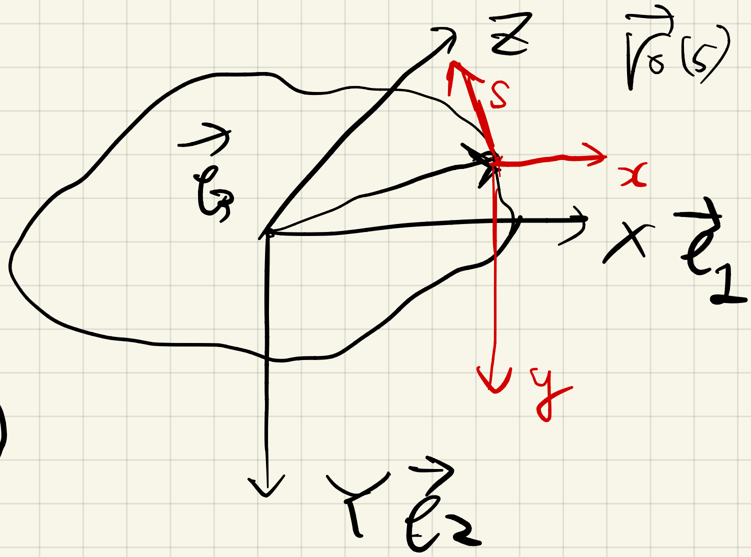
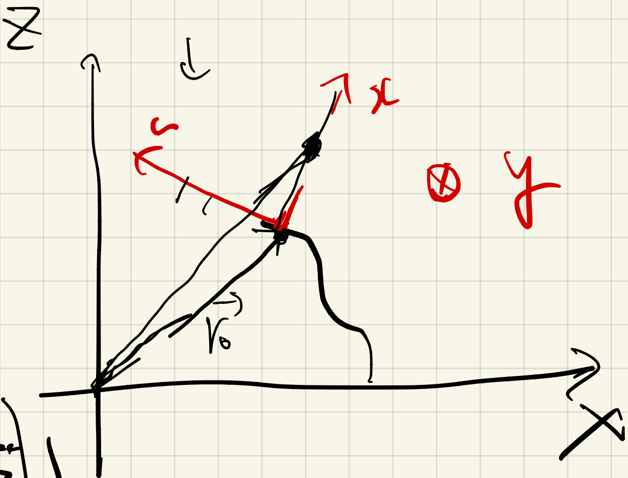
座標変換



$$\dot{\hat{y}} = 0$$

$$\vec{r}_0 = (X_0(s), 0, Z_0(s))$$

$$\hat{s} = (\dot{X}_0(s), 0, \dot{Z}_0(s))$$



$$\hat{x} = (-\rho(s)\ddot{X}_0(s), 0, -\rho\ddot{Z}_0(s))$$

$$\dot{\hat{x}} = \left(-\left(\frac{d\rho}{ds}\ddot{X}_0 + \rho\dddot{X}_0\right), 0, -\left(\frac{d\rho}{ds}\ddot{Z}_0 + \rho\dddot{Z}_0\right)\right)$$

$$\hat{x}\cdot\dot{\hat{x}} = \frac{1}{2}[(\hat{x}\cdot\hat{x})^{\cdot}] = 0$$

$$\hat{s}\cdot\dot{\hat{x}} = -\dot{\hat{s}}\cdot\hat{x} = \frac{1}{\rho(s)} \quad \text{より}$$

$$\hat{y}\cdot\dot{\hat{x}} = 0 \quad (\text{仮定})$$

$$\dot{\hat{x}} = \frac{1}{\rho(s)}\cdot\hat{s}(s)$$

$$= \left(\frac{\dot{X}_0}{\rho(s)}, 0, \frac{\dot{Z}_0}{\rho(s)}\right)$$

$$(\vec{r} - \vec{r}_0)\cdot\hat{s} = 0$$

$$\dot{X}_0(X - X_0) + \dot{Z}_0(Z - Z_0) = 0$$

まず基底$(\vec{e}_1, \vec{e}_2, \vec{e}_3) \to (\hat{x}, \hat{y}, \hat{s})$

変換則

$$\hat{x} = -\rho(s)(\ddot{x}_0 \vec{e}_1 + \ddot{z}_0 \vec{e}_3) \qquad \hat{y} = \vec{e}_2$$

