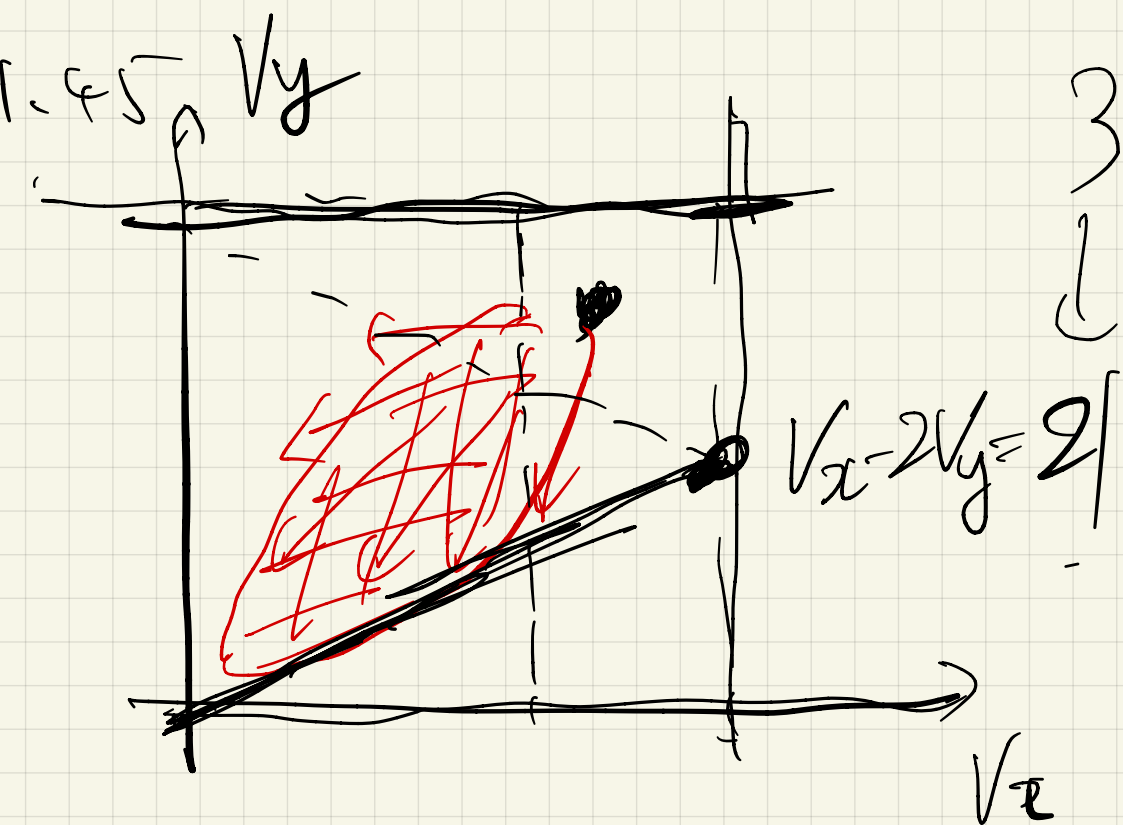
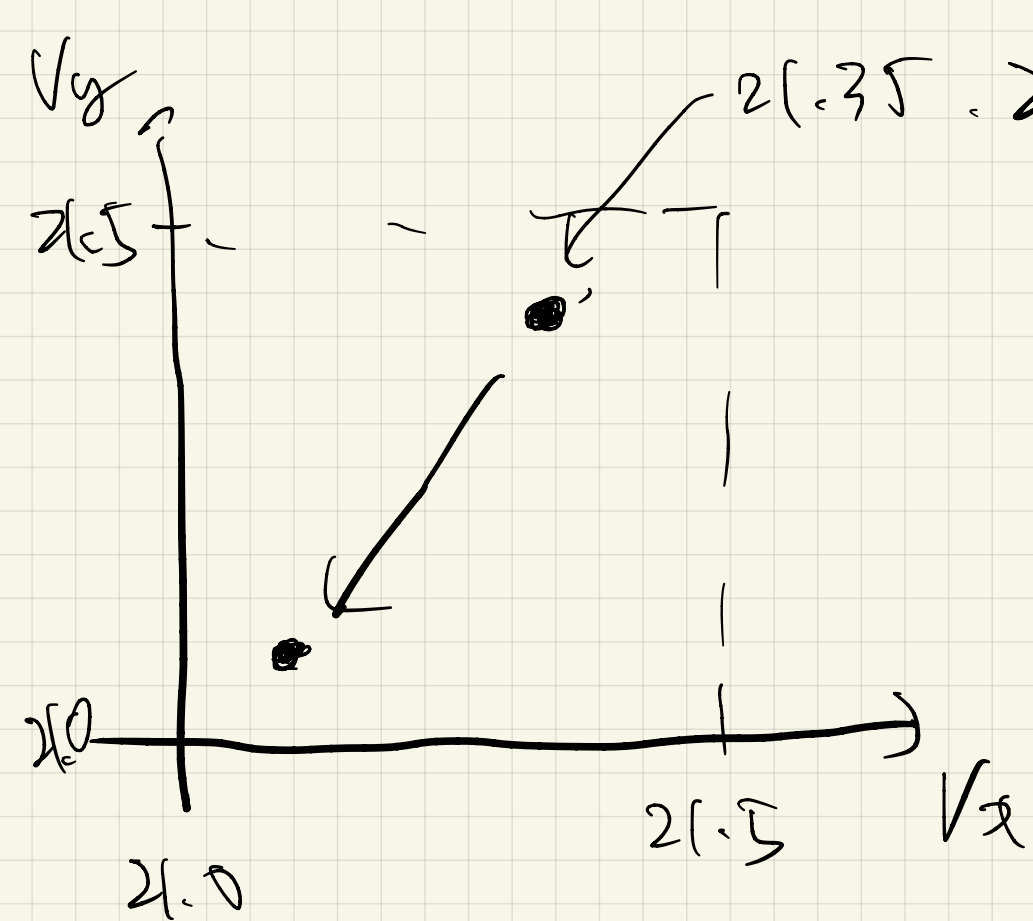


図 2 の α $f_2 - \dots$
 別の値をとる

理想状態

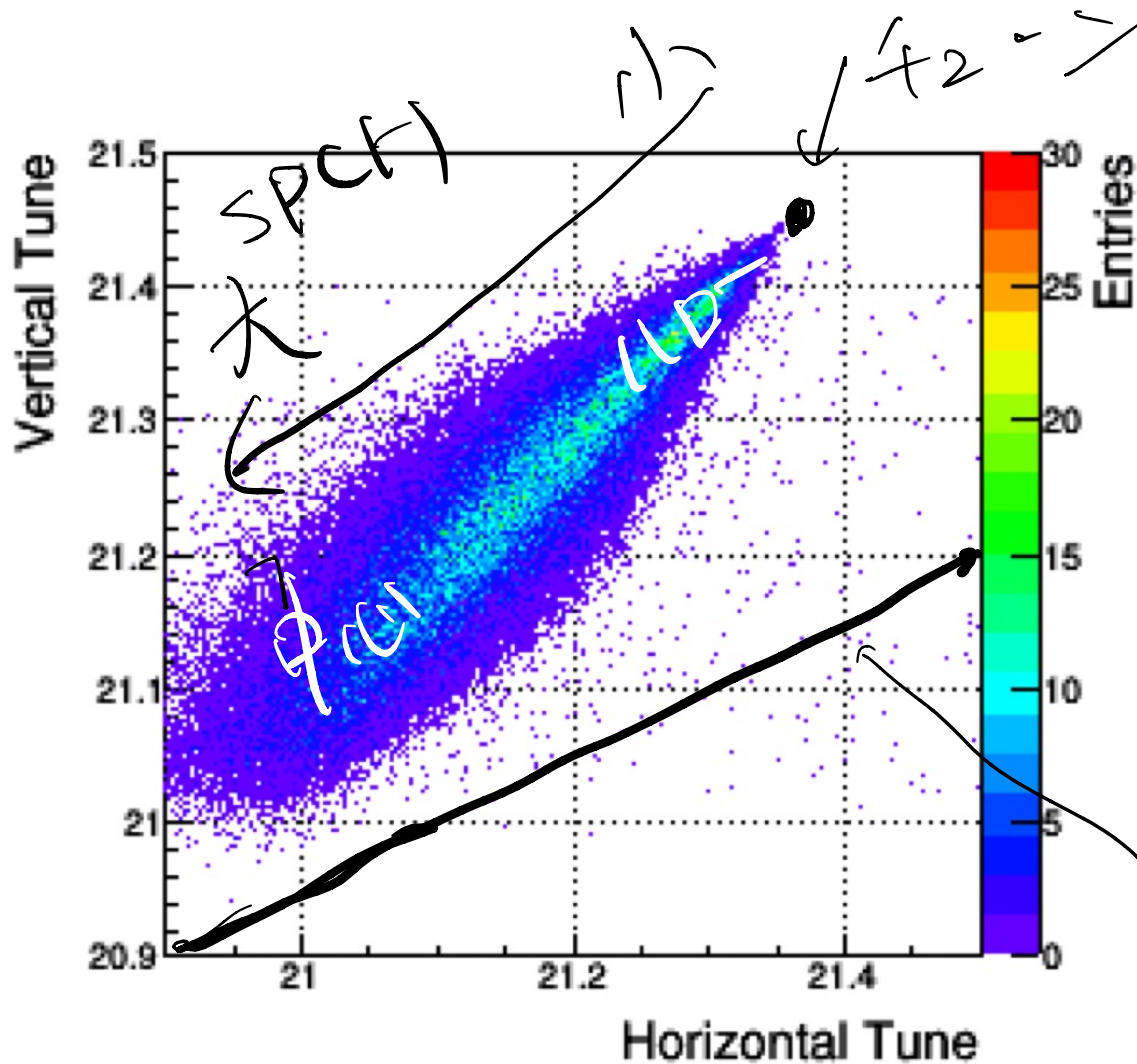
(一様分布)

