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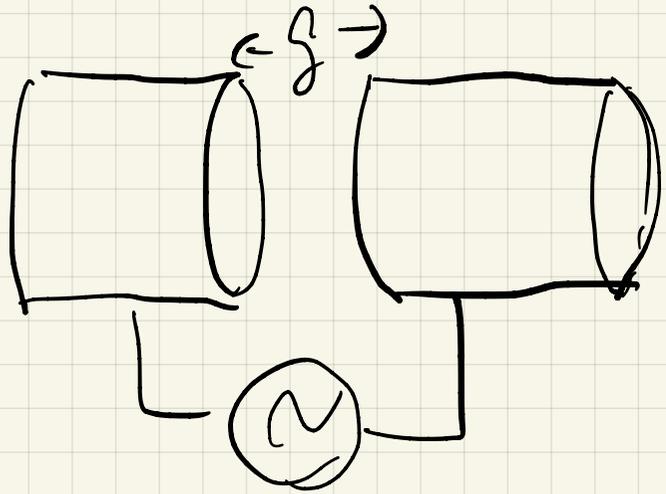
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# 縦方向の運動方程式 radio frequency



$f_{rev} \gg \text{RF 電圧の周期}$   
 加速した

$$E = E_0 \sin(2\pi f_{rf} t + \phi_s)$$

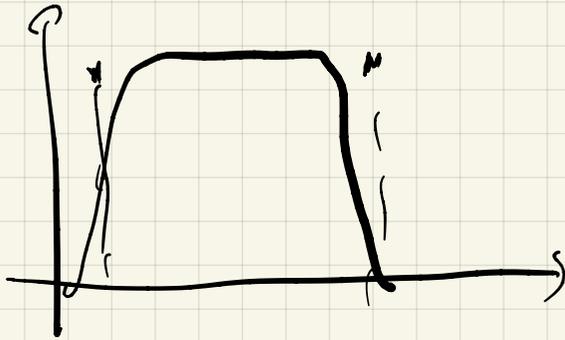
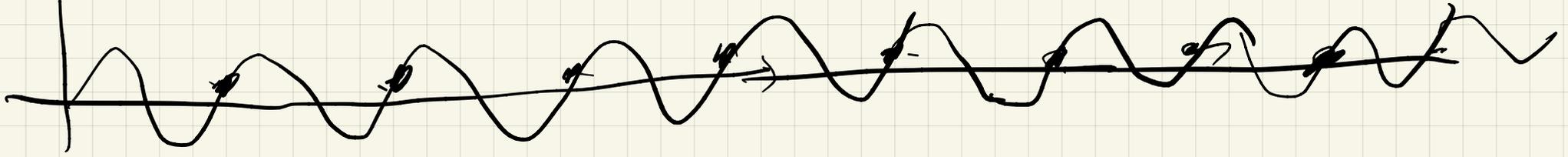
$$f_{rev} = \frac{\beta_0 c}{2\pi R_0}$$

$$2\pi R_0 = \underbrace{\text{周長}}_{\text{Average Radius}}$$

$$f_{rf} = \underbrace{h}_{\text{harmonic number}} c f_{rev} = \frac{h \beta_0 c}{2\pi R_0}$$

# $h=9$ J-PARC Main Ring

8 bunches  $\rightarrow$  7.5 GeV の正電子束の L' の軌道



17 -> 2 1/4 3 4 3 I \* r # ( 10 x 7 3 1 )

$$\Delta E = e E_0 \int_{-\frac{g}{2}}^{\frac{g}{2}} ds \sin \left( 2\pi f_{rf} \frac{s}{\beta_0 c} + \phi_s \right)$$

$$s = \beta_0 c t = e E_0 \frac{\beta_0 c}{\pi f_{rf}} \frac{\sin \pi f_{rf} g}{\beta_0 c} \sin \phi_s = e E_0 g \sin \phi_s \times T$$

$$T = \frac{\sin \frac{\pi f_{rf} g}{\beta_0 c}}{\frac{\pi f_{rf} g}{\beta_0 c}}$$

$$= e V \sin \phi_s$$

Effective Voltage

$$\frac{\sin x}{x} = 1 \quad x \ll 1$$

$$c(\alpha + \beta) - c(\alpha - \beta) = -2 \sin \phi$$

毎ターン  $\rightarrow V \sin \phi_s$  I のエネルギー損失 [GeV]

$$1 \text{ ターン} \rightarrow \frac{1}{f_{rev}} = \frac{2\pi R_0}{\beta_0 c} \text{ 秒} \quad V = 200 \text{ kV}$$

$$R_0 = 200 \text{ m}$$

$$\beta_0 = 1$$

$$\phi_s = 30^\circ$$

$$\frac{\beta_0 c}{2\pi R_0} V \sin \phi_s / \text{秒}$$

$$\sim 25 \text{ GeV / 秒}$$

1.4 秒  $\rightarrow$  3  $\rightarrow$  30 GeV J-PARC MR

1) → 4大数  $f_0$  のは  $\frac{1}{T_0} \int_0^{T_0} \phi(t) dt$  の平均値

粒子の方程式

synchrotron particle.