$$\hat{s} = \dot{x}_0 \vec{e}_1 + \dot{z}_0 \vec{e}_3$$

$$\vec{e}_1 = \frac{\dot{z}\hat{x} + \rho \ddot{z}_0 \hat{s}}{\rho(\ddot{z}_0 \dot{x}_0 - \dot{z}_0 \ddot{x}_0)}$$

$$\vec{e}_3 = \frac{\dot{x}_0 \hat{x} + \rho \ddot{x}_0 \hat{s}}{\rho(\ddot{x}_0 \dot{z}_0 - \dot{x}_0 \ddot{z}_0)} = -\frac{\dot{x}_0 \hat{x} + \rho \ddot{x}_0 \hat{s}}{\rho(\ddot{z}_0 \dot{x}_0 - \dot{z}_0 \ddot{x}_0)}$$

$$\vec{e}_2 = \hat{y}$$

ベクトルの成分の変換則は

$$\vec{A} = A_x \hat{x} + A_y \hat{y} + A_s \hat{s}$$

$$= (-\rho(s)\ddot{x}_0 A_x + \dot{x}_0 A_s)\vec{e}_1 + A_y \vec{e}_2$$

$$(-\rho(s)\ddot{z}_0 A_x + \dot{z}_0 A_s)\vec{e}_3$$

$$= A_1 \vec{e}_1 + A_2 \vec{e}_2 + A_3 \vec{e}_3$$

$$A_1 = \left(-\rho(s)\ddot{X}_0 A_x + \dot{X}_0 A_s\right)$$

$$A_2 = A_y$$

$$A_3 = \left(-\rho(s)\ddot{Z}_0 A_x + \dot{Z}_0 A_s\right)$$

微分は

$$\frac{\partial F(X, Y, Z)}{\partial X} = \frac{\partial F}{\partial x}\frac{\partial x}{\partial X} + \frac{\partial F}{\partial y}\frac{\partial y}{\partial X} + \frac{\partial F}{\partial s}\frac{\partial s}{\partial X}$$

$$x = x(X, Z) \quad y = Y \quad s = s(X, Z)$$

とすると.

$$\vec{r} = \vec{r}_0 + x\cdot\hat{x} + y\hat{y} \quad \text{なので}$$

$$\left(\frac{1}{\rho}\right)^2 \hat{x}\cdot\hat{x}$$

$$X = X_0 - \rho(s)\cdot x\ddot{X}_0$$

$$Z = Z_0 - \rho(s)\cdot x\ddot{Z}_0$$

$$-\rho\left[(X - X_0)\ddot{X}_0 + (Z - Z_0)\ddot{Z}_0\right] = \rho^2 x\left(\ddot{X}_0^2 + \ddot{Z}_0^2\right)$$

$$= x$$

$$\frac{\partial x}{\partial X} = -\dot{\rho}\frac{\partial s}{\partial X}\left[(X-X_0)\ddot{X}_0 + (Z-Z_0)\ddot{Z}_0\right]$$

$$-\rho\left[\left(1-\dot{X}_0\frac{\partial s}{\partial X}\right)\ddot{X}_0 + (X-X_0)\dddot{X}_0\frac{\partial s}{\partial X} - \dot{Z}_0\frac{\partial s}{\partial X}\ddot{Z}_0 + (Z-Z_0)\dddot{Z}_0\frac{\partial s}{\partial X}\right]$$

$$= -\rho\ddot{X}_0 + \rho\frac{\partial s}{\partial X}\left(\dot{X}_0\ddot{X}_0 + \dot{Z}_0\ddot{Z}_0\right) - \frac{\partial s}{\partial X}\left(\left(\dot{\rho}\ddot{X}_0 + \rho\dddot{X}_0\right)(X-X_0) + \left(\dot{\rho}\ddot{Z}_0 + \rho\dddot{Z}_0\right)(Z-Z_0)\right)$$

$$= -\rho\ddot{X}_0 + \frac{\partial s}{\partial X}\left(\underbrace{\frac{\dot{X}_0}{\rho(s)}(X-X_0) + \frac{\dot{Z}_0}{\rho(s)}(Z-Z_0)}_{\to 0}\right)$$

$$= -\rho\ddot{X}_0$$

$$\dot{X}_0\ddot{X}_0 + \dot{Z}_0\ddot{Z}_0 = -\frac{1}{\rho(s)}\hat{s}\cdot\hat{x} = 0$$