現実

(ガウシアン)

$f_2 - \dots$ 図 2: $t = 4$ 調整が一番早い

観測値の分布 = $F_2 - F_1$ の分布



この分布は
測定の誤差
= $F_2 - F_1$ の分布!!

F_1 F_2 の?

$$V_2 - 2V_y = 21$$

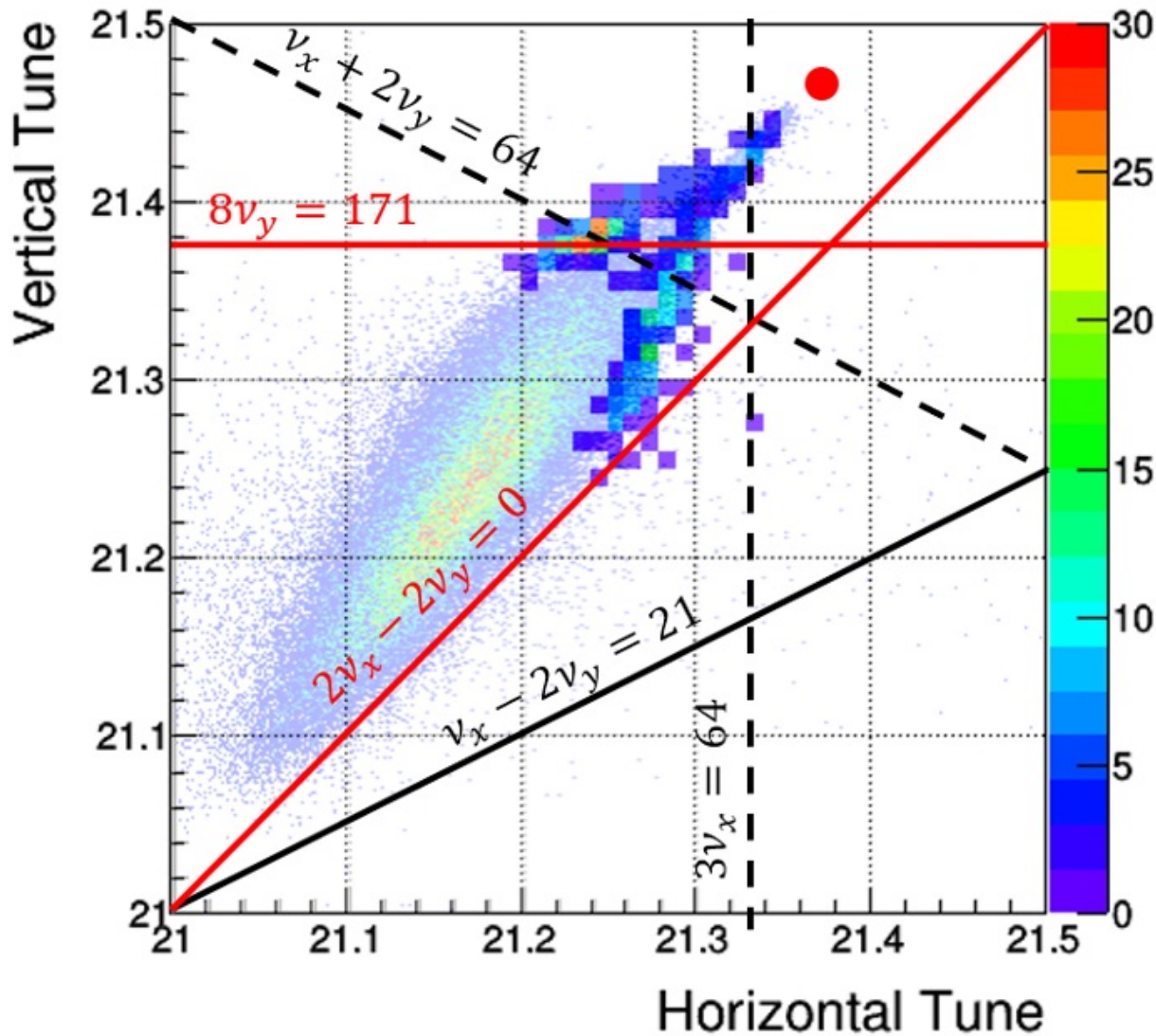
この問題が何を求めているのか

x^4, x^2y^2, y^4, \dots (べき乗) $(f = A^2)$

これら自身 $p(x, y)$. $\text{Max}\{f = \text{value}\}$ $\in \mathbb{C}^2$ 共鳴の

原因 $r = r_0$

$f'' - 4DZ$ の分布 (縦横)
 $\Rightarrow \approx 2V - \dots$



$8V_y = 171$ e^{30}

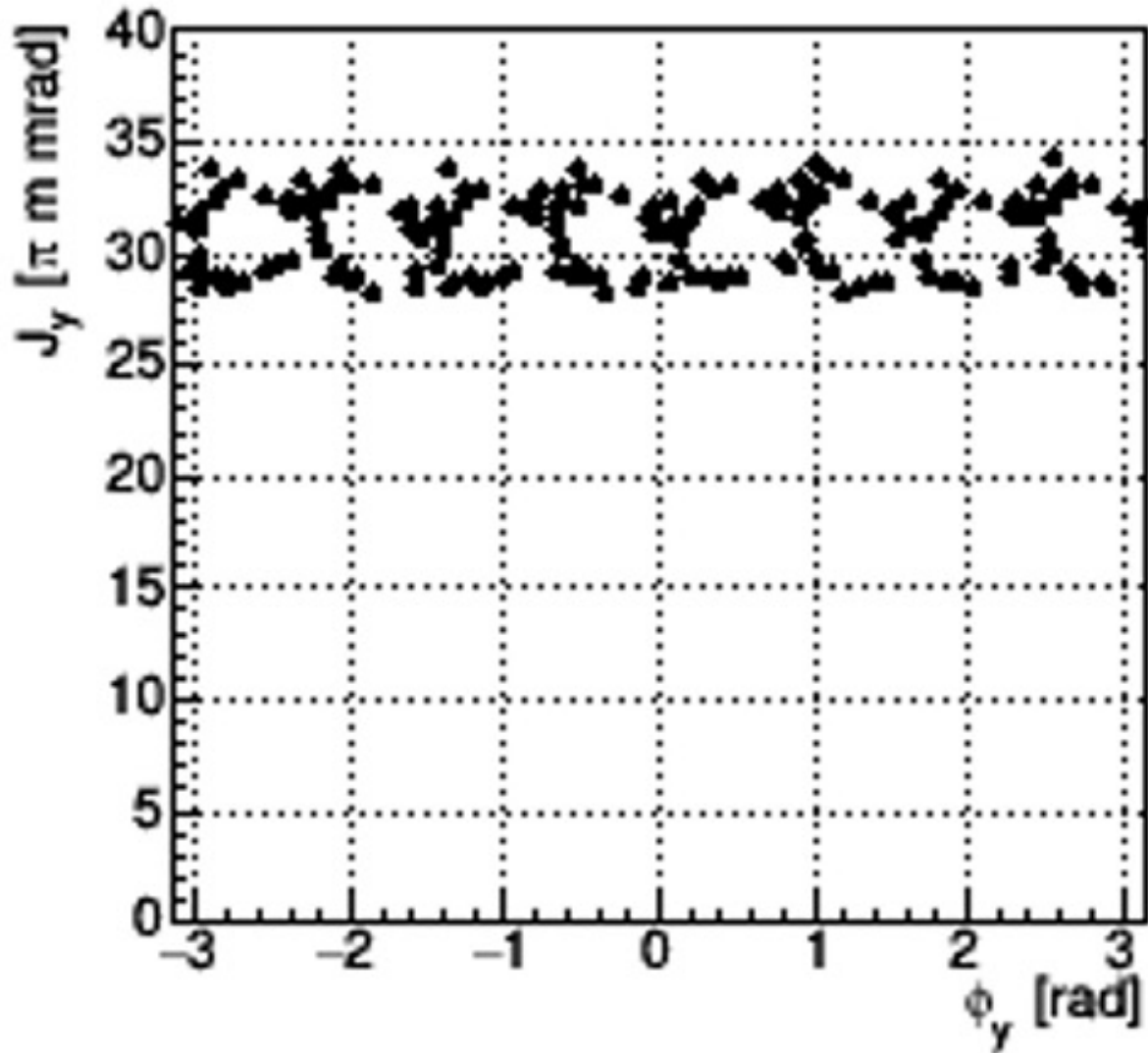
$2V_x - 2V_y = 0$
 の $f'' - 4DZ$ の分布

16 種類 縦横

$f'' - 4DZ$

\rightarrow $f'' - 4DZ$ の分布
 (= $f'' - 4DZ$)