$$f_{rf} = f_0 + \Delta f$$

$$\phi = \phi_s + \Delta \phi$$

$$p = p_0 + \Delta p \quad E = E_0 + \Delta E$$

$$\omega_0 = \frac{\beta_0 c}{R_0}$$

$$\omega_{rev} = 2\pi f_{rev} = \omega_0 + \Delta \omega$$

$$\frac{d}{dt} \left( \frac{\Delta E}{\omega_0} \right) = \frac{1}{\omega_0} (\dot{E} - \dot{E}_0) - \frac{\dot{\omega}}{\omega_0^2} (E - E_0)$$

$$\frac{\Delta \omega}{\Delta t}$$

$$= \frac{1}{\omega_0} (\dot{E} - \dot{E}_0) - \frac{\Delta \omega}{\omega_0^2} \frac{\Delta E}{\Delta t} \cdot E$$

$$= \left( \frac{1}{\omega_0} - \frac{\Delta \omega}{\omega_0^2} \right) \dot{E} - \frac{\dot{E}_0}{\omega_0}$$

$$\frac{\omega_0 \left( 1 - \frac{\Delta \omega}{\omega_0} \right)}{\omega_0^2} = \frac{1}{\omega_0 \left( 1 + \frac{\Delta \omega}{\omega_0} \right)} \approx \frac{1}{\omega_0 + \Delta \omega} \approx \frac{1}{\omega}$$

$$\approx \frac{\dot{E}}{\omega} - \frac{\dot{E}_0}{\omega_0} = \frac{1}{2\pi} eV (\sin \phi - \sin \phi_s)$$

$$\frac{d}{dt} \left( \frac{\Delta E}{\omega_0} \right) = \frac{1}{2\pi} e \Gamma (\sin \phi - \sin \phi_s)$$

$$\frac{\Delta p}{p} = \delta = \frac{\Delta E}{\beta^2 E}$$

$$p = m \gamma v$$
$$E = m \gamma c^2$$

$$\delta = \frac{\omega_0}{\beta^2 E} \frac{\Delta E}{\omega_0}$$

$$\delta = \frac{\omega_0}{2\pi \beta^2 E} e \Gamma (\sin \phi - \sin \phi_s)$$

↪  $\phi_s < 1$  と仮定

$$\dot{\phi} = -h(\omega - \omega_0) = -h \Delta \omega$$

$$\frac{\omega R}{\omega_0 R_0} = \frac{\beta}{\beta_0}$$

$$\frac{\Delta \omega}{\omega_0} = \frac{\omega}{\omega_0} - 1$$

$$\omega_0 = 2\pi \frac{\beta_0 c}{2\pi R_0}$$

$$= \frac{R_0 \beta}{R \beta_0} - 1$$

$\Rightarrow$   
 $\left\{ \begin{array}{l} \text{速度: 丈 } \beta > \beta_0 \\ \Rightarrow \omega \text{ 大 } \pm \text{ なる} \\ \text{周回が速い} \end{array} \right.$

$R$  は  $\frac{v}{\omega}$  の  $\frac{1}{\omega}$  成分

$\left\{ \begin{array}{l} R: \text{ 丈 } R > R_0 \\ \text{周回が速い} \end{array} \right.$

$$R = R_0 (1 + \alpha_0 \delta + \alpha_1 \delta^2 \dots)$$

$$\alpha_c = \frac{1}{R_0} \frac{dR}{dS} = \alpha_0 + \frac{1}{r_0^2} S$$

↳ Momentum Compression factor

$$S = \frac{P - P_0}{P_0} = \frac{\beta r}{\beta_0 r_0} - 1 \quad \frac{\beta}{\beta_0} = \frac{1 + S}{\sqrt{1 + 2S\beta_0^2 + S^2\beta_0^2}}$$

$$\frac{\Delta \omega}{\omega_0} = (1 - \alpha_0 S) \left(1 + \frac{1}{r_0^2}\right) S = 1 + \frac{1}{r_0^2} S \dots$$

$$= -\left(\frac{1}{r_0^2} - \frac{1}{r_0}\right) S$$

$$\Delta \omega = -\omega_0 \left( \frac{1}{r_T^2} - \frac{1}{r_0^2} \right) \delta$$

slippage

$r_0 > r_T$  運動量が大きいほど周回が速い  
 $r_0 < r_T$  速い...

$r_T =$  純虚数 J-PTRC main

加速の途中  $r_0 < r_T \rightarrow r_0 > r_T$

加速

"

$$\dot{\phi} = -h \omega_0 \left( \frac{1}{r_T^2} - \frac{1}{r_0^2} \right) \delta$$

$$= h \omega_0^2 \eta \frac{1}{\beta^2 E} \frac{\Delta E}{\omega_0}$$

$$\eta = \frac{1}{r_T^2} - \frac{1}{r_0^2} \quad \text{slippage}$$

J-PARC MR.  
 $\eta < 0$

$$\dot{\phi} = \frac{\partial H}{\partial S} \quad \dot{\delta} = - \frac{\partial H}{\partial \phi} \quad 2_T = \delta$$

$$H = \frac{\hbar \omega_0}{2\pi \beta^2 E} eV (\cos \phi + \phi \sin \phi) + \frac{1}{2} h \omega_0 \eta \delta^2$$