同様に

$$\frac{\partial x}{\partial Z} = -\rho\ddot{X}_0$$

次に　$s = s(X, Z)$　X, Z で微分

$$\dot{X}_0(X - X_0) + \dot{Z}_0(Z - Z_0) = 0$$

$$\ddot{X}_0 \frac{\partial s}{\partial X}(X - X_0) + \dot{X}_0\left(1 - \frac{\partial s}{\partial X}\dot{X}_0\right)$$

$$+ \ddot{Z}_0 \frac{\partial s}{\partial X}(Z - Z_0) - \dot{Z}_0 \dot{Z}_0 \frac{\partial s}{\partial X} = 0$$

$$\frac{\partial s}{\partial X}\left[\underbrace{\ddot{X}_0(X - X_0) + \ddot{Z}_0(Z - Z_0)}_{-\frac{x}{\rho}} - \underbrace{(\dot{X}_0^2 + \dot{Z}_0^2)}_{1}\right] = -\dot{X}_0$$

$$\frac{\partial s}{\partial X} = \frac{\dot{X}_0}{1 + \frac{x}{\rho}}$$

同様に　$\frac{\partial s}{\partial Z} = \frac{\dot{Z}_0}{1 + \frac{x}{\rho}}$ 〃

まとめ

$$\frac{\partial x}{\partial X} = -\rho \ddot{X}_0 \quad \frac{\partial x}{\partial Y} = 0 \quad \frac{\partial x}{\partial Z} = -\rho \ddot{Z}_0$$

$$\frac{\partial y}{\partial X} = 0 \quad \frac{\partial y}{\partial Y} = 1 \quad \frac{\partial y}{\partial Z} = 0$$

$$\frac{\partial s}{\partial X} = \frac{\dot{X}_0}{1 + \frac{x}{\rho}} \quad \frac{\partial s}{\partial Y} = 0 \quad \frac{\partial s}{\partial Z} = \frac{\dot{Z}_0}{1 + \frac{x}{\rho}}$$

これらを全て使用すると

$$\left(\frac{\partial A_3}{\partial Y}-\frac{\partial A_2}{\partial Z}\right)\vec{e}_1=(\mathrm{rot}\,\vec{A})_x$$

$$=\left(\frac{\partial}{\partial Y}\left(-\rho\ddot{z}_0A_x+\dot{z}_0A_s\right)-\frac{\partial}{\partial Z}A_y\right)\frac{\dot{z}_0\hat{x}+\rho\ddot{z}_0\hat{s}}{\rho(\ddot{z}_0\dot{x}_0-\dot{z}_0\ddot{x}_0)}$$

$$=\left(-\rho\ddot{z}_0\frac{\partial A_x}{\partial y}+\dot{z}_0\frac{\partial A_s}{\partial y}-\frac{\partial A_y}{\partial x}(-\rho\ddot{z}_0)-\frac{\partial A_y}{\partial s}\frac{\dot{z}_0}{1+\frac{x}{\rho}}\right)\quad\square$$

$$\left(\frac{\partial A_1}{\partial Z}-\frac{\partial A_3}{\partial X}\right)\vec{e}_2=(\mathrm{rot}\,\vec{A})_y$$

$$\Bigg[\left(-\dot{\rho}\ddot{X}_0A_x-\rho(s)\underset{A_x}{\dddot{X}_0}\right)\frac{\dot{z}_0}{1+\frac{x}{\rho}}-\rho(s)\ddot{X}_0\left(\frac{\partial A_x}{\partial x}(-\rho\ddot{z})+\frac{\partial A_x}{\partial s}\frac{\dot{z}_0}{1+\frac{x}{\rho}}\right)$$