$g v_y = 171$ 方向の粒子の運動



ϕ - J 分布

多次共鳴

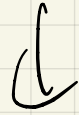
$$H = \frac{1}{2} J_y^2 \text{ (order)}$$

詳細は議論する

$J_y = 17$ の 2 共鳴
 の 2 共鳴

Image Charge の効果

自由電荷 = 電荷分布 (電流も)



媒質は金属 グラトの内

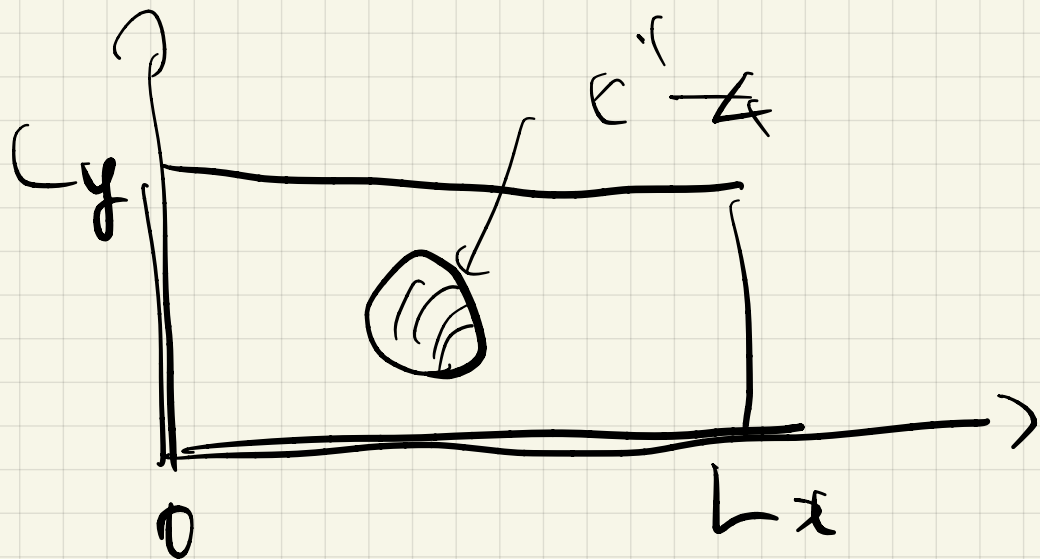
→ 境界条件の電場と磁場で

電場

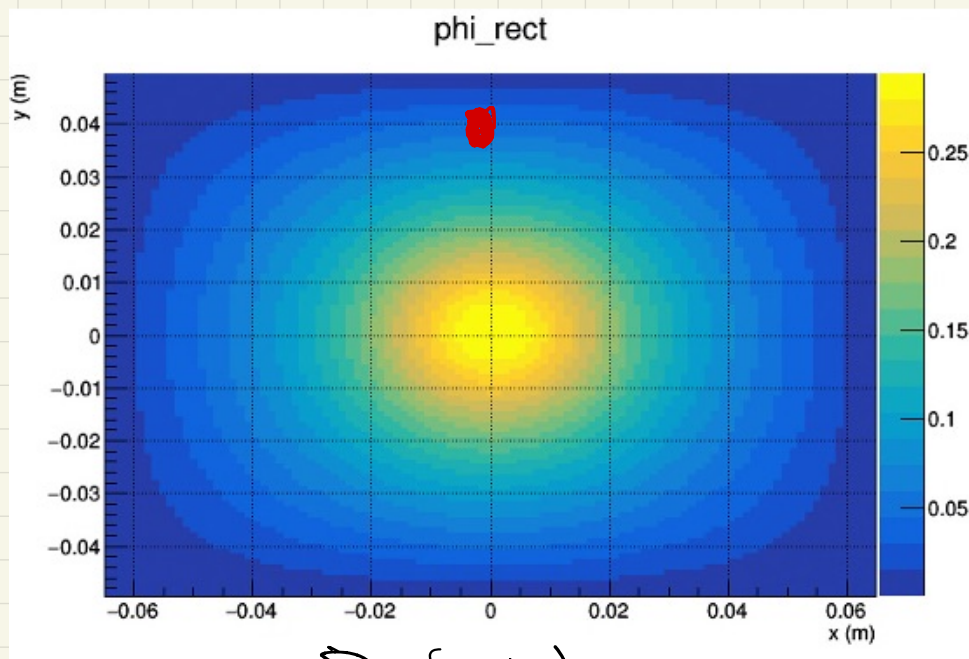
電場 E の端 \rightarrow

球 \rightarrow 電位 ϕ の ∇ の ∇ 体中

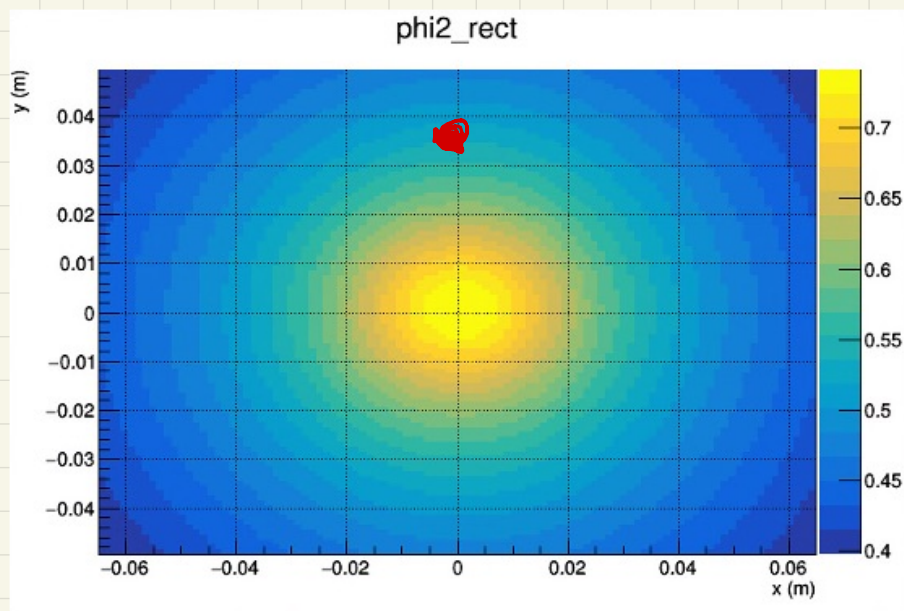
一定



$$\phi(x, \pm L) = \phi\left(\pm \frac{L}{2}, y\right) = 0 \quad \text{const.}$$

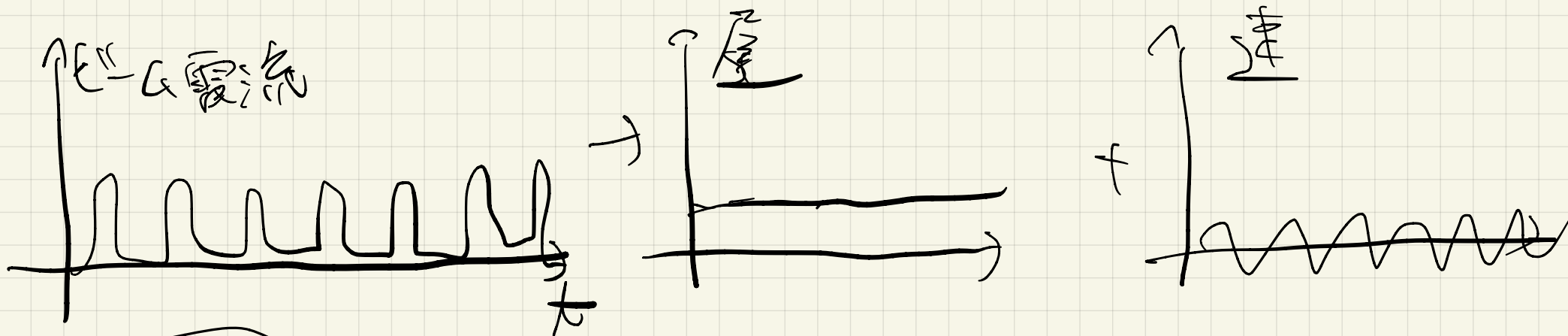


BC $\phi(x')$

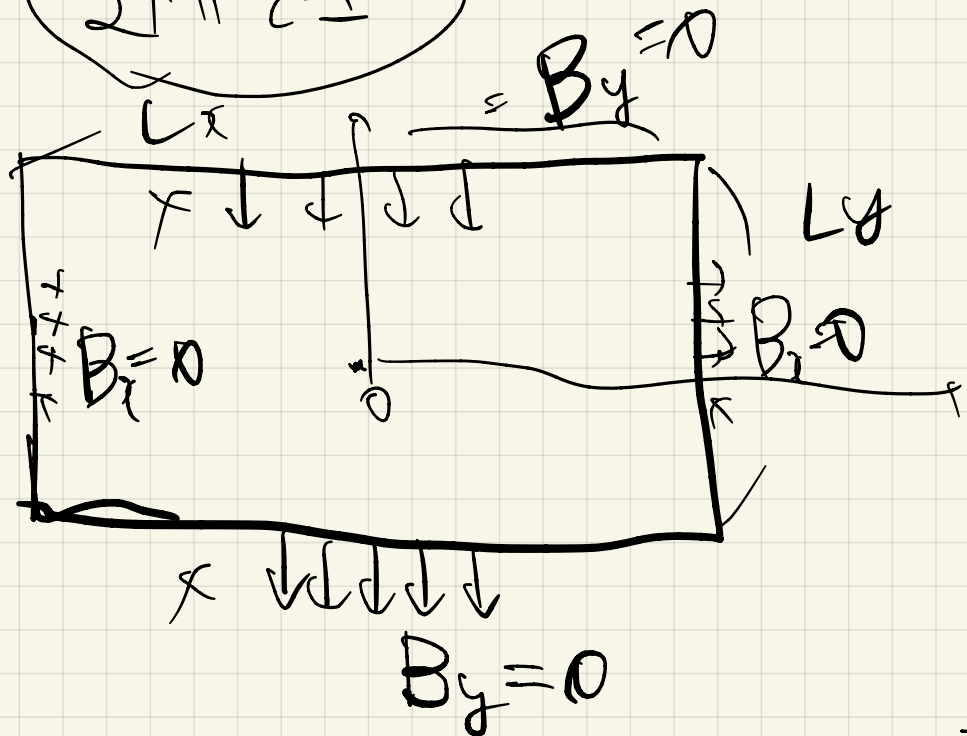


BC $\phi(x, L)$

磁場の場合の周波数は ω である。



$\nabla \cdot \mathbf{E} = \rho$



$$B_x\left(\pm \frac{L_x}{2}, y\right) = \frac{\partial A_s}{\partial y} = 0$$

$$B_y\left(x, \pm \frac{L_y}{2}\right) = -\frac{\partial A_s}{\partial x} = 0$$

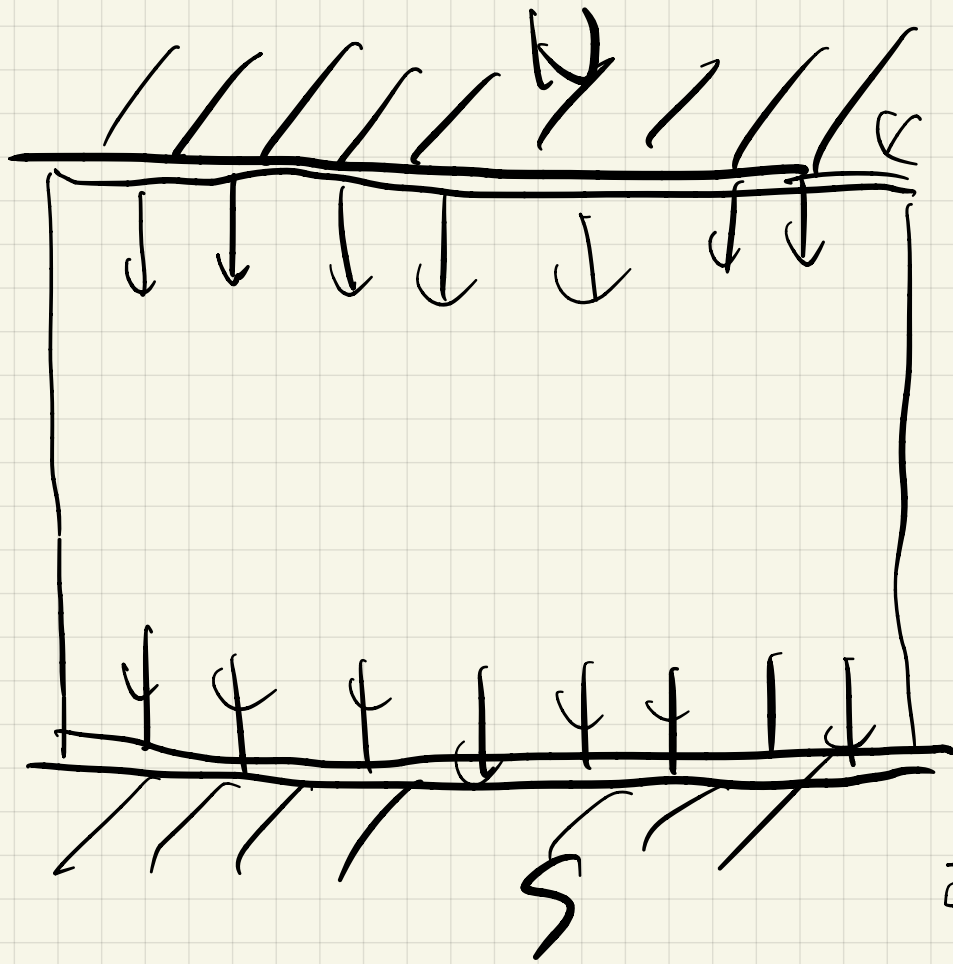
$$A_s\left(\pm \frac{L_x}{2}, y\right) = \text{const}$$