$$\phi - \phi_s = \Delta\phi$$

$$\cos(\Delta\phi + \phi_s) + (\phi_s + \Delta\phi) \sin\phi_s$$

$$= \left(1 - \frac{\Delta\phi^2}{2}\right) \cos\phi_s - \Delta\phi \sin\phi_s + (\phi_s + \Delta\phi) \sin\phi_s$$

$$= -\cos\phi_s \frac{\Delta\phi^2}{2}$$

$$H = \frac{1}{2} h \omega_0 g^2 - \frac{\omega_0}{4\pi\beta^2 I} eV \cos\phi_s \frac{\Delta\phi^2}{2}$$

$$\eta < 0 \quad \cos \phi_S > 0 \quad -\frac{\pi}{2} < \phi_S < \frac{\pi}{2}$$

$$\gamma_0 < \gamma_T$$

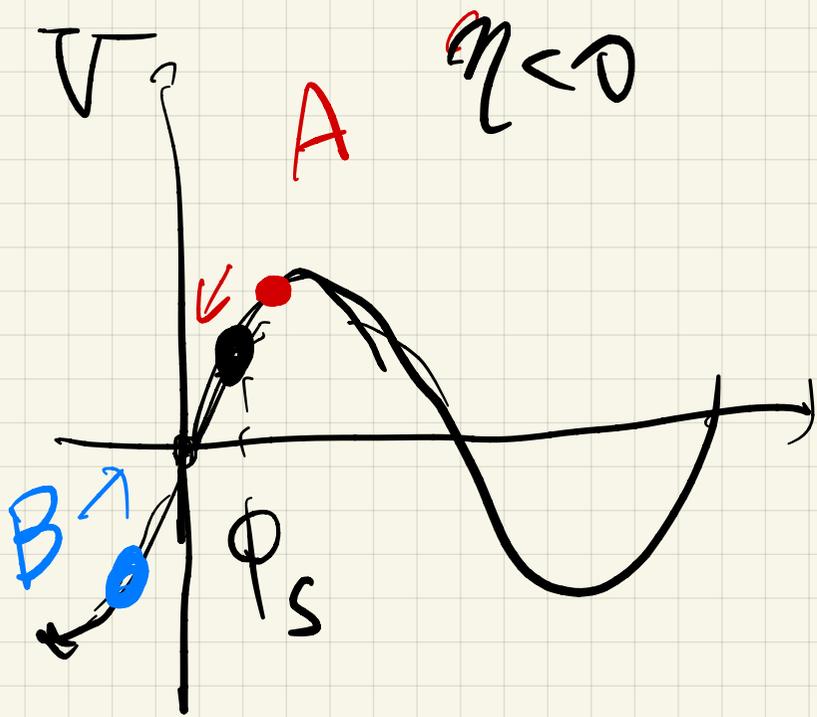
$$\eta > 0 \quad \cos \phi_S < 0 \quad = \frac{\pi}{2} \leq \phi_S \leq \pi$$

$$\gamma_0 > \gamma_T$$

$$\pi \leq \phi_S \leq \pi$$

η 符号

$$\Delta \phi = \frac{h \omega_0 \eta e V}{2 \pi \beta^2 E} \cos \phi_S \Delta \phi$$



$A \quad \eta < 0$

粒子 A

同位相

= 運動量

→  $\phi_s$  の値は 3.5V 以下  
加速電圧

粒子 B

同位相

= 運動量大

→  $\phi_s$  の値は 3.5V 以上  
加速電圧

結果として

$\phi_s$  値が 3.5V 以下

(運動解)

これを  $\gamma = \beta c$  の振動

$$h = 9 \\ v = 150 \text{ km} \left. \begin{array}{l} \text{JPARC} \\ \text{MR} \end{array} \right\}$$

$\gamma = \beta c$  の角周波数

$$R_s = \frac{h \omega_0^2 |q| \cos \theta_s l e V}{2 \pi \beta^2 E}$$

$$E = \gamma m c^2$$

粒子の速度  $v = \beta c$

中の電磁振動

J-PARC MR  $\gamma = 2$  と

$$\beta_0 = 0.97$$

$$3 \text{ GeV} \frac{R_s}{2 \pi} = 3 \text{ GeV Hz}$$

$$\frac{R_s}{\omega_0} = 0.002$$

$$\text{周回周波数} \frac{\beta_0 c}{15.875} = 48.5 \text{ kHz}$$

5007 1 回周動

$$\dot{\phi} = 0 \quad \dot{\delta} = 0$$

$$\hookrightarrow h\omega_0 \left( \frac{1}{r_T^2} - \frac{1}{r_0^2} \right) \delta = 0$$

$$\hookrightarrow \int = \frac{\omega}{2\pi\beta^2 E} eV (\sin\phi - \sin\phi_s) = 0$$