$$+\ddot{X}_0\frac{\dot{z}_0}{1+\frac{x}{\rho}}A_s+\dot{X}_0\left(\frac{\partial A_s}{\partial x}(-\rho\ddot{z}_0)+\frac{\partial A_s}{\partial s}\frac{\dot{z}_0}{1+\frac{x}{\rho}}\right)\Bigg]$$

$$-\Bigg[\left(-\dot{\rho}\ddot{Z}_0A_x-\rho(s)\dddot{Z}_0A_x\right)\frac{\dot{X}_0}{1+\frac{x}{\rho}}-\rho(s)\ddot{Z}_0\left(\frac{\partial A_x}{\partial x}(-\rho\ddot{X})+\frac{\partial A_x}{\partial s}\frac{\dot{X}_0}{1+\frac{x}{\rho}}\right)\cdots\Bigg]$$

$$---.$$

$\frac{\dot{x}}{\rho} = -\dot{\rho}\ddot{x} - \rho(s)\dddot{x}$ などを使う

$$= -\rho\frac{1}{\left(1+\frac{x}{\rho}\right)}\left(\ddot{x}_0\dot{z}_0 - \dot{x}_0\ddot{z}_0\right)\frac{\partial A_x}{\partial s}$$

$$+\left(\ddot{x}_0\dot{z}_0 - \dot{x}_0\ddot{z}_0\right)\frac{1}{1+\frac{x}{\rho}}A_s$$

$$+\rho\left(\ddot{x}_0\dot{z}_0 - \dot{x}_0\ddot{z}_0\right)\frac{\partial A_s}{\partial x}$$

$$\begin{cases} \ddot{x}_0\dot{z}_0 - \dot{x}_0\ddot{z}_0 \\ \frac{\hat{x}}{\rho(s)}\times\hat{s} = \frac{\hat{y}}{\rho(s)} = (\ddot{x}_0, 0, \ddot{z}_0)\times(\dot{x}_0, 0, \dot{z}_0) \\ \quad = (0, \ddot{z}_0\dot{x}_0 - \ddot{x}_0\dot{z}_0, 0) = \left(0, \frac{1}{\rho}, 0\right) \\ \ddot{x}_0\dot{z}_0 - \dot{x}_0\ddot{z}_0 = -\frac{1}{\rho} \end{cases}$$

$$\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_3}{\partial x}\right)e_2 = \left(\frac{1}{1+\frac{x}{\rho}}\frac{\partial A_x}{\partial s} - \frac{A_s}{\rho\left(1+\frac{x}{\rho}\right)} - \frac{\partial A_s}{\partial x}\right)\hat{y}$$

$$= \frac{1}{1+\frac{x}{\rho}}\left(\frac{\partial A_x}{\partial s} - \frac{\partial\left[\left(1+\frac{x}{\rho}\right)A_s\right]}{\partial x}\right)\hat{y}$$

$$\left(\frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial Y}\right)\vec{e}_3$$

$$= \left(\frac{\dot{x}}{1+\frac{x}{\rho}}\frac{\partial A_y}{\partial s} - \rho\ddot{x}\frac{\partial A_y}{\partial x} + \rho\ddot{x}_0\frac{\partial A_x}{\partial y} - \dot{x}_0\frac{\partial A_s}{\partial y}\right)\left[-\left(\dot{x}_0\hat{x} + \rho\ddot{x}_0\hat{s}\right)\right]$$

$$\left(\frac{\partial A_3}{\partial Y} - \frac{\partial A_2}{\partial Z}\right)\vec{e}_1 + \left(\frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial Y}\right)\vec{e}_3$$

$$= \hat{x}\left[\frac{\partial A_y}{\partial s}\left(\frac{(-\dot{x}_0^2 - \dot{z}_0^2)}{1+\frac{x}{\rho}}\right) + \frac{\partial A_s}{\partial y}\right]$$

$$\hat{s}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$= \hat{x}\left(\frac{\partial A_s}{\partial y} - \frac{1}{1+\frac{x}{\rho}}\frac{\partial A_y}{\partial s}\right)$$

$$+ \hat{s}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_y}{\partial y}\right)$$