$$A_s\left(x, \pm \frac{L_y}{2}\right) = \text{const}$$

遅い成分

Bend

弾性体



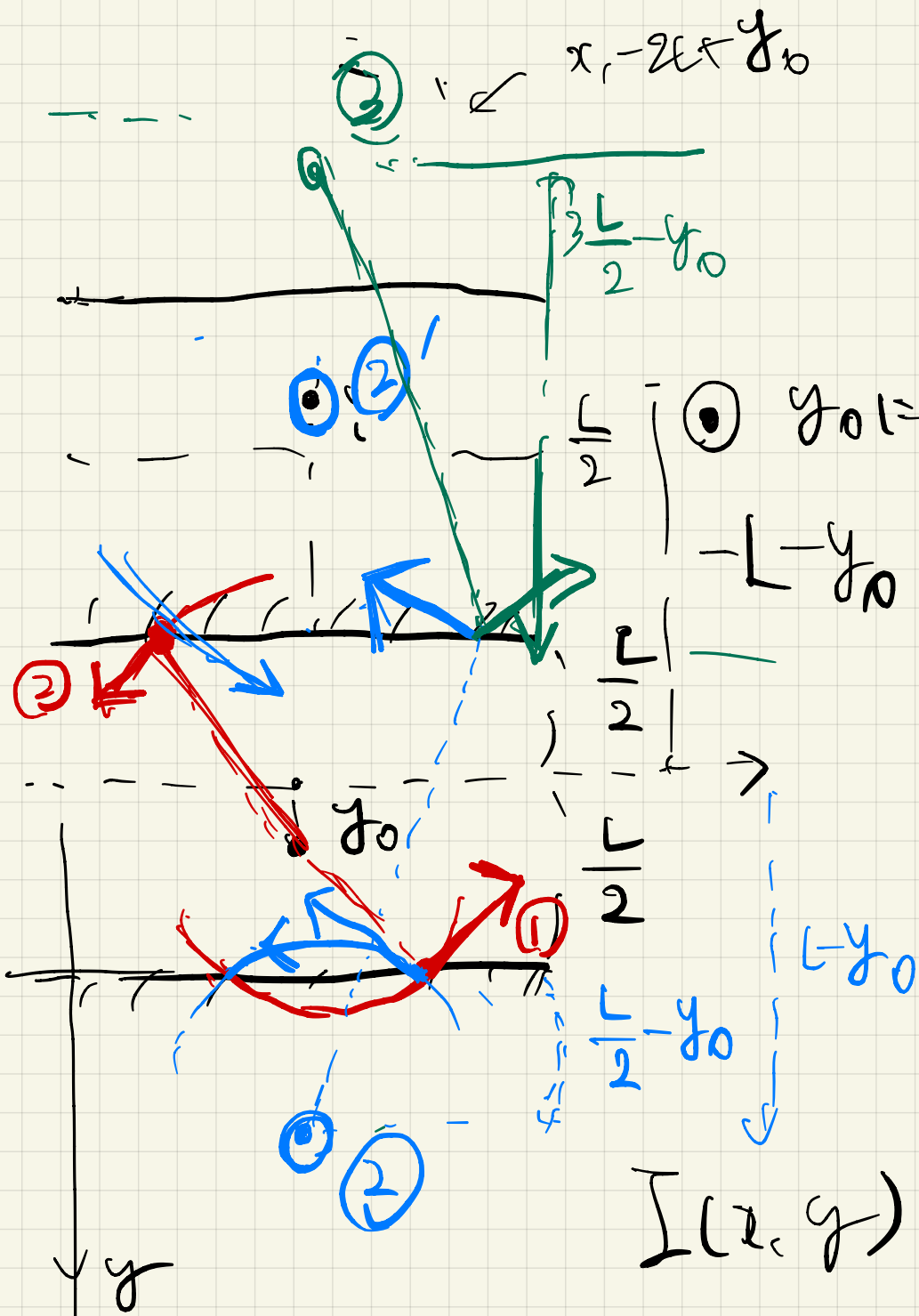
磁極

磁場は磁極
= 垂直

磁極

ε¹ ε² ε³ ε⁴ ?

鏡像電流

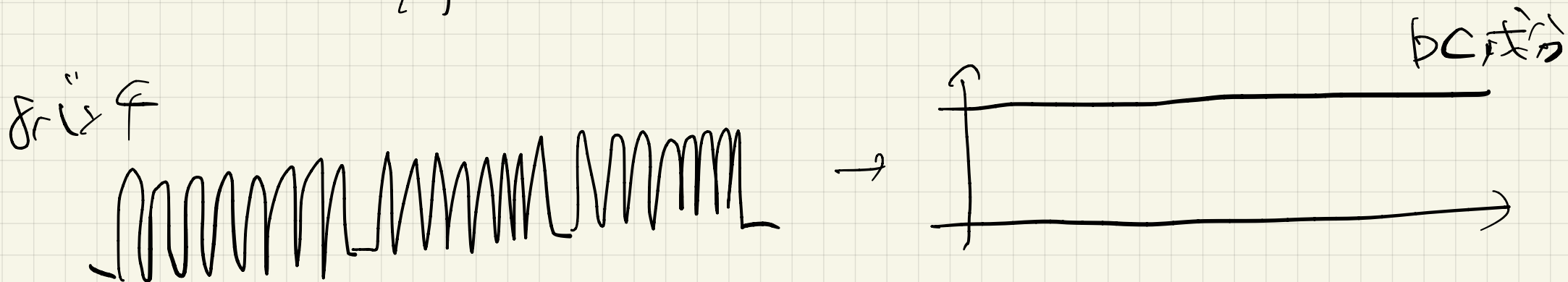


$\odot y_0 =$ 實流
 \ominus 鏡像電流

$$\begin{aligned}
 I_{\text{image}} = & I(x, L-y) \\
 & + I(x, -L-y) \\
 & + I(x, 2L+y) \dots
 \end{aligned}$$