$$\underline{\underline{\phi = \phi_s}}$$

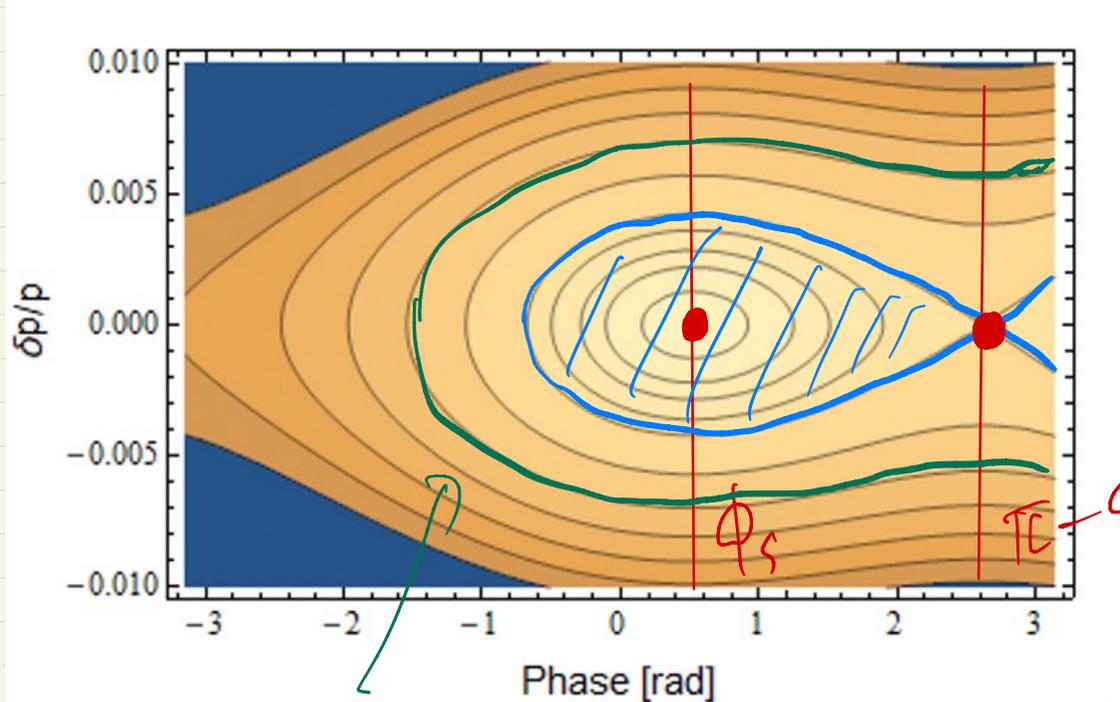
Stable fix.  
point.