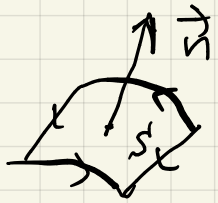
$\mathrm{rot}\,\vec{A}$

$$= \frac{1}{\left(1+\frac{x}{\rho}\right)}\left[\frac{\partial\left[\left(1+\frac{x}{\rho}\right)A_s\right]}{\partial y} - \frac{\partial A_y}{\partial s}\right]\hat{x}$$

$$+ \frac{1}{\left(1+\frac{x}{\rho}\right)}\left[\frac{\partial A_x}{\partial s} - \frac{\partial\left[\left(1+\frac{x}{\rho}\right)A_s\right]}{\partial x}\right]\hat{y}$$

$$+ \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\cdot\hat{s}$$

別解 Rotation (Vectorの回転)とは何だったか?



$$\lim_{S\to 0}\left[\frac{1}{S}\int_{\partial S}\vec{A}\cdot d\vec{r}\right]\vec{n}$$

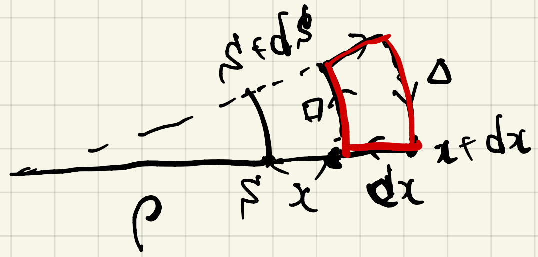
厳密ではないと思う

∂S 境界

Frenel serret

(s, x) 平面のとき

$\int_{\partial S}$



$\rho : ds = \rho + x : \square$

$\square = (1+\frac{x}{\rho})ds$

$\Delta = (1+\frac{x+dx}{\rho})ds$

線積分は.

$$\square = A_s(x,y,s)(1+\frac{x}{\rho})ds + A_x(x,y,s+ds)dx - A_s(x+dx,y,s+ds)(1+\frac{x+dx}{\rho})ds - A_x(x+dx,y,s)dx$$

$$A_x(x,y,s+ds) = A_x + \frac{\partial A_x}{\partial s}ds$$

$$A_s(x+dx,y,s+ds) = A_s + \frac{\partial A_s}{\partial x}dx + \frac{\partial A_s}{\partial s}ds$$

$$A_x(x+dx,y,s) = A_x + \frac{\partial A_x}{\partial x}dx$$

$$\square = \frac{\partial A_x}{\partial s}dx\,ds - \frac{\partial A_x}{\partial x}dx\,dx - \frac{\partial A_s}{\partial x}(1+\frac{x}{\rho})dx\,ds$$

$$- A_s\frac{1}{\rho}ds\,dx\,,,$$

$$- \frac{\partial A_s}{\partial s}(1+\frac{x}{\rho})ds\,ds$$

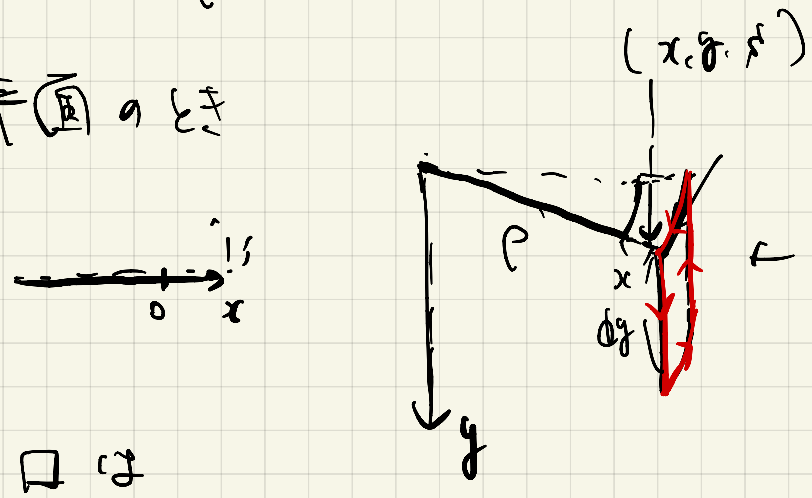
$$\mathrm{rot}\,\vec{A} = \frac{1}{\left(1+\frac{x}{\rho}\right)}\left(\frac{\partial A_x}{\partial s} - \frac{A_s}{\rho} - \frac{\partial A_s}{\partial x}\left(1+\frac{x}{\rho}\right)\right)$$