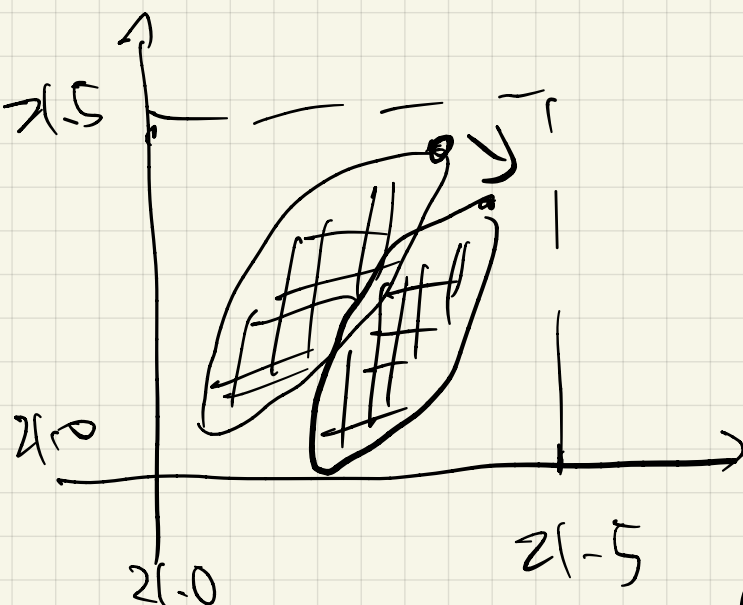
無限級數

DC成分 = 0 \rightarrow 他のバイン F の交点になる



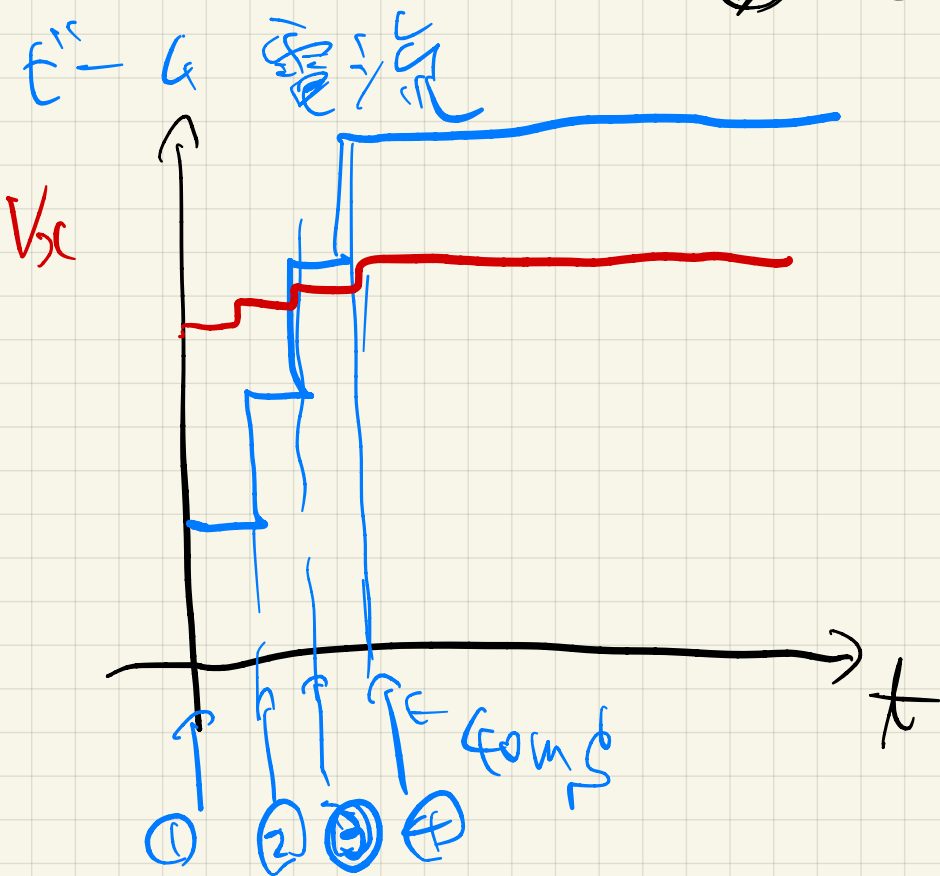
積分電流 $I = \int$

f_2 分布自体を知らず



$\int_{t_1}^{t_2} I dt \rightarrow \hat{f}_2$

① $\int_{t_1}^{t_2} I dt \rightarrow \hat{f}_2$



J-PARC MR

40ms 毎に 2 punches

入射

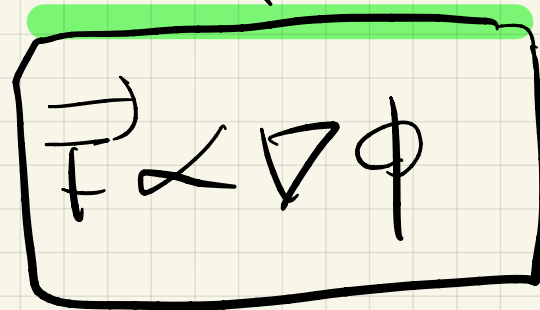
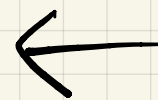
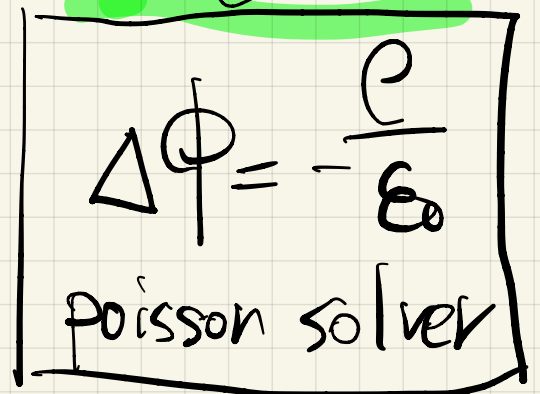
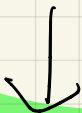
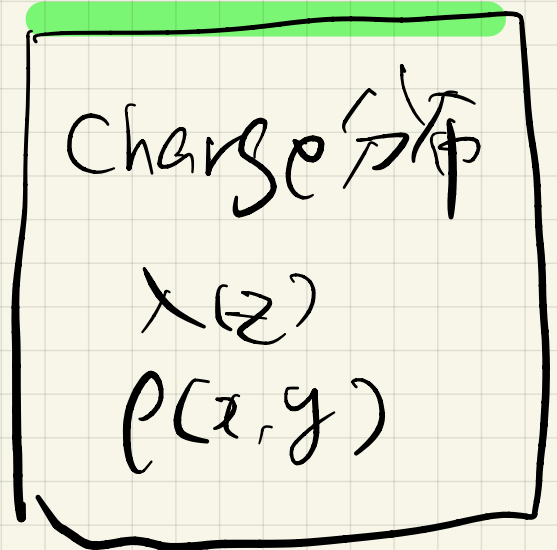
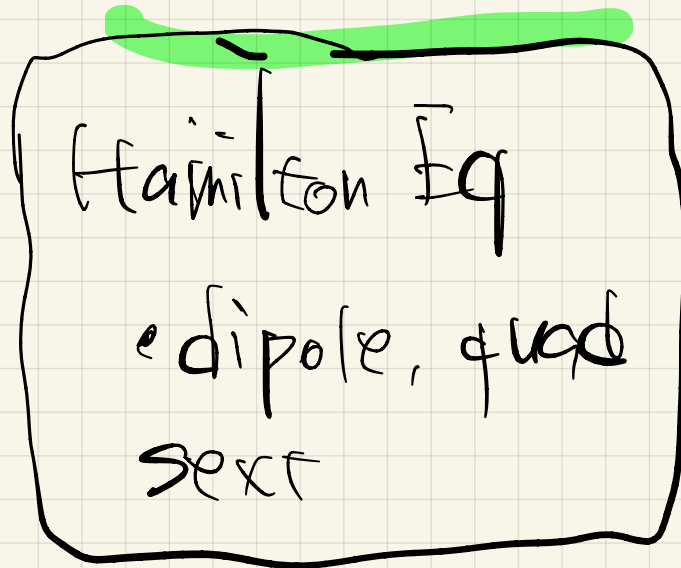
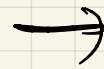
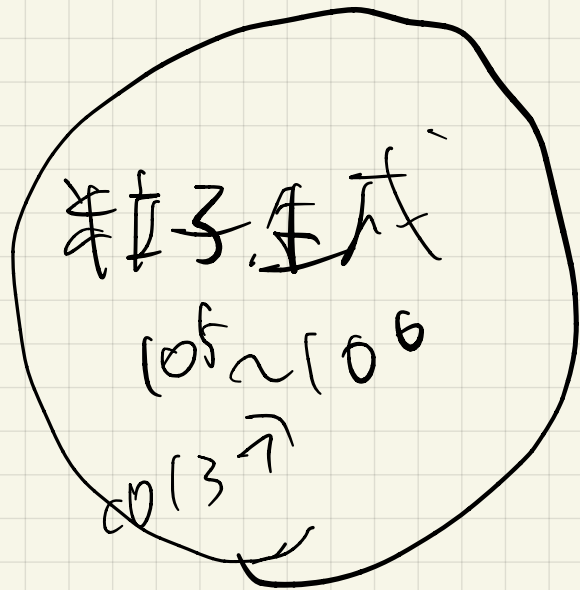
V_2 のパルス数 $I = f$, 2変数

→ 4本程 入射時に

4パルス変えられ

Multi Particles と 2° の \vec{F} に $\vec{v} = \vec{v}_0 + \vec{v}_1 + \vec{v}_2 + \dots$