$$\underline{\underline{\phi = \pi - \phi_s}}$$

unstable

fixed point

実際は Hamiltonian の等高線を書き出すと  
 不安定な固定点 (unstable fixed point) を通る等高線がある



不安定

unstable fixed point

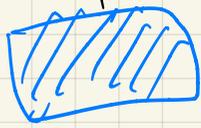
を通る等高線

separatrix

より外側の粒子は不安定

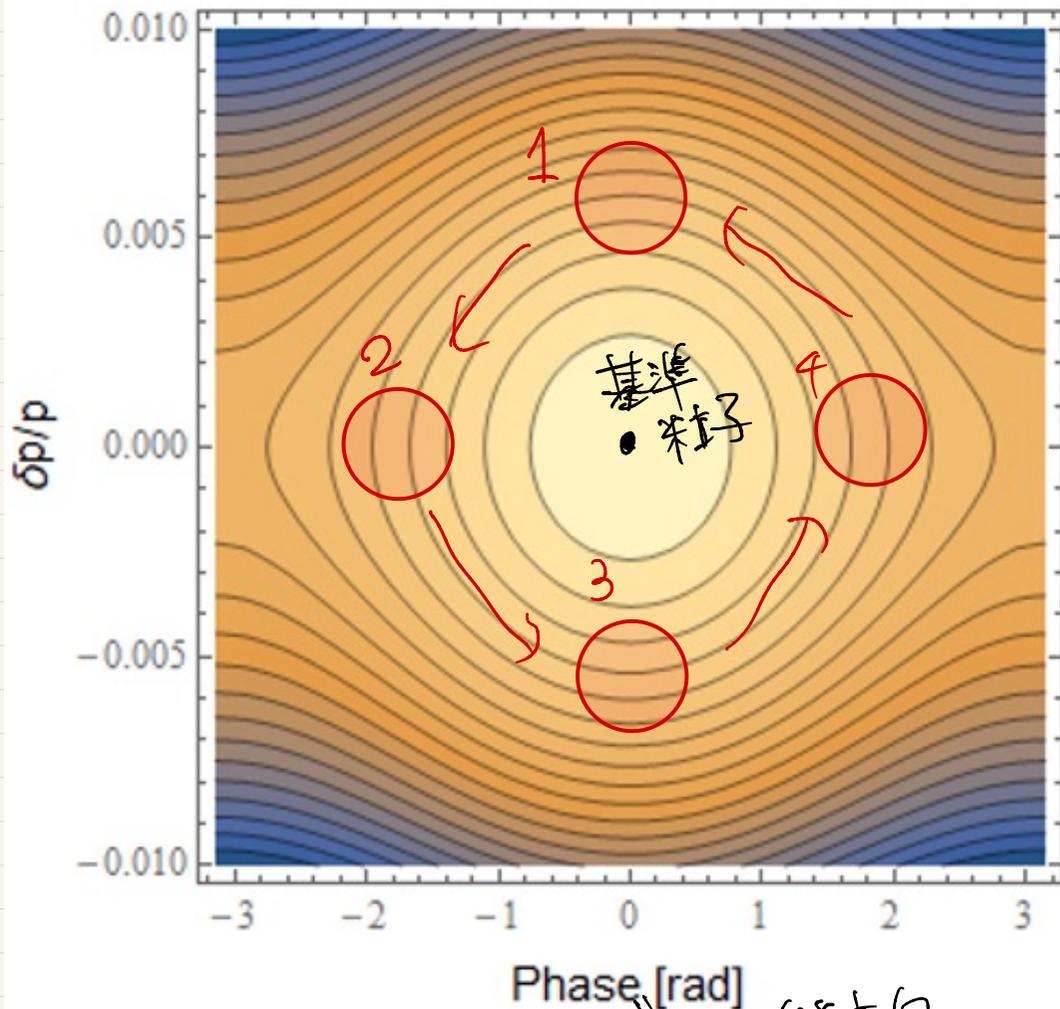
内側の粒子は安定

内側の領域を RF bucket と言う



# 安定領域の運動を定性的に調べる (1)

$\phi_s = 0$  とする (加速しない)



→  $\frac{1}{2\pi}$  縦方向位置

運動が「安定状態」

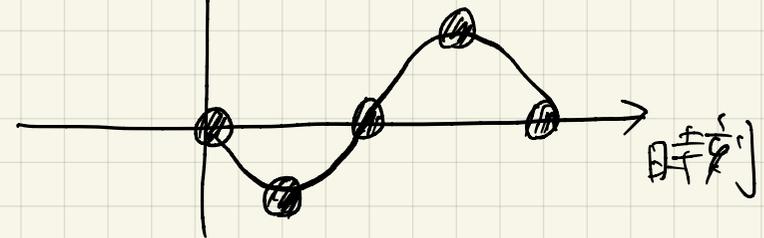
RFバケツに入射すると

H 等高線にそって ( $\phi, \delta p$ )

平面を回転する



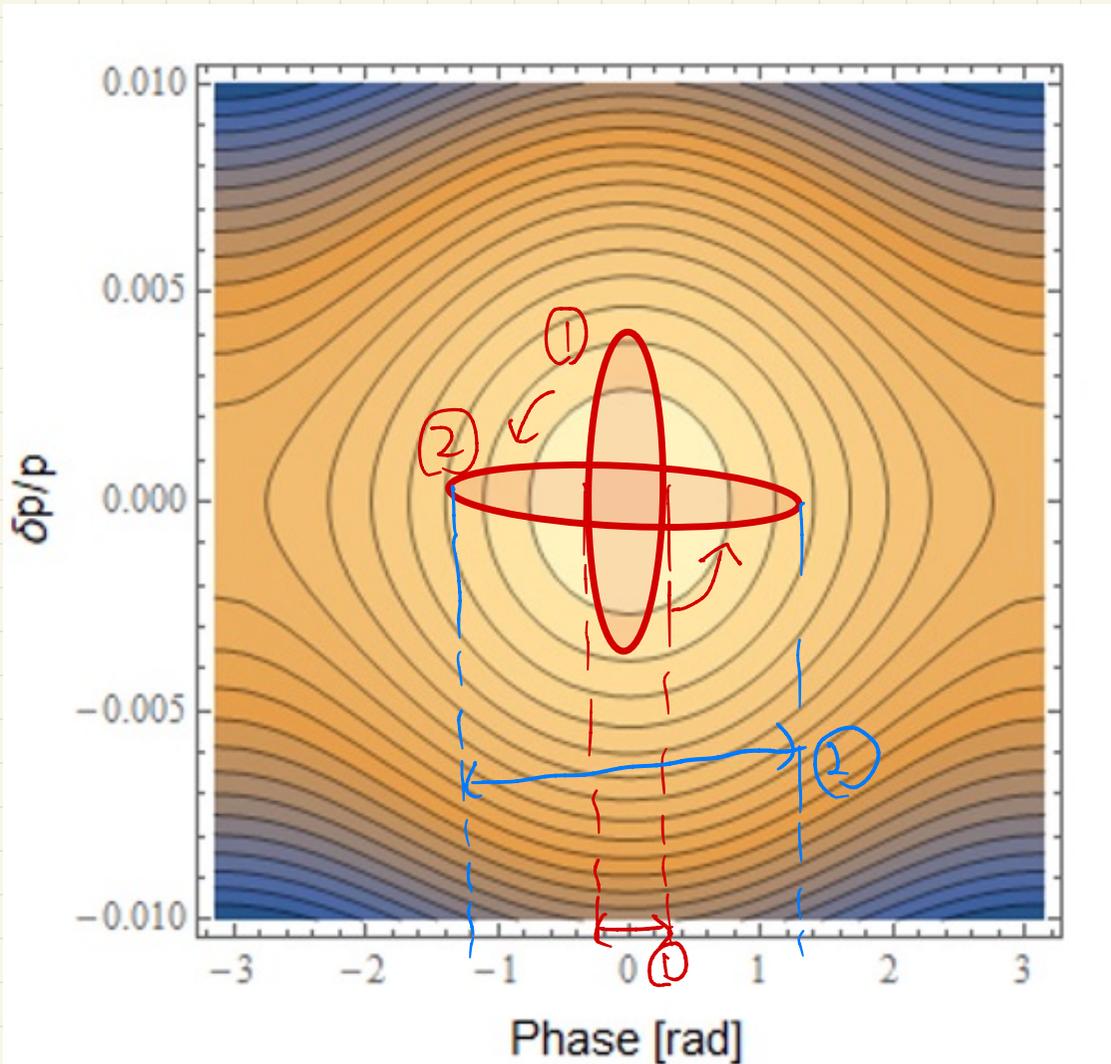
縦方向位置 (Wall Current monitor for  $\phi_s$ )