$$- \frac{1}{1+\frac{x}{\rho}}\frac{\partial A_x}{\partial x}\underbrace{\frac{dx}{ds}}_{\to 0} - \frac{\partial A_s}{\partial s}\underbrace{\frac{ds}{dx}}_{\to 0}$$

$$= \frac{1}{\left(1+\frac{x}{\rho}\right)}\left(\frac{\partial A_x}{\partial x} - \frac{\partial \left(1+\frac{x}{\rho}\right)A_s}{\partial x}\right)\hat{y}$$

OK

(y, s) 平面のとき



面積

$dy\,ds\left(1+\frac{x}{\rho}\right)$

線積分□は

$$\square = A_y(x,y,s)\cdot dy + A_s(x,y+dy,s)\left(1+\frac{x}{\rho}\right)ds - A_y(x,y+dy,s+ds)\,dy$$

$$- A_s(x,y,s+ds)\left(1+\frac{x}{\rho}\right)ds$$

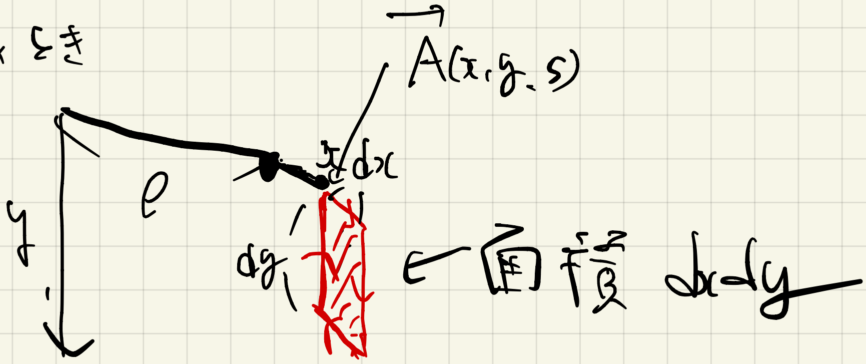
$dy\,dy$ $ds\,ds$ などは消してよい。

$$\square = -\frac{\partial A_y}{\partial s}dy\,ds + \frac{\partial A_s}{\partial y}\left(1+\frac{x}{\rho}\right)dy\,ds$$

$$\mathrm{rot}\,\vec{A} = \frac{1}{\left(1+\frac{x}{\rho}\right)}\left(\frac{\partial\left(1+\frac{x}{\rho}\right)A_s}{\partial y} - \frac{\partial A_y}{\partial s}\right)\hat{x}$$

OK

(x, y)平面のとき



線積分□は

$$\square = A_x(x,y,s)\,dx + A_y(x+dx,y,s)\,dy - A_x(x+dx,y+dy,s)\,dx - A_y(x,y+dy,s)\,dy$$

$$= -\frac{\partial A_x}{\partial y}dxdy + \frac{\partial A_y}{\partial x}dxdy$$

$$\mathrm{rot}\,\vec{A} = \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{s}$$

まとめると、(一般的に)

$$\mathrm{rot}\,\vec{A} = \frac{1}{\left(1+\frac{x}{\rho}\right)}\left[\frac{\partial\left(1+\frac{x}{\rho}\right)A_s}{\partial y} - \frac{\partial A_y}{\partial s}\right]\hat{x}$$

$$+ \frac{1}{\left(1+\frac{x}{\rho}\right)}\left[\frac{\partial A_x}{\partial s} - \frac{\partial\left(1+\frac{x}{\rho}\right)A_s}{\partial x}\right]\hat{y}$$

$$+ \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{s}$$