Particle in Cell PIC



$\Delta S = \ln \frac{K}{F}$

J-PARC 1507.5m

2000 A / 1周

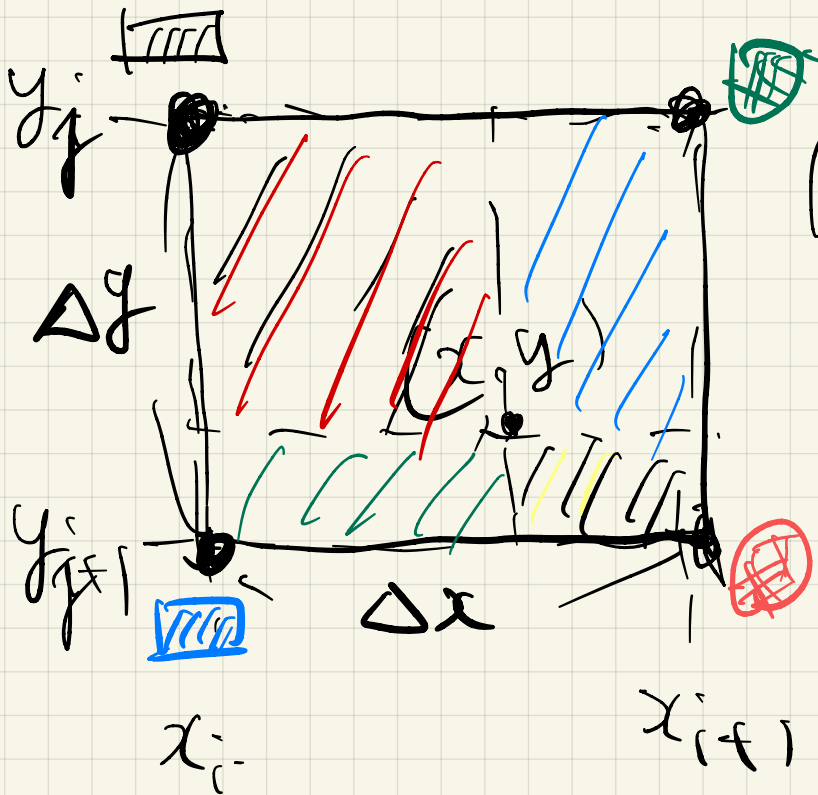
100ms \rightarrow 20000 turn

Charge 分布

直線座標

$i = 0, \dots, 99$

$j = 0, \dots, 99$



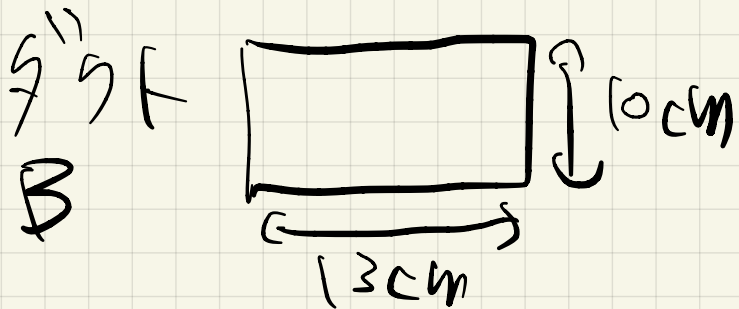
$$\rho(x, y) = \rho_{ij}$$

$$\rho(x_i, y_j) = e \frac{(x_{i+1} - x)(y_{j+1} - y)}{\Delta x \Delta y}$$

$$\rho(x_i, y_{j+1}) = e \frac{(x_{i+1} - x)(y - y_j)}{\Delta x \Delta y}$$

$$\rho(x_{i+1}, y_j) = e \frac{(x - x_i)(y_{j+1} - y)}{\Delta x \Delta y}$$

$$\rho(x_{i+1}, y_{j+1}) = e \frac{(x - x_i)(y - y_j)}{\Delta x \Delta y}$$

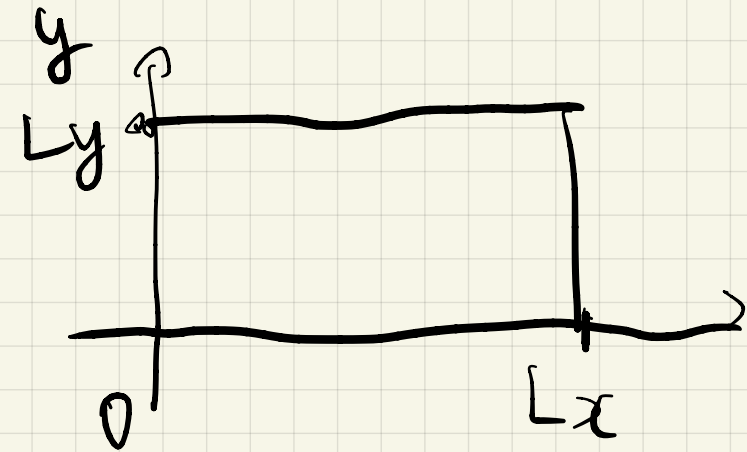


→ 7" 11" 1" 2" (mm)

11° $P \Delta =$ 方程式 2次元の場合.