$\frac{\Omega_s}{2\pi}$  の振動が観測される

そのポールの振動

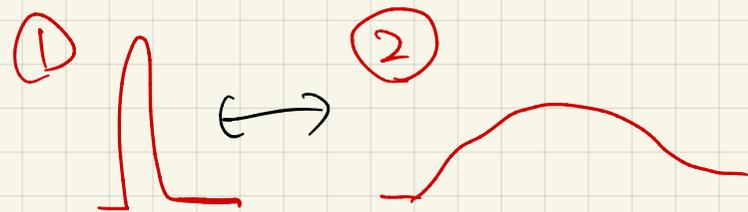
# 安定領域、運動を定性的に調べる (2)



↑ 縦方向分布

H等高線の形とちがった分布のビームを注入する

ビームの縦方向分布が  
 細くなる、たり、太くなる、たりをくり返す。  
 Wall Current Monitorの波形



振動の周期は2倍

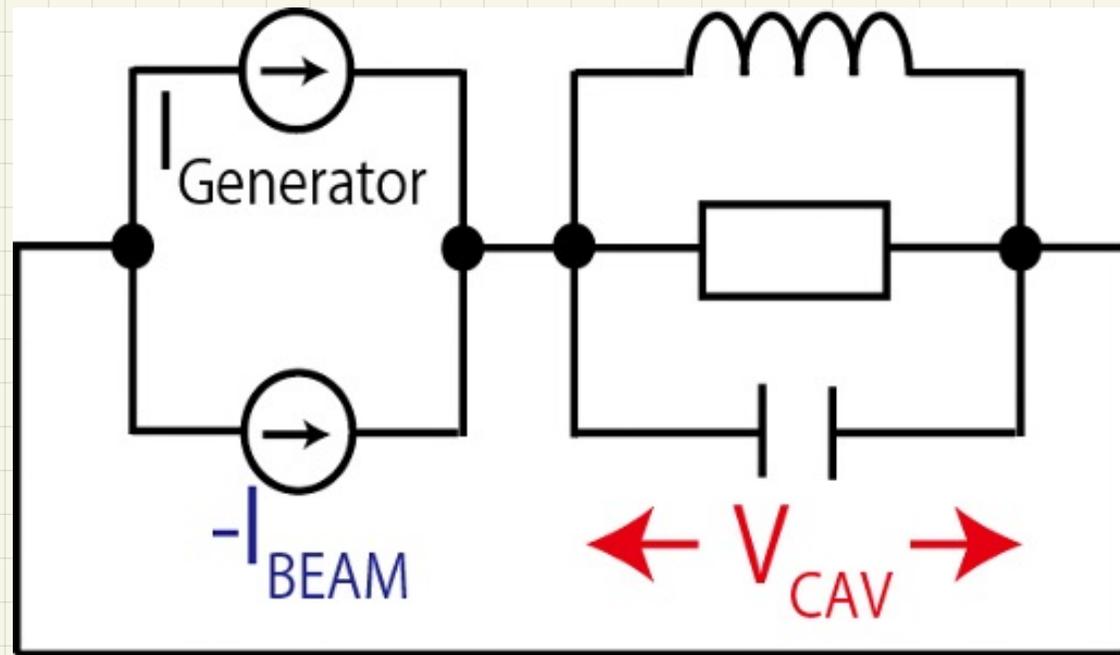
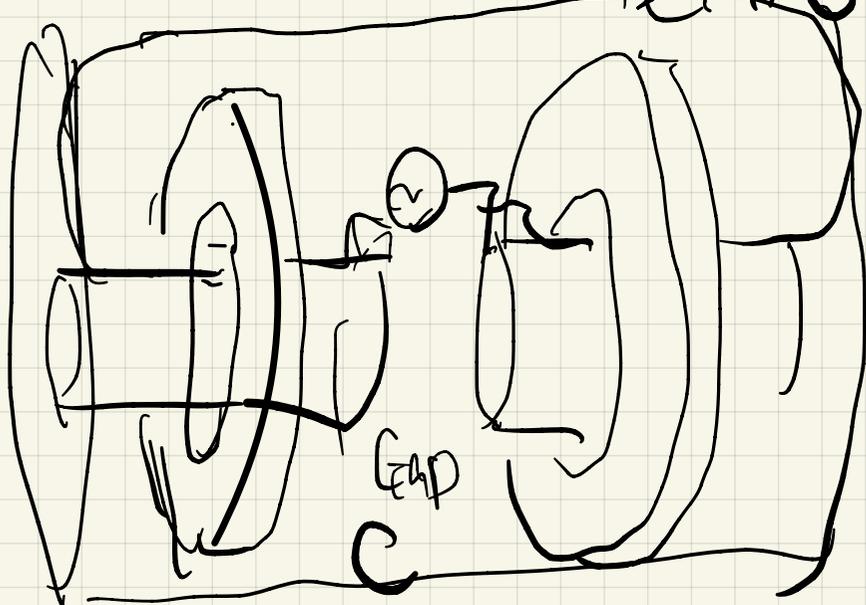
4極振動