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) u(x, y) = f(x, y)$$

境界条件付きの解 \subset .



$$\begin{aligned} u(x, Ly) &= u(x, 0) \\ &= u(Lx, y) = u(0, y) = 0 \end{aligned}$$

と可なり.

2
19世紀フーリエ変換 (DFT) Discrete Fourier

Transform

$$\rightarrow \hat{V}_{PA} = \sum_{l=0}^{N-1} \sum_{l'=0}^{N-1} e^{-i\frac{Pl}{N}T} e^{-i\frac{Pl'}{N}T} V_{l,l'} \quad N^2 \text{点}$$

↑ 計算量が膨大すぎる

DFT FFT (Fast Fourier Transform)

↑ 高速アルゴリズム 4 C++ python

$$O(N^4) \rightarrow (N^2 \log N^2)$$

$\hat{x} - \hat{y}$

$$\int \left(\frac{d}{dx} f(x) \right) e^{-i\omega x} = - \int f(x) e^{-i\omega x}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) u(x, y) = f(x, y) \quad \in \mathbb{R}^2$$

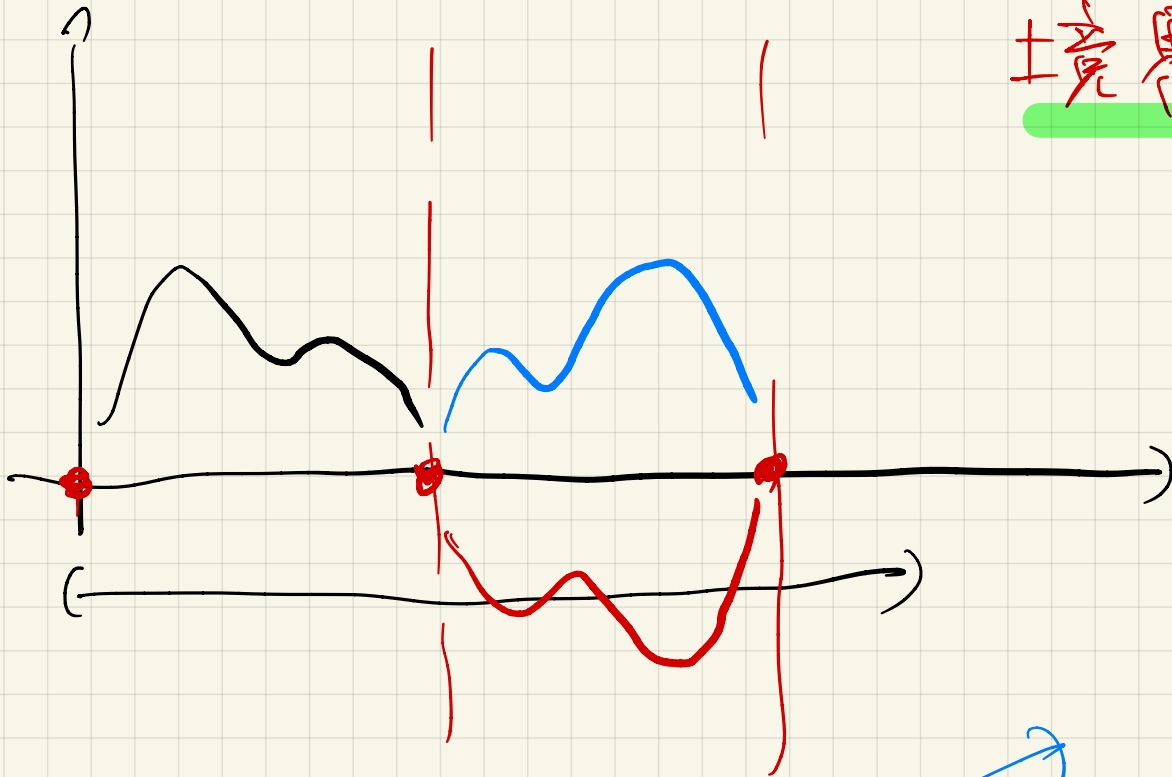
$$\frac{dG}{dt} = \frac{G_{i+1} - G_{i-1}}{\Delta t}$$

$$\rightarrow \frac{u_{i-1, j} - 2u_{i, j} + u_{i+1, j}}{\Delta x^2} + \frac{u_{i, j-1} - 2u_{i, j} + u_{i, j+1}}{\Delta y^2} = f_{i, j}$$

$i, j = 1, 2, \dots, m$ $\neq i = 0, 3$

Odd extension $u \rightarrow V \quad f \rightarrow F$

奇函数数据表



even extension \rightarrow

边界条件

$$u(0) = u(L) = 0$$

Dirichlet 条件

$$u'(0) = u'(L) = 0$$

$1/2 = \text{条件}$

$\vec{v} =$

$$\begin{pmatrix}
 0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\
 0 & u_{r1} & u_{r2} & \dots & u_{r,m} & 0 & -u_{r,m} & \dots & -u_{r1} \\
 \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
 0 & u_{m,r1} & u_{m,r2} & \dots & u_{m,m} & 0 & -u_{m,m} & \dots & -u_{2r} \\
 0 & 0 & 0 & \dots & 0 & 0 & \dots & \dots & 0 \\
 0 & -u_{m,r1} & u_{r2} & \dots & -u_{m,m} & 0 & u_{m,m} & \dots & -u_{m,r1} \\
 0 & -u_{m-1,r1} & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
 0 & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
 0 & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
 0 & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
 -u_{r1} & -u_{r2} & \dots & -u_{r,m} & 0 & u_{r,m} & \dots & \dots & u_{r1}
 \end{pmatrix}$$

$$l, l' \quad l = 0, \dots, 2m-1, 2m, 2m+1$$

$$l' = 0, \dots, 2m-1, 2m, 2m+1$$

$$\frac{\psi_{l-1, l'} - 2\psi_{l, l'} + \psi_{l+1, l'}}{\delta x^2} + \frac{\psi_{l, l'-1} - 2\psi_{l, l'} + \psi_{l, l'+1}}{\delta y^2} = F_{l, l'}$$

$$\psi_{0, l'} = \psi_{l, 0} = \psi_{m+1, l'} = \psi_{l', m+1} = 0$$

$$\text{DFT}_l (g_l)_{\mathbf{r}} = \sum_{l=0}^{N-1} g_l e^{-i \frac{2\pi \mathbf{r} \cdot \mathbf{l}}{N}}$$

$$N = 2m+1 + 1$$

$$= 2(m+1)$$

$$\text{DFT}_l (V_{l-1,l'} - 2V_{l,l'} + V_{l+1,l'})_R$$

$$= U_{1,l'} + e^{-i \frac{2\pi R}{2(m+1)}} (-2U_{1,l'} + U_{2,l'})$$

$$+ e^{-i \frac{2\pi \cdot 2R}{2(m+1)}} (U_{1,l'} - 2U_{2,l'} + U_{3,l'}) - \dots$$

$$= 0 + U_{1,l'} \left(1 - 2e^{-i \frac{2\pi R}{2(m+1)}} + e^{-i \frac{4\pi R}{2(m+1)}} \right) =$$

$$+ U_{2,l'} \left(e^{-i \frac{2\pi R}{2(m+1)}} - 2e^{-i \frac{2\pi \cdot 2R}{2(m+1)}} + e^{-i \frac{6\pi R}{2(m+1)}} \right)$$

||

$$= 4 \sin^2 \frac{\pi R}{2(m+1)}$$

$$\text{DFT}_l (g_l)_R = \sum_{l=0}^{N-1} g_l e^{-i \frac{2\pi R l}{N}}$$

$$N = 2m + 2$$

$$\begin{aligned}
&= -4 \sin^2 \frac{R\pi}{2(m+1)} \left(u_1 e^{i\pi} + u_2 e^{i\frac{2\pi R}{2(m+1)}} \right. \\
&\quad \left. + u_3 e^{i\frac{4\pi R}{2(m+1)}} + \dots \right) \\
&= \underline{-4 \sin^2 \frac{R\pi}{2(m+1)}} \sum_{l=0}^{2m+1} \underline{e^{-i\frac{Rl}{2(m+1)} 2\pi} V_{l,l'}}
\end{aligned}$$

$$\text{DFT}_{l'} \left(\text{DFT}_l \left(V_{l-1,l'} - 2V_{l,l'} + V_{l+1,l'} \right) \right) \&$$

$$= -4 \sin^2 \frac{R\pi}{2(m+1)} \cdot \text{DFT}_{l'} \left(\text{DFT}_l \left(V_{l,l'} \right) \right) \&$$

$$\text{DFT}_{l'} \left(\text{DFT}_l \left(\frac{V_{l-1,l'} - 2V_{l,l'} + V_{l+1,l'}}{\delta x^2} + \frac{V_{l,l'+1} - 2V_{l,l'} + V_{l,l'+2}}{\delta y^2} \right) \right)_{p,q}$$

$$= -4 \left(\frac{1}{\delta x^2} \frac{\sin^2 \frac{p\pi}{2(m+1)}}{\sin \frac{p\pi}{2(m+1)}} + \frac{1}{\delta y^2} \frac{\sin^2 \frac{q\pi}{2(m+1)}}{\sin \frac{q\pi}{2(m+1)}} \right) \text{DFT}_{l'} \left(\text{DFT}_l (V_{l,l'}) \right)_{p,q}$$

$$= \text{DFT}_{l'} \left(\text{DFT}_l (F_{l,l'}) \right)_{p,q}$$

$$\text{DFT}_L(\text{DFT}_L(\text{F}_{rel})_P)_Q$$

$$= \frac{\text{DFF}_L(\text{DFT}_L(\text{F}_{rel})_P)_Q}{-4 \left(\frac{1}{8^2} \sin^2 \frac{P\pi}{2(m+1)} + \frac{1}{8^2} \sin^2 \frac{Q\pi}{2(m+1)} \right)}$$

$$i \text{DFT}_P(Q)_L = \frac{1}{2\pi} \sum_{p=0}^{N-1} G_p e^{\frac{i2\pi pL}{L}}$$

$$\vec{T}_{m,m'} = -i \text{DFT}_P(i \text{DFT}_Q \left(\begin{matrix} \phantom{\vec{T}_{m,m'}} \\ \phantom{\vec{T}_{m,m'}} \\ \phantom{\vec{T}_{m,m'}} \end{matrix} \right)_{m,m})$$

$$\begin{aligned}
 u(x, 0) &= f_1(x) \\
 u(x, L) &= f_2(x) \\
 u(0, y) &= f_3(y) \\
 u(L, y) &= f_4(y)
 \end{aligned}$$

境界条件が
任意の関数であって

$$\frac{\cancel{u_{0,l}'} - 2u_{1,l}' + u_{2,l}'}{\delta x^2} + \frac{u_{1,l'-1} - 2u_{1,l}' + u_{1,l'+1}}{\delta y^2} = F_{1,l}'$$

↑
0
対称

$$\frac{0 - 2u_{1,l}' + u_{2,l}'}{\delta x^2} + \frac{u_{1,l'-1} - 2u_{1,l}' + u_{1,l'+1}}{\delta y^2}$$

→ 境界値 0 のときは

$$= F_{1,l}' - \frac{u_{0,l}'}{\delta x^2}$$
 ← 電荷分布
 11030

Free Space の解

$$\Delta G(\vec{r}) = \delta(r)$$

$$G(r, \vec{r}) = \frac{1}{2\pi} \log(\vec{r} - \vec{r}')$$

引数 → 距離

$$\underbrace{\nabla \int \rho(\vec{r}') G(\vec{r} - \vec{r}') d\vec{r}'}_{\Phi} = \rho(r)$$

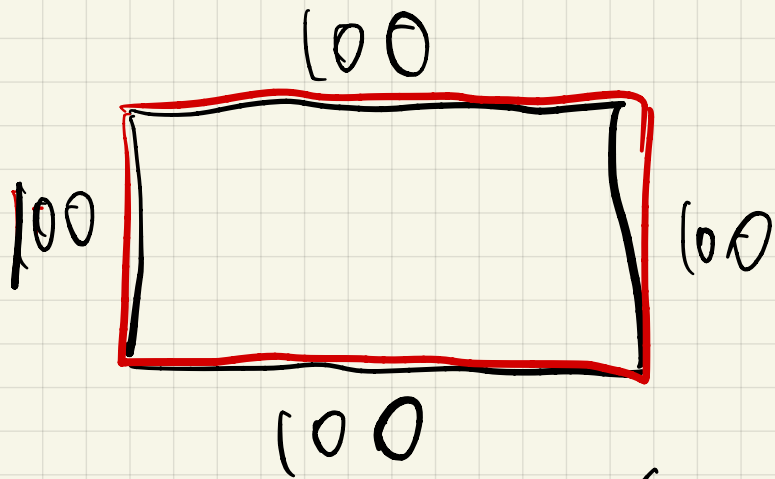
$$\Phi = \int \rho(\vec{r}') G(\vec{r} - \vec{r}') d^3\vec{r}'$$

$$\Phi_{m,n} = \sum \rho_i r_i \frac{1}{2\pi} \log \sqrt{(x_n - x_i)^2 + (y_n - y_i)^2}$$

$$m, n = 100 \times 1000$$

$$i, j = 100 \times 100$$

10000 x ~~(10000)~~
 回の計算
 → 時間 O()



境界 f = 1 +

グリーン関数を用いる

$O(400 \times \cancel{10000})$

↓
 $v(x, 0)$ $v(0, x)$
 $v(x, L_y)$ $u(0, L_x)$

* $O(10000 \log 10000)$

↳ DFTを用いた
 解法

$$\Phi(x, y) \leftrightarrow U_{i,j}$$

$$\lambda \succ \lambda' \Leftrightarrow \Phi(x, y)$$

$$\sum f(x, y)$$

$$\Rightarrow F \propto -\nabla \Phi(x, y)$$

$$i, j+2 \quad i+1, j+2$$

$$i, j+1 \quad i+1, j+1 + i+2, j+1$$

$$i, j \quad i+1, j \quad i+2, j$$

$$\Phi(x, y) = \sum_{m=0}^3 \sum_{n=0}^3 U_{i+m-1, j+n-1} \frac{3!}{m!(3-m)!} \frac{3!}{n!(3-n)!} \left(\frac{x-x_i}{\delta x} \right)^m \left(\frac{y_{i+n}-y}{\delta y} \right)^{3-n}$$

木を便、 τ

$$\vec{F} \propto -\text{grad } \Phi(x, y)$$

($= F^c$) $\vec{F} \propto \vec{\tau} \in \vec{\tau} = \vec{\tau}$ 3

$$\phi(x, y) = \int_0^{\infty} dt \frac{\exp\left(-\frac{x^2}{2\sigma_x^2 + t} - \frac{y^2}{2\sigma_y^2 + t}\right)}{\sqrt{(2\sigma_x^2 + t)(2\sigma_y^2 + t)}}$$

これは、

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\phi = -\frac{\rho(x, y)}{\epsilon_0}$$

$$\rho(x, y) = \frac{Ne}{2\pi\sigma_x\sigma_y} e^{-\frac{x^2}{2\sigma_x} - \frac{y^2}{2\sigma_y}}$$

$$\Delta G(\vec{r}) = -\rho(\vec{r} - \vec{r}')$$

$$G(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{1}{|\vec{r} - \vec{r}'|}$$

$$\rho(\vec{r}') \Delta G(\vec{r}) = -\rho(r') \delta(\vec{r} - \vec{r}')$$

$$\Delta \underbrace{\int d\vec{r}' \rho(\vec{r}') \cdot G(\vec{r})}_{\phi} = -\rho(r)$$

$$\frac{1}{4\pi|\vec{r}-\vec{r}'|} = \frac{1}{2\pi^{\frac{3}{2}}} \int_0^{\infty} dq e^{-|\vec{r}-\vec{r}'|^2 q^2}$$

$$\text{für } r = r' \Rightarrow dt = -\frac{2}{q^3} dq \quad \frac{1}{2\sqrt{\pi}} \frac{1}{|\vec{r}-\vec{r}'|}$$

$$\frac{1}{4\pi|\vec{r}-\vec{r}'|} = \frac{1}{4\pi^{\frac{3}{2}}} \int \frac{1}{t^{\frac{3}{2}}} e^{-\frac{|\vec{r}-\vec{r}'|^2}{t}} dt$$

$$\Phi = \frac{1}{4\pi^{\frac{3}{2}} \epsilon_0} \int_0^{\infty} dt \frac{1}{t^{\frac{3}{2}}} \int dV' \lambda \frac{Ne}{2\pi\sigma_x\sigma_y} e^{-\frac{x^2}{2\sigma_x} - \frac{y^2}{2\sigma_y}} e^{-\frac{|\vec{r}-\vec{r}'|^2}{t}} dt$$

$$\phi = \frac{1}{4\pi \epsilon_0} \int_0^{\infty} dt \int_{-b}^b dx' e^{-\frac{x^2}{2\sigma_x} - \frac{(t-x')^2}{t}} \int_{-b}^b dy' \quad \text{y version}$$

からの積分

これは二重積分