$E'' - 4R - F''$

加速時間

→ トライアード電流

~~$E'' - 4R$  電流~~



$$\frac{1}{Z} = \left( \frac{1}{R} + \frac{1}{i\omega L} + i\omega C \right)$$

$$Z = R \frac{1}{1 + i\frac{R}{L}\omega \left( \omega^2 - \frac{1}{LC} \right)}$$

$$Z = \frac{1}{1 + i \frac{RC}{\omega} \left( \omega - \frac{1}{\sqrt{LC}} \right) \left( \omega + \frac{1}{\sqrt{LC}} \right)} \quad \omega \approx \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{1 + i \frac{2RC}{\omega} \left( \omega - \frac{1}{\sqrt{LC}} \right)}$$

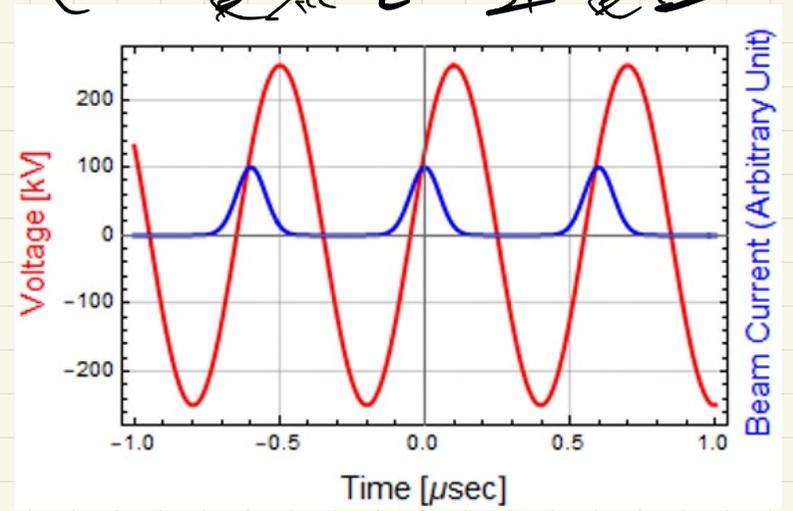
$$= \frac{1 + i 2RC \left( \omega - \frac{1}{\sqrt{LC}} \right)}{1 - i 2RC \left( \omega - \frac{1}{\sqrt{LC}} \right)}$$

$$= \frac{1 + i 2RC \left( \omega - \frac{1}{\sqrt{LC}} \right)}{1 + \left( 2RC \left( \omega - \frac{1}{\sqrt{LC}} \right) \right)^2} = \underline{\underline{R \cos \varphi e^{-i\varphi}}}$$

↳  $\tan \varphi$

$$\varphi = \arctan \frac{\varphi''}{\varphi'} = \arctan \frac{2RC}{1}$$

電流と電圧



$$V_{cav} = V_0 e^{-i(\frac{\pi}{2} - \phi_s)}$$

$$I_{beam} = I_B e^{i\phi}$$

$$\neq \vee I_{beam} \sim 0$$

$$I_0 = \frac{V}{R} e^{-i(\frac{\pi}{2} - \phi_s)} = I_0 e^{-i(\frac{\pi}{2} - \phi_s)}$$

$$V e^{-i(\frac{\pi}{2} - \phi_s)} = R I_0 e^{-i(\frac{\pi}{2} - \phi_s)}$$

$$I_{beam} \neq 0$$

この電圧は  $I_{beam}$  の何らかの  
 関数の必要がある

$I_{beam} \neq 0$  のとき

$$R I_0 e^{i(\frac{\omega}{2} - \phi_s)}$$

空間平均電流

$$= R \cos \theta e^{-i\theta} (I_G e^{i\theta} - I_B e^{i\theta})$$

↑  
RF 電源  
の電流

↑  
 $I_G$  の電流

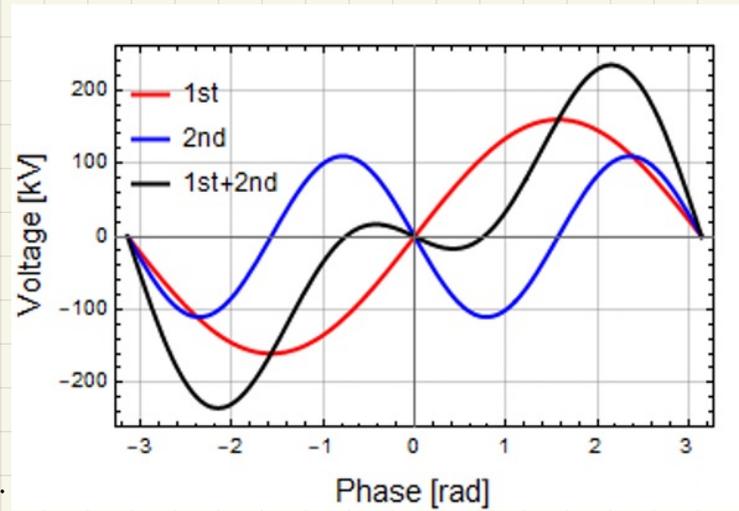
$I_G$  の電流が大きい

$I_B \rightarrow$  大

$I_G \rightarrow$  大

→  $\frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{2} V_1 \sin(\omega t) + \frac{1}{2} V_2 \sin(2\omega t) dt$  の平均値  $\bar{v}$

$$\rightarrow \frac{1}{2\pi} \int_{-\pi}^{\pi} V_2 \sin(2\omega t) dt \text{ は } 0 \text{ である (} \phi_s = 0 \text{ とする)}$$



$$\bar{v} = \frac{1}{2\pi} \int_{-\pi}^{\pi} (V_1 \sin \phi - V_2 \sin 2\phi) d\phi$$

$$V \sin \phi \rightarrow V_1 \sin \phi - V_2 \sin 2\phi \text{ の}$$

平均値は  $\frac{1}{2\pi} \int_{-\pi}^{\pi} V_1 \sin \phi d\phi$  である

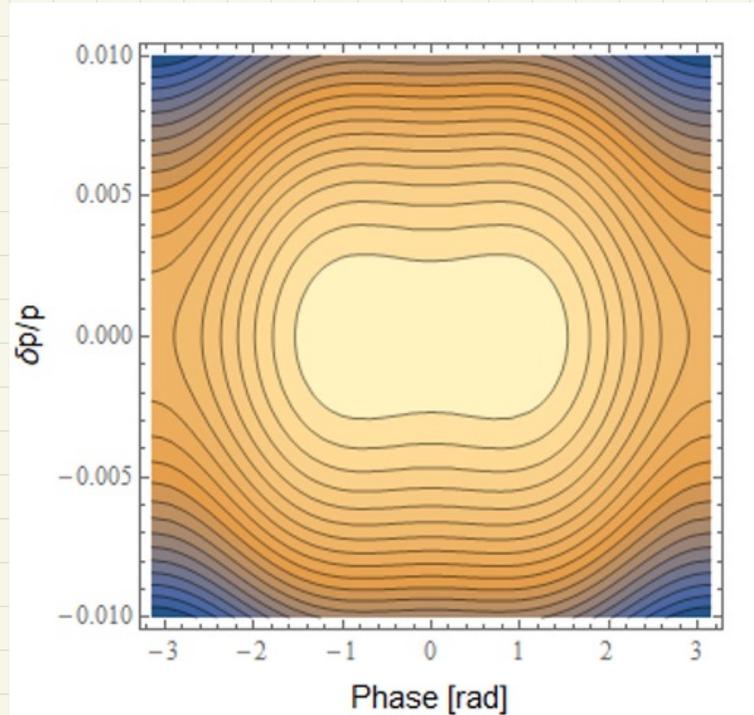
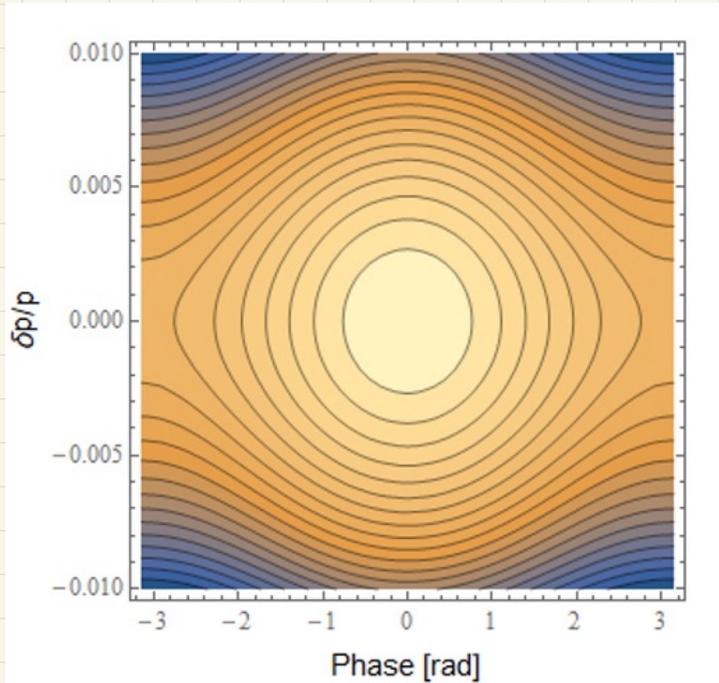
1

effective Hamiltonian

$$H = \frac{\omega_0}{2\pi\beta^2 E} e(V_1 \cos\phi - V_2 \cos 2\phi) + \frac{1}{2} \hbar \omega_0 \eta \delta^2$$

2nd order

2nd order



电荷密度在垂直方向上分布不均匀，电荷密度在平行方向上分布均匀。

電子銃の電流波形

